Pontifícia Universidade Católica do Rio de Janeiro

Rafael Lincoln Pereira Mattos

The Role of Inequality in the Response of Consumption to a Credit Deepening

Dissertação de Mestrado

Masters dissertation presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor : Prof. Carlos Viana de Carvalho
Co-advisor: Prof. Fernando Jerico Mendo Lopez

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To my grandfather, Pedro Lincoln, for our debates
on economic ideas and policies that have
nurtured my love for economics

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Abstract

Lincoln Pereira Mattos, Rafael; Viana de Carvalho, Carlos (Advisor); Jerico Mendo Lopez, Fernando (Co-Advisor). **The Role of Inequality in the Response of Consumption to a Credit Deepening**. Rio de Janeiro, 2024. 91p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

We study how income and wealth inequality affect the transmission mechanism of a credit deepening to consumption in a heterogeneous-agents incomplete markets model. After a one-time unexpected shock that increases the borrowing capacity of households, there is a short-run consumption "boom" and a subsequent persistent "bust" due to household leverage. At the household level, consumption response is driven by two channels: a *direct* channel of credit, which we characterize by a novel statistic - the Intertemporal Credit MPC, and the *indirect* channel led by budget and intertemporal substitution effects. Heterogeneity in responses is caused by the household wealth position, and these results also hold at the aggregate level, with wealth inequality amplifying consumption cycles through the *direct* channel of credit, while the *indirect* leads to a quicker recovery. Moreover, based on aggregate consumption, sectoral credit, and inequality cross-country data, we find robust evidence on consumption Boom & Bust cycles, and weak evidence of inequality as a state-dependent amplifier of these cycles

Keywords

Income and Wealth Inequality; Heterogeneous Agents; Credit Deepening; Credit MPCs; Household Debt; Consumption.

Resumo

Lincoln Pereira Mattos, Rafael; Viana de Carvalho, Carlos; Jerico Mendo Lopez, Fernando. **O Papel da Desigualdade na Resposta do Consumo Agregado ao Aprofundamento de Crédito**. Rio de Janeiro, 2024. 91p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Estudamos como a desigualdade de renda e riqueza afetam o mecanismo de transmissão de uma expansão do crédito para o consumo em um modelo de mercados incompletos com agentes heterogêneos. Após um choque inesperado que aumenta a capacidade de endividamento das famílias, há um "boom" de consumo de curto prazo seguido por um "bust" persistente devido ao endividamento das famílias. No nível das famílias, a resposta do consumo é impulsionada por dois canais: um canal de crédito *direto*, que caracterizamos por uma nova estatística - o Intertemporal Credit MPC, e o canal *indireto*, liderado por efeitos orçamentários e de substituição intertemporal. A heterogeneidade nas respostas é causada pela posição de riqueza das famílias, e esses resultados também se mantêm no nível agregado, com a desigualdade de riqueza amplificando os ciclos de consumo através do canal de crédito *direto*, enquanto o canal *indireto* leva a uma recuperação mais rápida. Além disso, com base em dados de consumo agregado, crédito setorial e desigualdade entre países, encontramos evidências robustas de ciclos de Boom & Bust no consumo e evidências fracas de que a desigualdade atua como um amplificador dependente do estado desses ciclos.

Palavras-chave

Desigualdade de Renda e Riqueza; Agentes Heterogêneos; Aprofundamento de Crédito; Credit MPCs; Dívida das Famílias; Consumo.

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WID – World Inequality Database
GRID – Global Repository of Income Dynamics
MPC – Marginal Propensity to Consume
HJB – Hamilton-Jacobi-Bellman
KF – Kolmogorov Forward
IMF – International Monetary Fund
HtM – Hand-to-Mouth
VAR – Vector Auto-regression
LP – Local Projections
GFC – Global Financial Crisis
CRRA – Constant Relative Risk Aversion

1
Introduction

Increasing access to credit in the economy has been a global phenomenon in the past five decades (MüLLER; VERNER, 2023), particularly with the vertiginous rise of credit as a share of GDP in the 21^{st} century. This phenomenon is known as "credit deepening", which (CARVALHO et al., 2023) describe as the permanent increase in the level of private credit in the economy that results from institutional changes, and is often cited as a source of above-trend growth in emerging-market economies[1]. Given that most of this process was led by household credit, as argued by (MüLLER; VERNER, 2023), household demand for more consumption fostered above-trend growth.

Considering the widely mapped relationship between inequality and its effects on aggregate demand behavior, we turn to investigate whether the fact of emerging-markets having more inequality could drive above-trend consumption growth[2] in a credit deepening when compared to less unequal peers. This is motivated by the idea that in these countries, less developed credit markets and a higher share of constrained households facing income growth will benefit more from an expansion of credit. Moreover, it is also based on the widely studied importance of inequality in the determination of aggregate demand and propagation of macroeconomic shocks. Therefore, this paper seeks to address whether inequality affects the transmission mechanism of credit deepening to aggregate consumption, and if it does, scrutinize which sources of inequality matter, and through which channels do they act.

To tackle the question at hand, this paper relies on an incomplete-markets *Heterogeneous Agents* model with household heterogeneity in terms of preferences, income and wealth. Within this framework, a credit deepening is a one-time unexpected shock to the borrowing constraint of all households, which react in distinct ways. We take advantage of the rich setting of household heterogeneity to study the channels of a credit deepening to consumption and how they interact with different dimensions of inequality.

At the individual level, we show the role of inequality in the response of household consumption to a credit deepening with two main propositions. First, mapping that answers how much will a household adjust its current consumption given a marginal increase in the access to credit - a statistic which

[1] As in the case of Latin America in the 21^{st} century, as documented by (CARVALHO et al., 2023)

[2] In the 21^{st} century, emerging markets have been contributing more to global consumption growth than advanced economies. Mirae Asset apud IMF, available here

we henceforth call Credit MPC. And, second, an analytic characterization of the change in household consumption following the whole shock to the borrowing constraint, inspite of idiosyncratic income risk and incomplete credit markets - which is henceforth called the intertemporal Credit MPC. The Credit MPC is composed of the current marginal propensity to consume (MPC) of the household, a widely used statistic in the *Heterogeneous Agents* literature, and a second term that captures how the household adjusts to future consumption loss due to leverage.

Secondly, the intertemporal Credit MPC is composed by the response of consumption to current credit and future changes, and the latter is subject to precautionary effects over the discounted stream of the cost of future changes in credit conditions. Moreover, the intertemporal Credit MPC is directly proportional to the static Credit MPC, and the static response serves as a good approximation for the response to the whole credit deepening. Therefore, the intuition for the role of inequality in the partial equilibrium response of consumption becomes becomes simple, as inequality affects it through the distribution of credit MPCs and the share of constrained households. Hence, a source of inequality matters inasmuch as it affects these two components.

The intuition aforementioned describes the role of inequality in the transmission of credit to consumption through the *direct channel* of credit. However, heterogeneity also affects the response of prices in the economy, which in turn explains the *indirect channel* of credit. Thus, we build on the framework of (FARHI; OLIVI; WERNING, 2022) to build the full general equilibrium characterization of the household response of consumption to credit, prices and transfers, on a simple general equilibrium framework of (AIYAGARI, 1994). This characterization is novel in the literature, and sheds light on the heterogeneity of responses of consumption with respect to households liquid wealth position: households with low or negative wealth drive the response through direct channel of consumption, while wealthier households react to prices and transfers through budget and intertemporal substitution effects.

We extrapolate these results from micro to macro by analyzing the aggregate consumption IRF following a credit deepening, which we decompose by wealth percentiles and by the channels of credit to consumption (*direct* and *indirect*). These decompositions show that, first, there is heterogeneity in the aggregate consumption IRF conditional to the wealth percentile, and second that the *direct* channel of credit generates "Boom & Bust" consumption cycles in the short and long-run, respectively, while *indirect* channels are responsible for the long-run consumption rebound. Moreover, we also decompose each of *direct* and *indirect* responses by wealth percentiles, showing that the poorest households are responsible for the *direct* channel of credit, driving "Boom" & "Busts", while the wealthiest drive the *indirect* response of consumption. On top of the decompositions, we run counterfactual exercises in which we alter marginally the wealth inequality in economy by controlling for cross-section heterogeneity, and subjecting these counterfactual economies to a credit deepening. They conclude robustly that wealth inequality amplifies the response of consumption, generating wider consumption "Booms" & "Busts".

We base our analysis on high-quality micro data on income and wealth to develop quantitative assessments. Moreover, we take advantage of their cross-country availability and comparability to extend our exercise to a cross-country setting by calibrating our model to advanced and emerging markets. Such an approach is novel in the literature and made possible by the availability of micro data on income by the Global Repository of Income Dynamics (GRID) (GUVENEN; PISTAFERRI; VIOLANTE, 2022) as well as wealth data by the World Inequality Database (WID) (CHANCEL et al., 2021).

We conclude by investigating whether the quantitative results and the channels we investigated through our model have empirical validation. For such, we rely on an unbalanced panel dataset of 82 countries from 1961-2021, comprising data on sectoral credit, aggregate consumption and measures of income and wealth inequality. We find robust evidence on consumption boom & busts cycles following a credit shock, with household credit expansions predicting consumption growth within 3 to 4 years following the shock, and persistent recession that may last more than 6 years. Building on top of this evidence, we rely on regime-switching models to investigate whether there is amplification in the response for economies with higher income or wealth inequality. We find some evidence of state-dependent amplification, but results are not very robust.

Related Literature: This article finds its way into the literature of credit variation as a source of macroeconomic shocks, focusing on a credit deepening instead of a credit crunch, and making a bridge with inequality that was previously unexplored. It also uses state-of-the-art modeling of household heterogeneity calibrated with high-quality micro data to developing and developed economies, in contrast with previous models which were oriented towards developed economies and/or had limited heterogeneity. Most of the literature on the effects of credit as a macroeconomic shock was born in the aftermath of the Great Financial Crisis (GFC), with one branch of the literature focusing on spending multipliers the the liquidity trap - see (CHRISTIANO; EICHENBAUM; REBELO, 2009), (WOODFORD, 2010) and (WERNING, 2011); whilst other was focused households' balance sheet adjustment and consumer spending.

This branch was focused on explaining to which extent the rapid increase of household debt in the years leading up to the 2008 GFC was a factor that led to the recession and its slow recovery. (JONES; MIDRIGAN; PHILIPPON, 2011) argues that the role of credit was more of an amplification of macroeconomic shocks, as it played a lesser role in explaining changes in employment but a major role in a slow recovery. (MIAN; SUFI, 2011) use geographic variation to show how the home equity borrowing channel[3] drove the rise in household leverage when house prices soared before the GFC, while (MIAN; RAO; SUFI, 2013) estimate MPCs with respect to housing wealth and explores how a contraction in households borrowing capacity, caused by a fall in house prices, led to a slump on consumer spending and a rise of unemployment.

The canonical article of (EGGERTSSON; KRUGMAN, 2012) tries to conciliate the two branches in one that studies credit shocks as a driver of aggregate demand on business cycles, and does such by establishing a relation between credit crisis and liquidity trap. It was also one of the first to incorporate heterogeneity, albeit limited (the presence of borrowers and lenders with distinct discount rates), whose usage in the literature further evolved from that point onward. (JUSTINIANO; PRIMICERI; TAMBALOTTI, 2013) use a similar heterogeneity in the model (working with two types of households) with the intent of matching debt dynamics over a credit cycle to U.S. data, and does such by shocking preferences for housing services to generate credit cycles with leveraging and deleveraging.

In the spirit of understanding the GFC and housing crisis, (HUO; RÍOS-RULL, 2016) further incorporate heterogeneity by establishing a Bewley model with goods, labor and financial market frictions, as well as capital and housing as assets (despite housing also being in the utility function) but without transaction costs. In their model, Financial shocks increase the difficulty of house-collateralized credit, which rebounds on prices and depresses even more credit conditions. In this context, wealth distribution plays an important role in the effect of a credit shock depending on the quantity of households constrained. (GUERRIERI; LORENZONI, 2017) use a similar Bewley model with price rigidities, but one asset only and without collateralized debt, to delve into the effects of a one-time credit crunch. Similar to us, they capture two channels in the consumer's response to a credit shock: direct and precautionary channels. Following a credit shock, consumers in the lower end of the wealth distribution are forced to deleverage while others increase their precautionary

[3]borrowing with housing as collateral

savings, reducing consumption and increasing employment. The shock is also amplified by the movement in prices, and composition changes in magnitude when dissecting consumption of durables and non-durables.

Plentiful research has also been conducted on credit and debt as a source of amplification of macroeconomic shocks on aggregate demand, as in (CLOYNE; FERREIRA; SURICO, 2016), where the response of consumption to monetary policy is driven by households with mortgages as mortgagors hold sizable illiquid assets but small liquid assets, being more sensitive to changes in interest rates. (FLODÉN et al., 2016) instead examine how monetary policy is driven by households whose debt is linked to short-term rates and thus are more sensible to debt service. Others took a more empirical approach, as for instance (SCHULARICK; TAYLOR, 2012) use a historically long cross-country database to document that credit booms are indicative of a heightened risk of financial crisis. Whereas (MIAN; SUFI; VERNER, 2017) focus on how credit and household debt influence business cycles. They document that an increase in household debt predicts short-run growth booms and a subsequent reversal in debt and lower GDP growth, as well as other facts about household booms and possible theories behind them. However, (MüLLER; VERNER, 2023) look at the factors why some credit cycles lead to busts and others not. They document that credit expansions focused on firms of non-tradable sectors and households lead to busts, whereas those focused on firms of tradable sectors lead to sustained growth without financial risk.

As heterogeneity gets even more present in the literature that studies credit, household balance sheets and business cycles, MPCs have become a core statistic of the literature. (AUCLERT; ROGNLIE; STRAUB, 2018) demonstrates how this statistic is the key element that disciplines general equilibrium models with heterogeneous agents and nominal rigidities, and that dynamic effects are led by intertemporal MPCs. Given the growing role of MPCs, a question that arises is how to match empirical estimates of iMPCs successfully. (KAPLAN; VIOLANTE, 2022) lays out a trade-off: on one hand, one-asset models with ex-ante heterogeneityor behavioral preferences can generate MPCs as large in the data while maintaining consistent levels of aggregate wealth, but they generate an excessively polarized wealth distribution[4]. On the other, two-asset models with liquid and illiquid assets with a return differential can reconcile these tensions, but requiring a large return differential. For a comprehensive survey on MPCs and their usage in calibration and study of macro shocks in business cycles, see (KAPLAN; VIOLANTE, 2018) and

[4]In the sense that it understates the wealth attained by households in the middle of the distribution

(KAPLAN; VIOLANTE, 2022).

The rest of the article is organized as follows. In Section 2, we present the model and study analytically the relationship between credit, consumption and inequality at the household and aggregate level. In Section 3, we present the calibration of the model and analyze distribution of income, wealth and micro-level consumption behavior. Section 4 we study the response of aggregate consumption to a credit deepening, quantifying the response through each channel and using counterfactual analysis to assess the role of inequality in the transmission mechanisms. Section 5 we study whether short and long-run patterns of consumption “boom” and “bust” cycles are present after household debt shocks, and if they amplify with inequality as in the model. Finally, Section 6 concludes.

2
Model

In this section, we describe the general equilibrium model used to study how does aggregate and household-level consumption reacts to a credit deepening when controlling for different sources of heterogeneity. Time is continuous and the economy is closed with neither aggregate risk nor uncertainty. The risk is at the individual level, where a continuum of households faces idiosyncratic income risk and incomplete credit markets. They supply labor inelastically to consume and save in liquid assets (capital) for precautionary measures. Besides them, there are two agents in the economy: a representative firm that interacts with the continuum of households through capital and labor markets, hiring labor and capital to produce a final consumption good; and a government with an insurance role, which taxes labor earning and redistributes it through lump-sum transfers.

As one of the contributions of this work is to map which dimension of inequality is relevant in driving the relationship between credit and consumption, we employ a model with a rich setting in terms of the cross-section heterogeneity of households: There is preference heterogeneity through discount rates; individual income fixed effects drawn at birth; and multidimensional uninsurable idiosyncratic risk. These elements generate endogenous dispersion in the distribution of asset holdings of households and consumption-savings behavior, both of which are important for the propensity of households to take credit and consume out of it.

2.1
Households

The economy is populated with a continuum of infinitely-lived households with time-separable CRRA preferences. Each household is born with a fixed effect ω_i, $\forall i \in \mathbb{I}$, and a discount rate ρ_j, $\forall j \in \mathbb{J}$. Throughout his life, the household is subject to idiosyncratic income risk through its labor productivity z, which follows a Markov process and reflects permanent and transitory changes to income. Consequently, households have precautionary motives and save in the form of liquid assets (capital), but do so in the presence of incomplete markets. In addition, they supply labor inelastically, which is taxed by the government at the tax rate τ_t, but also receive lump-sum transfers T_t as a form of universal cash transfers.

Households are indexed by their individual fixed effect i, their discount rate j, their idiosyncratic labor productivity z and holding of liquid assets a, with preferences given by

$$\mathbb{E}_0\left[\int_0^\infty e^{-\rho_j t}u(c_{ijt})dt\right] \tag{2-1}$$

where the expectations are taken over the households′ realization of idiosyncratic productivity shocks. In the aggregate, idiosyncratic risks disappear, so there is no economy-wide uncertainty. Each household accumulates a liquid asset $a_{i,j,t}$ which, given consumption, follows the law of motion:

$$\begin{aligned} \dot{a}_{i,j,t} &= (1-\tau)w_t z_{i,t} + r_t a_{i,j,t} - c_{i,j,t} + T_t \\ a_{i,j,t} &\geq -\underline{a}_t \end{aligned} \tag{2-2}$$

where $\underline{a}_t \geq 0$ is an ad-hoc borrowing constraint due to incomplete credit markets. Households maximize (2-1) subject to prices of the economy $\{r_t, w_t\}$, their individual wealth law of motion and borrowing constraint (2-2), as well as the law of motion for idiosyncratic labor productivity $z_{i,t}$ (3-3) detailed in Section 3. Define the value function of the household of fixed effect type i and discount rate j as:

$$V_{i,j}(a,z,t) = \mathbb{E}_t\left[\int_t^\infty e^{-\rho_j s}u(c_{i,j,s})ds\right] \tag{2-3}$$

due to the fully markovian structure of the problem of the household, this value function satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \rho_j V_{i,j}(a,z,t) = \max_{c_{i,j}\geq 0} \Big\{ & u(c_{i,j,t}) + \partial_a V_{i,j}(a,z,t)s_{i,j}(a,z,t)\ldots \\ & \ldots + \mathcal{A}V_{i,j}(a,z,t) + \partial_t V_{i,j}(a,z,t) \Big\} \end{aligned} \tag{2-4}$$

where $\mathcal{A}$ is the infinitesimal generator for the income process related to z. Moreover, the HJB equation is subject to the following state constraint boundary conditions:

$$\partial_a V_{i,j}(\underline{a}, z) \geq u'(y_{i,t} + r_t\underline{a}_t)\ \forall\, z, i, t \tag{2-5}$$

where $y_{i,t} = (1-\tau_t)w_t z_i$ is the income of the household. The function $s_{i,j}(a,z)$ is the savings policy function, which is the optimally chosen drift of wealth, and $c_{i,j}(a,z)$ is the consumption policy function. These two can be obtained by the envelope condition, and are characterized by:

$$\begin{aligned} s_{i,j}(a,z,t) &= (1-\tau_t)w_t z_i + r_t a + T_t - c_{i,j}(a,z,t) \\ c_{i,j}(a,z,t) &= (u')^{-1}\left(\partial_a V_{i,j}(a,z,t)\right) \end{aligned} \tag{2-6}$$

From the individual decisions of agents with regards to consumption and savings in (2-4), and the dynamics of the idiosyncratic income risk in (3-3), the joint distribution of individual productivity and wealth for a given type $(i,j) \in \mathbb{I} \times \mathbb{J}$, $g_{i,j}(a,z,t)$ evolves in time as determined by the Kolmogorov-Forward (KF) equation:

$$\partial_t g_{i,j}(a,z,t) = -\partial_a(s_{i,j}(a,z,t)g(a,z,t)) + \mathcal{A}^* g_{i,j}(a,z,t) \tag{2-7}$$

where $\mathcal{A}^*$ is the adjoint operator of the infinitesimal generator $\mathcal{A}$. The Mean-Field Game system in (2-4), (2-5) and (2-7) dictates the evolution of micro-level consumption and saving decisions, as well as the evolution of joint distribution of wealth and prices of the economy[1]. In the aggregate, the measure of households $\mu_t(di, dj, da, dz)$ describes the state of economy at time t, and is comprised of aggregating all the distributions $g_{i,j}(a,z,t)$.

2.2 Credit Deepening, Household Consumption and Inequality

In this subsection, we present an analytic framework to understand how inequality affects household consumption following a credit deepening. We introduce a new statistic, the Credit MPC, which helps us examine (i) how much households change their consumption in response to changes in current and future borrowing constraints; and (ii) how the household's position in the income and wealth distribution influences the magnitude and direction of this response; and (iii) how the aggregate consumption response can be viewed as a weighted average of households given their position in the joint income and wealth distribution.

We define credit deepening as an economy-wide increase in households' access to credit, regardless of their individual characteristics, income, or assets. The formal definition is stated below:

Definition 1 (Credit Deepening) *A credit deepening is defined a path of borrowing constraints that increases the access of liquidity in the economy. In formal terms, let t be the period in which the credit deepening starts and T*

[1]The full forms of the HJB and KF equations with the income process (3-3) described in Section 3, as well as their respective derivations, are present in the appendix

when it ends, $T > t$. The path of borrowing constraints $\{\underline{\mathbf{a}}_{\mathbf{s}}\}_{\mathbf{s=t}}^{\mathbf{T}}$ such that $\forall s$ where $t \leq s \leq T$, $\underline{a}_s \geq \underline{a}_t$

2.2.1
Household response to credit deepening

To answer question (i), we use Proposition 1. It maps the response in the household consumption out of a marginal change in borrowing constraint to the current marginal propensity to consume (MPC) of the household corrected by a term which "captures" future consumption loss through debt. For simplicity, we drop time subscripts as this applies to any period.

Proposition 1 (Credit MPCs) *The household policy function $c_{i,j}(a, z)$ that solves the Household problem satisfies the following identity with respect to the borrowing constraint $\underline{a}$:*

$$\frac{\partial c_{i,j}}{\partial \underline{a}}(a, z; \underline{a}) = MPC_{i,j}(a, z; \underline{a}) - r\frac{\partial c_{i,j}}{\partial T}(a, z; \underline{a})$$

where $MPC_{i,j}(a, z) = \frac{\partial c_{i,j}}{\partial a}(a, z)$ is the current MPC of the household, and T is a lump-sum transfer.

Proof. See appendix A.1.1 ■

Therefore, the static response of consumption to credit is heterogeneous and driven by a key statistic of the literature, the current MPC. The second term represents the marginal propensity to consume out of a lump-sum transfer, weighted by the interest rate. This term reflects how household leverage affects consumption, as easing the borrowing constraint by one dollar in the current period is equivalent to forcing households to save the future value of one dollar in the next period to pay for the increased current debt. Figure 5 plots the distribution of Credit MPCs calibrated to the US, showing a more broad perspective of the forces behind the heterogeneity of responses.

Income inequality affects the distribution of MPCs through precautionary savings, as it implies potential income gains or losses from a shock, with higher income persistence implying lower mobility. However, these effects are scaled by wealth[2]. Therefore, individuals with more assets can better insure themselves

[2] Following (ACHDOU et al., 2022)'s Proposition 1 (MPCs and Saving at the Borrowing Constraint), under certain circumstances, we recover an analytic decomposition of the MPC close to the borrowing constraint, given by

$$iMPC_{i,j}(a, z) \sim r + \frac{1}{2}\sqrt{\frac{2\nu_{i,j}(z)}{a - \underline{a}}}$$

where $\nu_{i,j}(z)$ can be decomposed in two parts, and IES is the intertemporal elasticity of against second-order consumption effects. This aligns with the findings of (BENHABIB; BISIN; ZHU, 2015) and (ACHDOU et al., 2022), indicating that household wealth is the main determinant of the Credit MPC's magnitude.

Extending on Proposition 1, we characterize the response of consumption at a given period t to the whole credit deepening, $\{\underline{\mathbf{a}}_\mathbf{s}\}_{\mathbf{s=t}}^{\mathbf{T}}$.

Proposition 2 (Intertemporal Credit MPC) *Consider a perturbation to the path of borrowing constraints $\{\mathbf{d}\underline{\mathbf{a}}_\mathbf{s}\}_{\mathbf{s=t}}^{\mathbf{T}}$ induced by a credit deepening. The response of consumption at the time t to it is given by:*

$$dc(a,y,t) = \partial_{\underline{a}} c_t \left(d\underline{a}_t - \mathbb{E}_t \left[M_t^{T\wedge\tau} \int_t^{T\wedge\tau} e^{-\int_t^s r_u du} (r_s d\underline{a}_s) ds \right] \right)$$

where $\tau = \inf\{t \leq s \leq T | a_s = -\underline{a}_s\}$ is the stopping time when the agent hits the borrowing constraint, $T \wedge \tau = \min\{T, \tau\}$ and $M_t^{T\wedge\tau} = e^{-\int_t^{T\wedge\tau}(\rho - 2r_{t'} + \partial_a c) dt'}$ is the discount factor related to the marginal propensity of savings.

Proof. See appendix A.1.2 ■

Proposition 2 illustrates how current consumption responds to a credit deepening. It combines the static consumption response to an instantaneous change in borrowing constraints, as outlined in Proposition 1, with an intertemporal component. This intertemporal component captures how current consumption responds to future changes in borrowing constraints, considering the households' uncertainty regarding his own income.

$$dc(a,y,t) = \underbrace{(\partial_{\underline{a}} c_t) d\underline{a}_t}_{\substack{\text{Response of consumption} \\ \text{to instantaneous change}}} \underbrace{- \partial_{\underline{a}} c_t \mathbb{E}_t \left[M_t^{T\wedge\tau} \int_t^{T\wedge\tau} e^{-\int_t^s r_u du} (r_s d\underline{a}_s) ds \right]}_{\text{Response of current consumption to future changes}}$$

The magnitude of the intertemporal component is influenced by the Credit MPC at the time of the shock and dampens the current consumption response due to anticipated future changes in borrowing constraints. This effect is subject to the conjugation of two forces: first, the discounted stream of changes in the borrowing constraint given the path of interest rates, $\int_t^{T\wedge\tau} e^{-\int_t^s r_u du}(r_s d\underline{a}_s) ds$; and second, the discount factor $M_t^{T\wedge\tau}$ used by the

substitution:

$$\nu_{i,j}(z) = (\rho_j - r) \times IES \times c_{i,j}(\underline{a}, z) + \frac{\mathcal{A}(u'(c_{i,j}(\underline{a}, z)))}{u''(c_{i,j}(\underline{a}, z))}$$

where the first term is affected by ex-ante fixed-effects in income, and the second capturing income risk in earnings. These two components are driven by income inequality, but are scaled by wealth relative to the borrowing constraint.

household to discount these streams, reflecting precautionary effects driven by income and wealth uncertainty. The formula $M_t^{T\wedge\tau} = e^{-\int_t^{T\wedge\tau}(\rho - 2r_{t'} + \partial_a c)dt'}$ indicates that as the household approaches its borrowing limit, it discounts the future more heavily due to precautionary effects, thus reducing its response to future changes in borrowing constraints. Therefore, given the current Credit MPC, more constrained households also have higher intertemporal Credit MPCs.

To better illustrate the factors influencing the consumption response to a credit deepening, we take a simplified example of this response in partial equilibrium to a permanent shock to the borrowing constraint, leading to a new steady state $\underline{a}'$:

Lemma 1 *Consider a perturbation to the path of borrowing constraints* $\{\mathbf{d\underline{a}_s}\}_{\mathbf{s=t}}^{\mathbf{T}}$ *induced by a credit deepening that follows the ordinary differential equation* $d\underline{a}_t = \nu(\underline{a}' - \underline{a}_t)dt$. *The response of consumption at the time* t *in partial equilibrium, maintaining prices and transfers fixed, is given by:*

$$dc(a,z,t) = (\partial_{\underline{a}} c_t) d\underline{a}_t - (\partial_{\underline{a}} c_t) r\nu e^{-rt} \mathbb{E}_t \left[e^{-\int_t^{T\wedge\tau} \partial_a c_{t'} dt'} \int_t^{T\wedge\tau} e^{(\rho-r)s} (\underline{a}' - \underline{a}_s) ds \right]$$

where τ *is the stopping time of when the agent hits the borrowing constraint,* $\underline{a}'$ *the borrowing constraint at the new steady state in* T *,* r *the partial equilibrium interest rate and* ν *a parameter that governs the speed of convergence of the path of borrowing limits.*

Proof. See appendix A.1.3 ■

Lemma 1 highlights the various factors influencing the response of current consumption to future changes. These factors include interest rates, which affect both discounting and level effects, and the speed of convergence parameter ν, which determines the autocorrelation of household credit. A faster convergence leads to a more pronounced dampening effect on consumption response. Additionally, conditional on the expected stopping time τ, the term $e^{-\int_t^{T\wedge\tau} \partial_a c_{t'} dt'}$ reflects additional discounting of changes in borrowing constraints due to precautionary effects that increases the response of consumption to the credit deepening.

Precautionary savings' impact on the intertemporal consumption response to future changes in borrowing constraints can be understood as a reevaluation of objective probabilities regarding the household's future income and wealth states. This reevaluation is achieved through the prudence-adjusted measure of probabilities, as demonstrated in Lemma 2:

Lemma 2 (Prudence-adjusted Intertemporal Credit MPC) *Define the Prudence-adjusted measure $\mathbb{Q}^I$ that turns the process $(\partial_a c)u''(c)$ a martingale:*

$$(\partial_a c)u''(c) = \mathbb{E}_t^{\mathbb{Q}^I}[(\partial_a c_{T\wedge\tau})u''_{T\wedge\tau}]$$

thus, under the measure $\mathbb{Q}^I$, the Intertemporal Credit MPC is given by:

$$dc(a,y,t) = \partial_{\underline{a}} c_t \left(d\underline{a}_t - \mathbb{E}_t^{\mathbb{Q}^I}\left[\int_t^{T\wedge\tau} e^{-\int_t^s r_u du}(r_s d\underline{a}_s)ds\right]\right)$$

Proof. See appendix A.1.4 ■

As defined by (FARHI; OLIVI; WERNING, 2022), the Prudence-adjusted measure $\mathbb{Q}^I$ can be interpreted as reweighing states of the world by the curvature of utility with respect to income. This measure typically overweighs "bad" states compared to a risk-neutral and common probability measures. Therefore, an additional dollar in a state where consumption is low generates a larger consumption response under incomplete markets. In our context of credit deepening, this implies that when the agent is constrained and there is a credit deepening, the response of current consumption to future changes dampens the overall response. It follows because the agent puts more weight on future bad states where he we will be constrained, already increasing his consumption response beforehand

2.2.2
Heterogeneity of responses across the wealth distribution

Through Proposition 2, we have shown that keeping all prices constant, the household position in the joint income-wealth distribution matters for its response following a credit deepening. This partial equilibrium channel is important, but is only quantitatively relevant for the households in the lower-end of the wealth distribution. Extending Proposition 2, we incorporate the full response of consumption in a general equilibrium setting. Hence, capturing the effects of a credit deepening through prices that are relevant for wealthy individuals: the intertemporal substitution and budget effects.

Proposition 3 (General Equilibrium Response of Consumption) *Consider a perturbation of prices and borrowing limits generated by a credit deepening, $\{d\Gamma_s\}_{s=t}^T = \{d\underline{a}_s, dr_s, dw_s, dT_s\}_{s=t}^T$. The general equilibrium response* *of consumption to a credit deepening is given by:*

$$dc(a,y,t) = \underbrace{\partial_{\underline{a}} c_t d\underline{a}_t - \partial_{\underline{a}} c_t \mathbb{E}_t^{\mathbb{Q}^I}\left[\int_t^{T\wedge\tau} e^{-\int_t^s r_u du}(r_s d\underline{a}_s) ds\right]}_{\text{Partial Equilibrium Credit Effect}}$$

$$+ \underbrace{\overbrace{\partial_a c_t \mathbb{E}_t^{\mathbb{Q}^I}\left[\int_t^{T\wedge\tau} e^{-\int_t^s r_u du}(dr_s a + dy_s)\right]}^{\text{Budget Effects}} - \overbrace{\varepsilon_t c_t \mathbb{E}_t^{\mathbb{Q}}\left[\int_t^{T\wedge\tau} e^{-\int_t^s \partial_a c_{t'}} dr_s ds\right]}^{\text{Intertemporal Substitution effects}}}_{\text{General Equilibrium Effect Through Prices}}$$

where $\mathbb{Q}^I$ and $\mathbb{Q}$ are the prudence-adjusted and marginal-utility adjusted measures, $\varepsilon_t = \frac{u'}{u''c}$ is the (local) intertemporal elasticity of substitution and $dy_s = dw_s z + dT_s$ is the change in income due to wages and transfers.

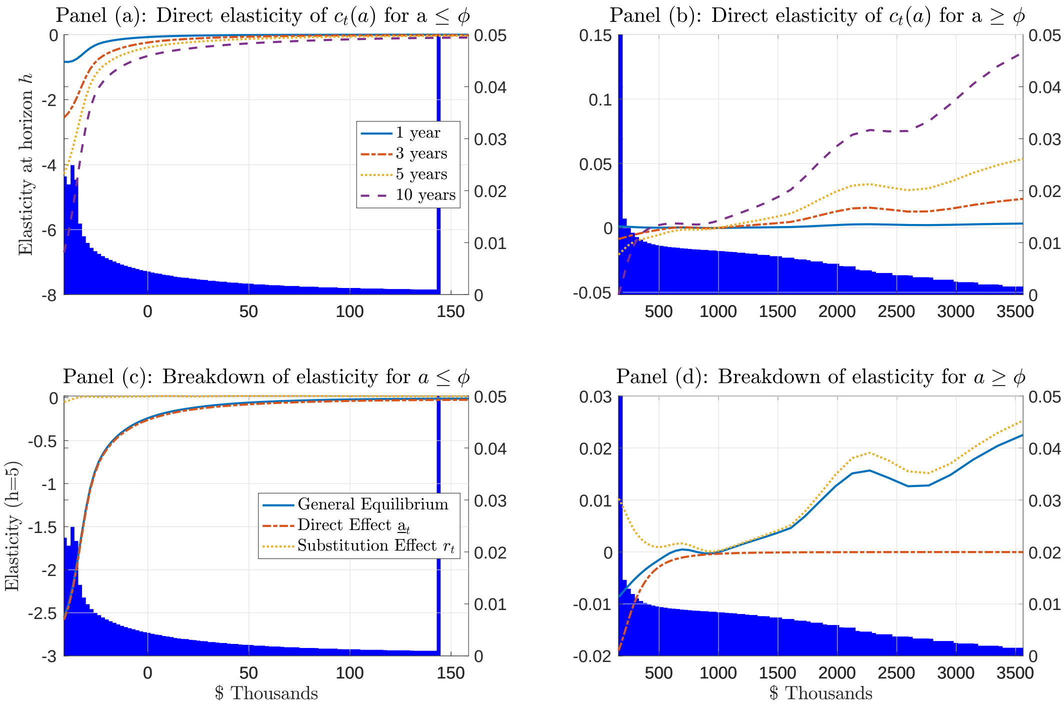
Proof. See appendix A.1.5 ■

Proposition 3 describes the complete response of consumption in general equilibrium to a credit deepening. This response mirrors the aggregate equilibrium response of consumption discussed in Section 2.3 and is crucial for understanding how inequality influences consumption dynamics. In households at the lower end of the wealth distribution, the most significant component of the response comes from partial equilibrium credit effects. This is because of their low asset holdings, which limit consumption due to negative wealth and intertemporal substitution effects through interest rates. Credit MPCs also play a significant role in this response.

Conversely, for wealthy households, the response of consumption is primarily driven by general equilibrium effects through prices. This is because their lower credit MPCs and more prudence-adjusted discounting of future streams of changes in borrowing constraints lead to smaller partial equilibrium responses of consumption. Additionally, high asset holdings increase the wealth and intertemporal substitution effects of wealthy households, making them more responsive to changes in interest rates $\{dr_s\}_{s=t}^T$.

An illustration on how responses of consumption in general equilibrium to the credit deepening are conditional on the liquid wealth distribution is shown at Figure 2.1. On Panels (a) and (b), we compute the consumption elasticity to the borrowing constraint at different time horizons ($1, 2, 5$ and 10 years after the shock) of the credit deepening while maintaining the liquid wealth distribution before the shock displayed in the background. We obtain these elasticities by solving the general equilibrium model of Section 3 and calculating them through the consumption policy function. To facilitate viewing purposes, we separate poorer (below liquid wealth $\phi = 142.9$ US$ Thousands) and wealthy households in two different panels.

Figure 2.1: Elasticity of the Response of Consumption to Credit by Liquid Wealth Position



Note: On the Panel (a) and (b), I compute the elasticity of the response of consumption to credit for different horizons h, where the blue bars correspond to the consumption shares of each liquid wealth type - associated with the right y-axis. The difference remains that on panel (a), is the elasticity for liquid wealth below 142.9 Thousand US$; whereas on Panel (b), is for wealth levels above this threshold. On panels (c) and (d), we use the decomposition in Proposition 3 to breakdown the elasticity in intertemporal substitution effects and direct partial equilibrium effects.

At the time of the shock, the heavily constrained households spike their consumption in response to the credit relief. Consequently, they get indebted and lower their asset holdings below the depicted in Figure 2.1. For those that remain in the asset space previous to the shock, after one year the households reduce their consumption due to leverage effects and smaller Credit MPCs. These effects are mainly concentrated on households with negative liquid wealth, as wealthier households adjust positively their consumption. Moreover, these elasticities grow in time, indicating effects are stronger years after the beginning of the credit deepening.

Based on Proposition 3, we decompose the elasticity of consumption on direct partial equilibrium effects, led by $\underline{a}_t$, and the general equilibrium budget and intertemporal substitution effects led by r_t, while maintaining response conditional on the liquid wealth holdings of the household. On Panels (c) and (d), separating for poor and wealthy households, we can see that partial equilibrium effects - which is the direct channel of credit - drive the response of consumption for the households with negative or low liquid wealth holdings. This effect is due to Credit MPCs being sizable for these households, while asset holdings low enough not to induce substantial budget and intertemporal substitution effects. On the other hand, for very wealthy households, the opposite happens, as Credit MPCs mingle and budget and intertemporal substitution effects thrive.

2.3 Credit Deepening and Aggregate Consumption

So far, we have proven that the household-level response of consumption to a credit relief is heterogeneous and depends on the households' liquid wealth position. Poorer and indebted households react more to the direct partial equilibrium channel of credit, while wealthier households are more sensitive to prices, representing the indirect channels of credit. Our next step is extending the decomposition of Proposition 3 to aggregate consumption, aggregating all the responses of households.

We first connect the aggregate response to the object the computed in Proposition 3, highlighting the distribution's role. Then, we calculate its aggregate counterpart numerically following (KAPLAN; MOLL; VIOLANTE, 2018). Formally, define the general equilibrium path of prices and credit $\{\Gamma_t\}_{s=t}^{T}$ with $\Gamma_s = \{\underline{a}_s, r_s, w_s, T_s\}$ such that the aggregate consumption at period $t = 0$ is given by:

$$C_t(\{\Gamma_t\}_{t\geq 0}) = \int c_{i,j,t}(a, z; \{\Gamma_s\}_{s=t}^{T}) d\mu_t \tag{2-8}$$

where in $c_{i,j,t}(a, z; \{\Gamma_s\}_{s=t}^{T})$ and $\mu_t(da, dz, di, dj; \{\Gamma_s\}_{s=t}^{T})$ we emphasize the role of future and past prices on decisions and distributions. By differentiating both sides of the equation, we have:

$$dC_t(\{\Gamma_t\}_{t\geq 0}) = \int dc_{i,j,t}(a, z; \{\Gamma_s\}_{s=t}^{T}) d\mu_t \tag{2-9}$$

Equation (2-9) is crucial for understanding joint distribution's role in the aggregate consumption response. As wealth inequality increases, the wealth distribution's tails become thicker. Therefore, by Proposition 3, we know that there is a higher mass of households with consumption more sensible to credit, amplifying the aggregate consumption response. Since we cannot numerically compute the household response in (2-9), we proceed using the decomposition of the response of consumption featured in (KAPLAN; MOLL; VIOLANTE, 2018):

$$dC_t = \underbrace{\int_t^T \frac{\partial C_t}{\partial \underline{a}_s} d\underline{a}_s ds}_{\text{PE direct effect}} + \underbrace{\int_t^T \left(\frac{\partial C_t}{\partial r_s} dr_s + \frac{\partial C_t}{\partial w_s} dw_s + \frac{\partial C_t}{\partial T_s} dT_s \right) ds}_{\text{GE indirect effects}} \tag{2-10}$$

Where the first component captures the direct channel of the credit deepening over consumption, while the other components explain the indirect channel through secondary effects captured by the budget constraint and intertemporal substitution.

This decomposition differs slightly from the household counterpart in Proposition 3. Instead of aggregating each of the channels of credit to consumption with the general equilibrium evolution of the measure, we use the partial equilibrium evolution of the measure

$$\int_t^T \frac{\partial C_t}{\partial \underline{a}_s} d\underline{a}_s ds = \int_t^T \left(\int \frac{\partial c_{i,j,t}(a, z; \{\underline{a}_s, w_t, r_t, T_t\}_{s=t}^T)}{\partial \underline{a}_s} d\mu_t^{\underline{a}} \right) d\underline{a}_s ds \tag{2-11}$$

where $\mu_t^{\underline{a}} = \mu_t(da, dz, di, dj; \{\underline{a}_s, w_t, r_t, T_t\}_{s=t}^T)$ and $\frac{\partial c_{i,j,t}(a,z;\{\underline{a}_s,w_t,r_t,T_t\}_{s=t}^T)}{\partial \underline{a}_s}$ is the object in Proposition 2. We aggregate the partial equilibrium responses and compute their evolution based on the partial equilibrium evolution of the distribution. We present this exercise detailed for the full response and for the whole path $\{C_s\}_{s=t}^T$ in Section 4

2.4
General Equilibrium Framework

To assess the impacts of the indirect channels of a credit deepening on consumption and the role of inequality within this channel, we introduce a straightforward general-equilibrium framework. This choice is made because the model's complexity lies in the household side and its interaction with inequality.

Firms: We close the model in the tradition of (AIYAGARI, 1994), in which a representative firm hires labor and uses capital. The firm produces consumption goods with a Cobb-Douglas production function:

$$Y_t = F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha} \tag{2-12}$$

where K_t and L_t denote, respectively, aggregate capital and efficient labor units and $\alpha \in (0, 1)$ is the share of capital in production. We assume the presence of a competitive labor market. The problem of the representative firm is standard:

$$\max_{\{K_t,L_t\}} F\left(K_t, L_t\right) - w_t L_t - (r_t+\delta)K_t \tag{2-13}$$

Capital depreciates at the exogenous rate δ and since factor markets are competitive, the interest rate and the wage are given by:

$$w_t = \frac{\partial F\left(K_t, L_t\right)}{\partial L} \tag{2-14}$$

$$r_t = \frac{\partial F\left(K_t, L_t\right)}{\partial K} - \delta \tag{2-15}$$

Government: The government has the role of universal insurer of households as they face idiosyncratic income risk, partially insuring them against income fluctuations. It exerts this role by taxing linearly labor income through a tax rate τ_t, and rebating all of its funds as lump-sum transfers T_t. For each period, the government budget constraint must be satisfied:

$$\tau_t w_t L_t = \int T_t d\mu_t \tag{2-16}$$

Definition 2 (Competitive Equilibrium) *A competitive equilibrium is defined as the set of paths for household value functions and decisions* $\{c_{i,j,t}(a,z), s_{i,j,t}(a,z), V_{i,j,t}(a,z)\}_{t\geq 0}$*, input prices* $\{r_t, w_t\}_{t\geq 0}$*, fiscal variables* $\{\tau_t, T_t\}$*, measures* $\{\mu_t(da,dz,di,dj), g_{i,j}(a,z,t)\}_{t\geq 0}$*, aggregate quantities* $\{Y_t, C_t, L_t, K_t\}_{t\geq 0}$ *and borrowing constraints* $\{\underline{a}_t\}_{t\geq 0}$ *such that, given the exogenous stochastic process for* $\{z\}_{t\geq 0}$ *and cross-section heterogeneity* $(i,j)\in \mathbb{I}\times\mathbb{J}$*, at every period* t*:*

1. ***Households Optimize****: Given (i) prices* $\{r_t, w_t\}_{t\geq 0}$*; (ii) fiscal policy* $\{\tau_t, T_t\}_{t\geq 0}$*; and (iii) borrowing constraints* $\{\underline{a}_t\}_{t\geq 0}$*; the value functions* $\{V_{i,j,t}(a,z)\}_{t\geq 0}$ *solve the HJB (2-4) given the set of state constraints (2-5) and the infinitesimal generator* $\mathcal{A}$ *of the stochastic process* $\{z\}_{t\geq 0}$*. Moreover, policy functions* $\{c_{i,j,t}(a,z), s_{i,j,t}(a,z)\}_{t\geq 0}$ *satisfy (2-6)*

2. ***Firms Optimize****: Given prices* $\{r_t, w_t\}_{t\geq 0}$*, allocations* $\{L_t, K_t\}_{t\geq 0}$ *solve the Firm's problem (2-13)*

3. ***Fiscal Policy****: Fiscal policy* $\{\tau_t, T_t\}$ *satisfies the government budget constraint (2-16) at all periods*

4. ***Markets Clear****: Markets for goods, labor and capital clear*

(a) *Goods Market:* $Y_t = C_t + K_t$

(b) *Capital Market:* $K_t = \int a_t d\mu_t$

(c) *Labor Market:* $L_t = \int z_t d\mu_t$

5. ***Measures Satisfy Consistency Dynamics***: *For every pair* $(i, j) \in \mathbb{I} \times \mathbb{J}$, *distributions* $g_{i,j}(a, z, t)$ *satisfy the KF equations (2-7). Moreover, by aggregating* $(i, j) \in \mathbb{I} \times \mathbb{J}$, *we obtain measures* $\{\mu_t(da, dz, di, dj)\}_{t\geq 0}$

3
Calibration

There are two primary objectives in our calibration strategy. First, we aim to calibrate the model consistently across several countries[1] in our sample to compare average Credit MPCs and conduct cross-country quantitative exercises on credit expansions. Second, we seek to use assumptions about exogenous heterogeneity in household cross-sections and fiscal policy to match key moments of the income and wealth distribution. These moments are crucial as they determine two key model outcomes: (i) the distribution of Credit MPCs; and (ii) the aggregate measure of the economy, denoted as μ_t. Together, these outcomes enable us to provide a quantitatively accurate response of aggregate consumption to a credit deepening.

Table 3.1 displays the full set of parameters in the economy. The three initial parameters $\{\gamma, \alpha, \delta\}$ remain constant across all country calibrations and are based on estimates commonly used in the literature. The remaining parameters are internally calibrated to match moments of the income and wealth distribution. All the details of the internal calibration are explained as follows, while details on the data are explained in the appendix

3.1
Internal calibration strategy

The internal calibration strategy is carried out in two steps, taking advantage of the exogenous distribution of income: In the first step, we calibrate the parameters that govern the income process exogenously; to then use their estimates and calibrate moments of the wealth distribution in the steady-state of the model.

Fiscal Policy

Given the insurance role of fiscal policy in the model, we assume constant tax rates $\tau_t = \tau \ \forall t$, We then adjust these rates to align with the total size of cash transfers to households as a share of output.

[1]Due to time and computational constraints, we limit ourselves to only two countries in the current version of this working paper and do not run cross-country exercises.

Preference Heterogeneity

The concept of using heterogeneity in households' patience levels dates back to (KRUSELL; SMITH, 1997) and has been consistently employed in literature as a means to generate higher wealth inequality (KRUEGER; MITMAN; PERRI, 2016). This approach goes beyond merely reflecting varying degrees of impatience in the economy; it also encompasses differences in saving behaviors, life-cycle patterns, income growth expectations, risk aversion, and other factors that influence wealth accumulation goals across the population[2]. Our approach is similar to (CARROLL et al., 2017): The continuum of discount factors $\mathbb{J}$ is distributed uniformly between five different discount rates

$$\{\bar{\rho} - (\nabla_1 + \nabla_2), \bar{\rho} - \nabla_1, \bar{\rho}, \bar{\rho} + \nabla_1, \bar{\rho} + (\nabla_1 + \nabla_2)\} \tag{3-1}$$

where $\bar{\rho}$ is the average discount rate of the economy, used to match the target net wealth to net income ratio of the economy. Parameters $\{\nabla_1, \nabla_2\}$ determine the dispersion in heterogeneity of discount rates. These dispersion parameters provides us flexibility fitting the model's wealth distribution to the data, targeting net-wealth quintiles ($P0P20, P20P40, P40P60, P60P80, P80P100$), as in (CARROLL et al., 2017).

Income Process and Individual Fixed Effects

To achieve a highly unequal income distribution with risky earning dynamics, we adopt income process that introduces more risk than standard models based on AR processes with Gaussian shocks. Our approach, inspired by (GUVENEN et al., 2021), and utilizing annual data as in (GUVENEN; PISTAFERRI; VIOLANTE, 2022), incorporates a "buffer-stock" model of permanent and transitory income components as in (KAPLAN; MOLL; VIOLANTE, 2018), along with an individual fixed effect to capture heterogeneity in initial conditions, similar to (HEATHCOTE; STORESLETTEN; VIOLANTE, 2009).

Formally, the idiosyncratic productivity component of the household, z_{it}, is composed of three orthogonal components: (i) a fixed effect ω_i, drawn at birth specific to that household; (ii) a transitory component z_t^T; and (iii) a

[2] These can potentially be explicitly modelled by OLG and life-cycle models or by incorporating bequests and non-homothetic preferences, but this approach is significantly more simple and delivers quantitatively similar results

permanent component of income z_t^P:

$$\ln z_{it} = \omega_i + z_t^P + z_t^T \tag{3-2}$$

The permanent and transitory components of earnings are stochastic and each of them evolves separately, dictated by a compound jump-drift process with idiosyncratic jumps:

$$\begin{aligned} dz_t^T &= -\beta_{z^T} z_t^T dt + \eta_t dJ_{\eta,t} \;\; \eta_t \sim \mathcal{N}(0, \sigma_\eta^2) \\ dz_t^P &= -\beta_{z^P} z^P dt + \varepsilon_t dJ_{\varepsilon,t} \;\; \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2) \end{aligned} \tag{3-3}$$

which differs in frequency and amplitude of shocks, as well as the degree of persistence. The terms $\{\beta_{z^T}, \beta_{z^P}\}$ govern the mean-reverting characteristics of the process, directly determining how persistent are these shocks and each component of productivity. On the other hand, the jump components dJ_t have different arrival rates $\{\lambda_{z^T}, \lambda_{z^P}\}$ which determine how frequent is to receive a shock. Moreover, the sign and magnitude of the shock are also stochastic and determined by the normally-distributed random variables ε_t, η_t, each with variance $\{\sigma_\varepsilon^2, \sigma_\eta^2\}$. Therefore, these two will determine the amplitude of the shocks. The role of fixed effects ω_i is to capture variance in log-earnings in the income process. However, given that this moment is already within the set of moments targeted in the income process calibration, we can shut down the income fixed effect heterogeneity in this model as it would be redundant.

Table 3.1: Parameter Values for Country-Specific Calibration

	Scope	Description	Value	Source/Target
$1/\gamma$	Cross-country	IES	1	(KAPLAN; MOLL; VIOLANTE, 2018)
α	Cross-country	Capital share	0.33	(KAPLAN; MOLL; VIOLANTE, 2018)
δ	Cross-country	Depreciation rate	10% (p.a.)	Literature
Country-Specific Parameters				
$\underline{a}$	Country specific	Borrowing Limit	-2.9634 (16.3 Thousand US$)	HH Debt-to-GDP
$\bar{\rho}$	Country specific	Avg. Discount Rate	0.0432 (p.a.)	Net Wealth to Income ratio
$\{\nabla_1, \nabla_2\}$	Country specific	Discount rate Heterogeneity	$\{0.0088, 0.0028\}$ (p.a.)	Wealth Quintiles
τ	Country specific	Labor Income Tax	19.6%	Cash transfers to GDP
β_{z^T}	Country specific	Persistence Transitory inc.	0.011	Log Earnings dist.
β_{z^P}	Country specific	Persistence Permanent inc.	0.500	Log Earnings dist.
λ_{z^P}	Country specific	Arrival rate Permanent	0.197	Log Earnings dist.
λ_{z^T}	Country specific	Arrival rate Transitory	0.017	Log Earnings dist.
σ_ε	Country specific	Variance Permanent	0.816	Log Earnings dist.
σ_η	Country specific	Variance Transitory	1.025	Log Earnings dist.

Note: Pre-tax Income Gini reflects sample averages of administrative data, available in (GUVENEN et al., 2021)

3.2
Step 1: Income Process Calibration

In this step, we calibrate the set of parameters $\Theta = \{\beta_{z^P}, \beta_{z^T}, \sigma_\eta, \sigma_\varepsilon, \lambda_{z^P}, \lambda_{z^T}\}$ of the two-state income process (3-3) to match key moments of the empirical income distribution, ensuring realistic income dynamics. The targeted moments include the standard deviations of the distribution of log income innovations [3] and the one and five-year growth of log income[4], as well as the kurtosis and tail dispersion measures of these growth rates, such as the $P9050$ and $P5010$. These moments are crucial for identifying the components of permanent and transitory income.

All of these moments are available in the Global Repository of Income Dynamics (GRID) database (GUVENEN; PISTAFERRI; VIOLANTE, 2022), which contains a wide range of micro statistics on income inequality and dynamics. Its quality is based on two pillars: (i) it is build on micro panel data drawn from administrative records; and (ii) it is cross-country comparable. As argued by (BUSCH et al., 2022) and (GUVENEN; PISTAFERRI; VIOLANTE, 2022), some measures of idiosyncratic labor income risk[5] present business cycle variation, and may also present a trend. As the intent of our exercise is to emulate a steady-state instead of capturing short-term cycles, we abstract of trends and cycles and proceed by taking a sample average of each moment used for calibration. These values are displayed in the Table A.1

To carry on with this calibration, we first approximate the continuous-time continuous state processes (3-3) with continuous-time discrete-state processes. We use a non-linear grid for the permanent income state-space and the transitory income state-space[6], discretizing the infinitesimal generator of the processes as continuous time transition matrices based on finite difference approximations, provided the calibration of Θ. Given the transition matrices, we use the Kolmogorov-Forward equations of the stochastic processes to compute the ergodic distributions of permanent and transitory productivity components of the household. Moreover, we use the transition matrices and the

[3]These are obtained by taking the log income of the individual, regressing in a set of observable controls such as age, gender and education, and then taking the residual of this regression

[4]That is, the one and five-year change of the residuals of log income regressions

[5]Measures such as skewness of income growth have a very strong pro cyclical behavior, while volatility of income growth is mildly countercyclical. When decomposing skewness by measures of tail dispersion, such as $P9050$ and $P5010$, the latter presents strong countercylical behavior, while the former exhibits cyclical pattern

[6]We proceed as in (KAPLAN; MOLL; VIOLANTE, 2018), approximating the permanent income state-space with a 11-point nonlinear grid, and the transitory income state-space with a 3-point nonlinear grid. Grid spacing parametrization is the same as in (KAPLAN; MOLL; VIOLANTE, 2018)

ergodic distribution to simulate 5000 individuals over 20 years to reconstruct the distributions of log income and log earnings growth in time. Furthermore, as we observe only annual moments but simulate income quarterly, we aggregate income annually and then compute the simulated counterparts $\hat{\mu}(\Theta)_k$ of the observed empirical moments $\hat{\mu}_k$ Table A.1

For a uniform comparison of moments, we calculate the percentage deviation of the simulated moment from its empirical counterpart. For any given moment $k \in \mathcal{K}$:

$$F_k(\Theta) = \frac{\hat{\mu}(\Theta)_k - \hat{\mu}_k}{\hat{\mu}_k} \tag{3-4}$$

to then define the loss function from the given vector of percentage deviations. Let $F(\Theta) = (F_1(\Theta), \ldots, F_K(\Theta))'$ be the vector of stacked moments, over which we minimize to obtain the estimated parameters:

$$\widehat{\Theta} = \arg\min_{\Theta} F(\Theta)' W F(\Theta) \tag{3-5}$$

where W is a weighting matrix. We presuppose that each moment is equally informative, so we assign equal weights to all of them by choosing $W = I$. Finally, to solve the minimization problem, we employ a multi-start algorithm used in (MELLO; MARTINEZ, 2020), which is an adaptation of (GUVENEN et al., 2021): In the first stage of the algorithm, we randomly evaluate 10,000 initial parameter vectors (chosen based on a Sobol sequence). Afterward, based on the loss function, the 5% best guesses are selected and carried out for the second stage of the algorithm. In that stage, we perform a local search on the selected guesses using the Nelder-Mead simplex algorithm and select the $\widehat{\Theta}$ that minimizes equation (3-5). The estimated parameters for each country are displayed in Table A.3, and the numerical procedure for discretizing and estimating the income process is available at the appendix

3.3
Step 2: Wealth Calibration

In the second step, we use the calibrated parameters $\widehat{\Theta}$ from step one to target general moments of the wealth distribution. The goal is to recover as much information as possible about the entire wealth distribution. To calibrate the parameters $\Omega = \{\underline{a}, \rho, \nabla_1, \nabla_2, \tau\}$, we follow a procedure similar to (CARROLL et al., 2017) and use the following set of moments: (i) wealth shares of each quintile of the wealth distribution ($P0P20$, $P20P40$, $P40P60$, $P60P80$, $P80P100$); (ii) the net private wealth to net national income ratio;

(iii) unsecured household debt-to-GDP; and (iv) the annual real interest rate.

As with the first step, these data are available in the form of panel data. Wealth data, such as Mean Wealth to GDP ratio and Wealth Shares, are drawn from World Inequality Database (CHANCEL et al., 2021). For fiscal data, we use cash transfers as a share of GDP, which is available at the OECD database[7]. Moreover, for household debt, we focus on non-secured household debt, which is drawn from (HOENSELAAR et al., 2021), and for annual real interest rate we rely on central bank data of each respective country.

For the wealth shares by quintile series, we compute the average for the first four quintiles ($P0P20$, $P20P40$, $P40P60$, $P60P80$) over the sample period of 1990-2018, and calculate the last percentile $P80P100$ as a residue to ensure that the sum of percentiles is 1. For all other variables, we proceed similarly by computing sample averages. The only exception is household debt-to-GDP excluding mortgages, which has a single observation in the panel data (2018) due to data availability issues. All the data used is displayed in Table A.2 in the appendix.

To calibrate the parameters Ω, we compute the steady state of the model by solving the it globally with the finite difference method, as in (ACHDOU et al., 2022), and calculate the percentage deviation of the simulated moments from their empirical counterparts, as in (3-4). We then stack these moments and minimize (3-5). The estimated moments for each country are shown in Table A.4, and the numerical procedure for estimating the stationary equilibrium is available in the appendix.

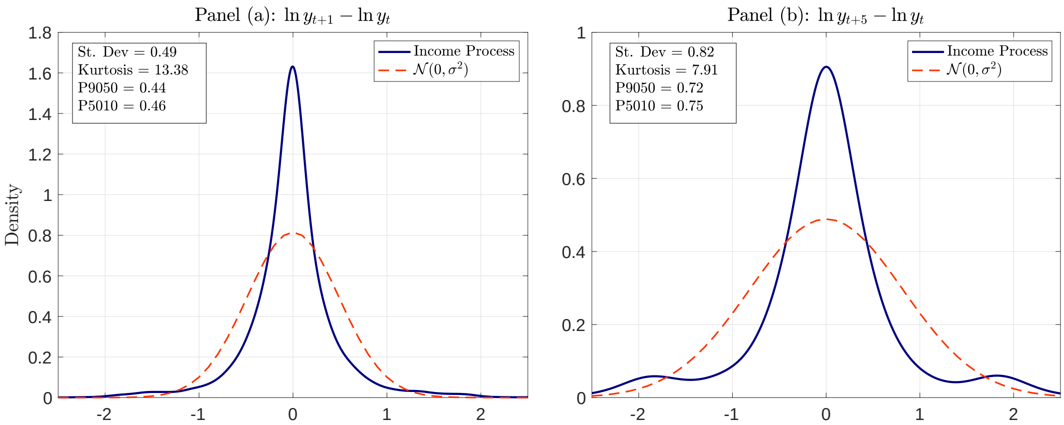
3.4 How well does the model fare with respect to data?

3.4.1 Earning Dynamics

The general estimation fit for selected countries are displayed in Table 3.2. Overall, the fitted earnings process matches the nine targeted moments of Table 3.2 well, and generates the observed level of income inequality within the data - as evidenced by the fit of untargeted moments, such as pre-tax income gini.

[7]https://data.oecd.org/socialexp/social-benefits-to-households.htm

Figure 3.1: Calibrated Distributions of One and Five Year Log Income Changes



Note: In Panel (a), distribution of 1 year log income changes and its respective high-order moments; while in Panel (b), distribution of 5 year log income changes. Both correspond to the stationary distributions of the process (3-3) calibrated to match micro moments of United States data, available in the Table 8. In the red-dotted lines, we have a gaussian distribution with the same variance as the leptokurtic distribution

Table 3.2: Earnings Process Estimation Fit

Targeted Moments	United States		Brazil	
	Model	Data	Model	Data
St. Dev. of Log Earnings	0.89	0.93	1.07	1.04
St. Dev. 1 year Log Earnings growth	0.48	0.56	0.59	0.68
St. Dev. 5 year Log Earnings growth	0.81	0.78	0.78	0.82
Kurtosis 1 year Log Earnings growth	13.37	12.86	8.42	8.53
Kurtosis 5 year Log Earnings growth	7.91	8.82	6.75	6.34
P9050 1 year Log Earnings growth	0.43	0.43	0.72	0.61
P5010 1 year Log Earnings growth	0.46	0.46	0.70	0.70
P9050 5 year Log Earnings growth	0.72	0.71	0.91	0.84
P5010 5 year Log Earnings growth	0.75	0.77	0.87	0.94
Untargeted Moment:				
Pre-tax Income Gini	0.46	0.47	0.58	0.55

Note: Pre-tax Income Gini reflects sample averages of administrative data, available in (GUVENEN et al., 2021)

By targeting higher-order moments of the distributions of log-earnings growth, we generate distributions of log-earnings growth that are *leptokurtic*, that is, with more mass concentrated around the mean and tails with respect to a normal distribution with same variance - as evidenced in Figure 3.1.

This stems from the fact that transitory earnings component concentrates mass around the mean with small but frequent shocks, whereas permanent earnings component concentrates mass on the tails with larger and infrequent shocks. In terms of the ergodic distribution of log earnings, these income dynamics will generate a skewed distribution that is a crucial element in generating the dispersion of liquid assets. Firstly, because earnings dispersion generates likewise wealth dispersion through saving decisions of households; secondly, as a leptokurtic distribution implies that households face substantially more income risk, increases wealth accumulation specially for the wealthiest

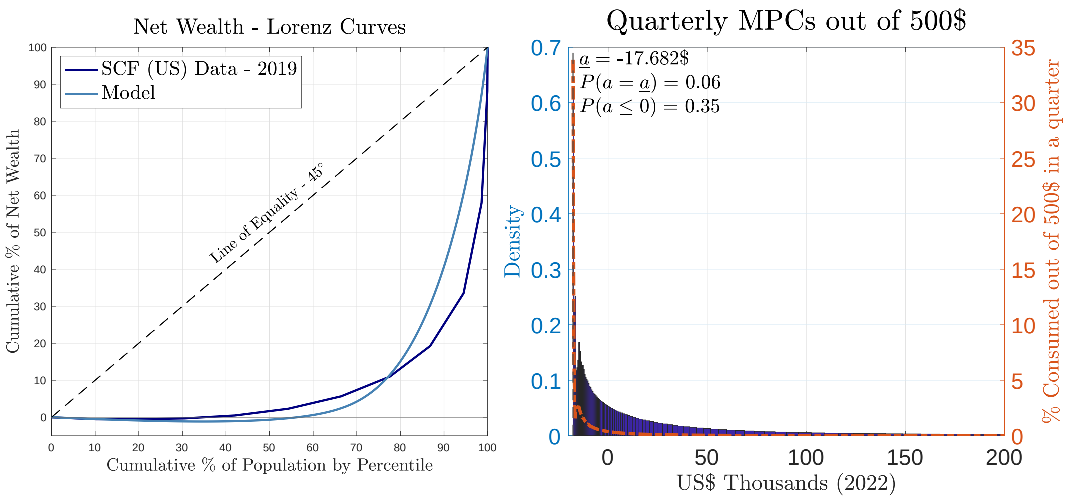
3.4.2 Wealth Distribution and micro-level consumption & credit behavior

Table 3.3: Wealth and Fiscal Data Estimation Fit

Targeted Moments	United States		Brazil	
	Model	Data	Model	Data
Mean Wealth to GDP ratio	2.74	4.88	2.33	3.61
$P0P20$	-0.82	-1.54	-0.85	-2.14
$P20P40$	-0.30	0.35	-0.72	1.23
$P40P60$	1.46	3.24	0.84	3.75
$P60P80$	13.80	12.06	8.82	9.50
$P80P100$	85.84	85.88	91.91	87.65
Cash transfers to GDP (%)	13.14	12.74	11.71	14.56
Household (unsecured) Debt-to-GDP	12.64	27.09	15.01	23.20
Real Interest Rate (p.a.)	2.00	1.803	4.11	5.461
Untargeted moments				
Average Quarterly MPC out of $500	8.97	-	11.08	-
Share of Households with negative wealth (%)	34.90	15	42.54	-
Share of HtM in the economy	41.96	14.2	48.73	-
Wealth Gini	0.794	0.867	0.831	0.886

In Table 3.3, we display the estimation fit for the targeted moments for selected countries. The model replicates fairly well wealth share quintiles ($P0P20$, $P20P40$, $P40P60$, $P60P80$) and wealth gini, and to a fair extent the size of fiscal policy in the economy and the real interest rate. However, aggregate moments of the wealth distribution such as mean wealth to GDP ratio and household debt are not well fit to the data as we would expect. Several reasons arise as to why we fail to account for them properly: (i) Over-identification of the simulated and matched moments with respect to structural parameters of the steady-state of the model; (ii) introducing household debt-to-GDP moment in the calibration is conflicting with the moment of mean wealth-to-GDP ratio, as our model is not of gross-positions, aggravating the "missing-middle" problem of (KAPLAN; VIOLANTE, 2022)[8], which can be clearly seen in the comparison of our model with US SCF[9] net wealth in the left plot of Figure 3.2

Figure 3.2: Calibrated Distributions of Wealth and Quarterly MPCs



Note: Distributions are calibrated to US data

In the right plot of Figure 3.2, we show the US model generated wealth distribution and the distribution of empirically observed MPCs. Two results are worth mentioning: first, we generate a skewed wealth distribution with a fat right-tail, as desired. Second, despite the higher mass of HtM households in the economy, the quantitative response of consumption generated by the model is far from being in line with the literature: the average quarterly MPC out of 500$[10] transfer is 8.97% for the US calibration, where empirical estimates range from 15% to 25% (JAPPELLI; PISTAFERRI, 2010). This result is expected, as one-asset models often fail to capture a high-average MPC without generating an excessively polarized wealth distribution (KAPLAN; VIOLANTE, 2022)

With respect to the object of interest in our study, we plot in the Figure 3.3 the Credit MPCs of Proposition 1 under the US steady-state calibration. In the panel (a), we plot the distribution of average credit MPCs with respect to the stationary distribution of z^T, and in panel (b) likewise, but with respect to the stationary distribution of z^P. This numerical exercise depicts micro-level consumption out of credit behavior much beyond the borrowing constraint,

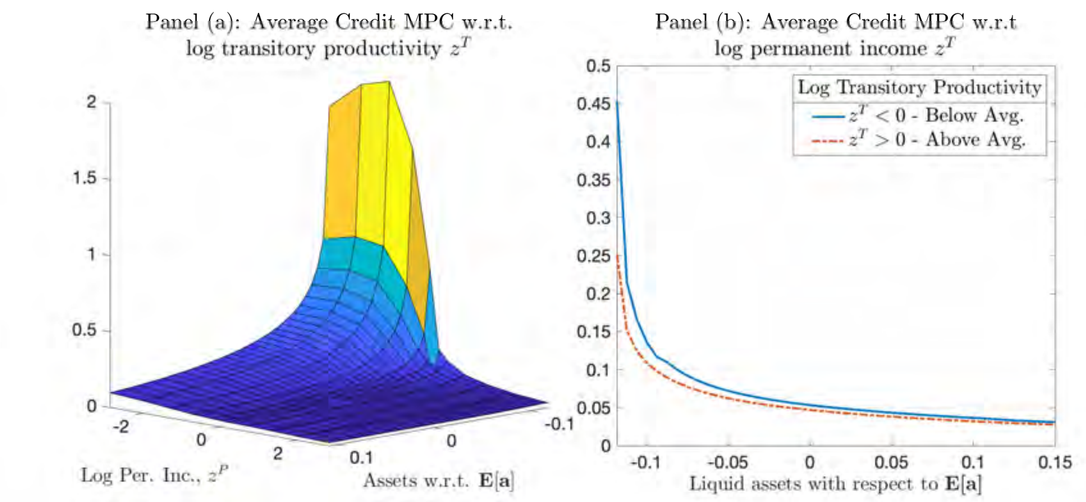
[8]When extensions of the one-asset heterogeneous agents model that incorporates ex-ante heterogeneity (as in our case) can generate sizeable responses of consumption while still remaining consistent with aggregate wealth data, but at the cost of severely understating asset holdings of households in the middle of the distribution

[9]United States 2019 Survey of Consumer Finances

[10]To calculate the Quarterly MPC of 500$, I first map the value of an asset to 2022 US$, to then calculate the object using the Feynman-Kac formula. For more details, see appendix

even though the region close to the borrowing constraint is the quantitatively relevant region.

Figure 3.3: Calibrated Distributions of Credit MPCs



Note: On the Panel (a), we compute MPCs averaging for log transitory productivity, $\mathbb{E}[MPC^{500\$}_{Qtr}|g^*(z^T)]$; whereas on Panel (b), we compute MPCs averaging for log permanent productivity, $\mathbb{E}[MPC^{500\$}_{Qtr}|g^*(z^P)]$. Both distributions are calibrated to US data

First, it tells us that close to the borrowing constraint, the distribution of income matters substantially for determining credit-consumption behavior. That is, the lower the income of the household[11], the higher the response of consumption to a credit deepening. And that this response will be even more increased if the household has no assets to smooth consumption. Second, the relevance is much bigger with respect to permanent income rather than the transitory income component. This result reflects the permanent income hypothesis (FRIEDMAN, 1957), as consumption behavior is much more sensible permanent income rather than transitory. Third and last, it strengthens the point made in Proposition 1: although income inequality is relevant for the consumption response to a credit deepening, its effects are mitigated by the relevance of the dispersion in household wealth. For already barely positive levels of assets, household credit MPC is almost insensible to permanent or transitory levels of income.

[11]That is, the credit MPC response is almost monotonically decreasing in permanent and transitory income for all levels of asset holdings. The only non-monotonicity implied by the exercise is at borrowing constraint for log permanent income, although this could be due to numerical error.

4
Quantitative Results

This section aims to examine the short and long-term responses of aggregate consumption to a credit deepening (1), elucidate the channels through which credit impacts aggregate consumption and explore how wealth inequality influences this response over time. To do such, we conduct a one-time unexpected credit shock that varies in persistence and duration: The persistence of the shock is to match quarterly-autocorrelation of household credit consistent with VAR-based empirical evidence (see Section 5), whereas the duration implies that the shock can either be transitory or permanent. In the following section, we focus on the transitory Credit Deepening. However, in the appendix we extend all the results to a permanent shock and we also include credit deepening exercises for the model calibrated to the Brazilian economy In the transitory Credit deepening experiment, there is an initial shock at $t = 0$ that alters the borrowing constraint from $\underline{a}_0$ to $\underline{a}'$, such that the path of borrowing constraints $\{\underline{a}\}_{t\geq 0}$ follow the trajectory dictated by the Ordinary Differential Equation (ODE) back to the previous steady-state:

$$d\underline{a}_t = \nu(\underline{a}_0 - \underline{a}_t)dt \tag{4-1}$$

We calibrate $\underline{a}' = 28.33$ US$ Thousands [1] so the model-calibrated US economy undergoes a trajectory that doubles household debt-to-gdp at the new steady-state when the shock is permanent, and $\nu = 0.25$ implying a quarterly autocorrelation of household credit of $e^{-\nu} = 0.78$[2].

To shed light on the channels that drive the response of consumption through time, we rely on the decomposition in (2-10) by (KAPLAN; MOLL; VIOLANTE, 2018) to measure how much of the response is driven by pure credit effects and by indirect effects through prices, as intertemporal substitution and income effects. We then dissect each of these responses of consumption with respect to the wealth distribution. First, by investigating the aggregate consumption response by wealth percentiles (that is, the aggregate consumption IRFs conditional on a wealth percentile) in time, and second by decomposing by wealth percentile each of the partial and general equilibrium responses of consumption to credit and prices. This decomposition shows us

[1] In the model wealth grid, this stands for $\underline{a}' = 4.75$ given that the US$ to asset ratio is given by 5.96 Thousand US$ for the US calibrated economy

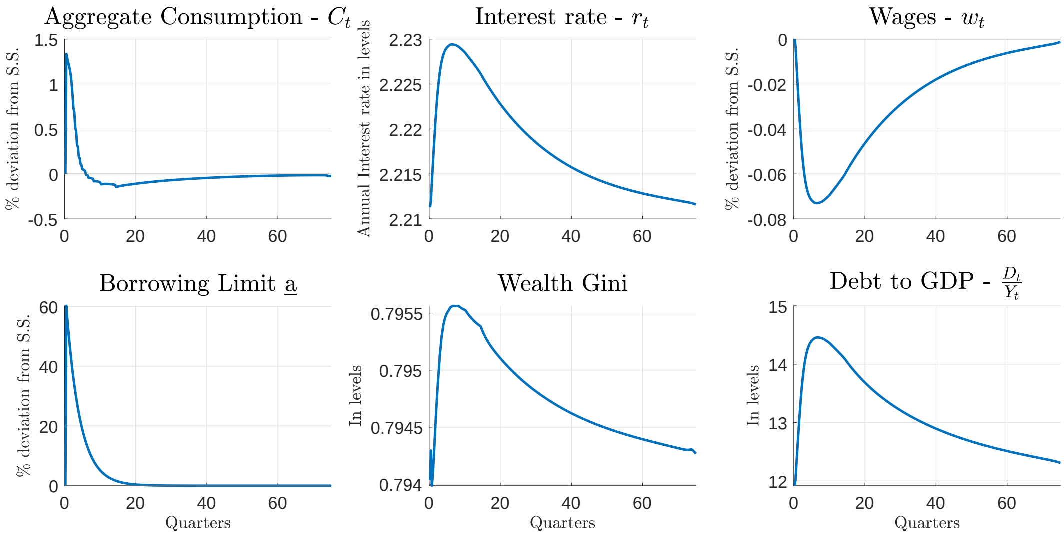
[2] This value implies the mean-reversion of the shock after 28 quarters, in line with VAR-based evidence on section 5

the role of heterogeneity in each of the channels of credit to consumption

Finally, in our last experiment, we show the role of inequality in the response of consumption to a credit deepening by altering the initial level of income or wealth inequality in the economy. We do such by changing marginally the calibration of one of the sources of ex-ante heterogeneity or fiscal policy targeting a 1 point increase in wealth gini, to then run transitory credit shocks and see the impact of inequality on the initial income and wealth distribution over the whole path of aggregate consumption.

4.1 Impulse responses to a Credit Deepening

Figure 4.1: General IRFs to a Transitory Credit Shock



We display the Generalized Impulse Response Functions (GIRFs) in Figure 4.1 after a transitory credit deepening (4-1), whereas Figure A.2 displays the GIRFs for the permanent (A-18) credit deepening. In response to a credit deepening, regardless of the duration - transitory or permanent, aggregate consumption is stimulated in the short run as households have access to more credit, decreasing their liquid asset positions in order to consume more. This response is short-lived, and related to the duration of the credit shock: as the shock is permanent, the longer the short-run consumption "boom" lasts. Under our calibration for the United States economy, the positive consumption boom can last from 2 to 3 years. As a consequence of the decreasing liquid asset positions, poorer households lever and have lower wages and wealthier households benefit of higher interest rates (2-14). These forces altogether generate higher wealth inequality, albeit the increase is small - as seen by evolution of Wealth Gini.

After the initial boom, aggregate consumption falls below its initial level, leading to a long-lasting reduction in consumption - the consumption “bust”. This reduction is persistent because households are more levered in household debt and must deleverage under higher interest rates; and also is permanent in the case the credit shock increases the access to credit by households permanently. Moreover, in the case of a transitory shock, as credit conditions tighten once again to their previous level, households slowly readjust their consumption through a slow process of deleveraging, which makes aggregate prices and inequality revert to the initial values before the shock.

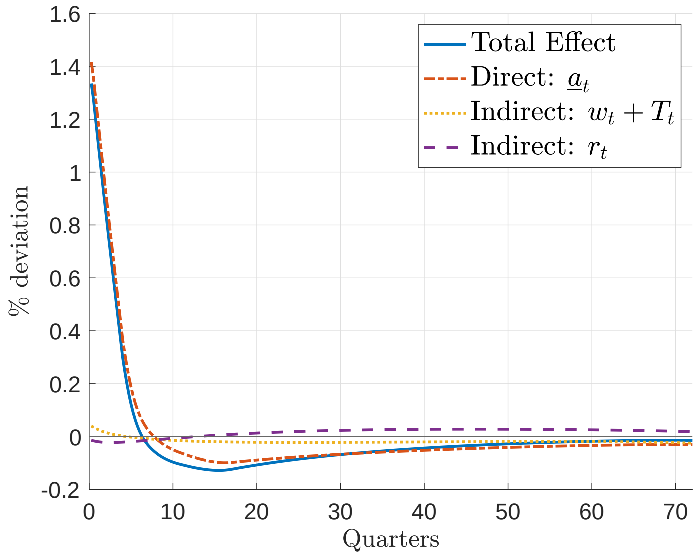
4.2 Transmission Mechanisms of a Credit Deepening to Aggregate Consumption

We now turn to decomposing the channels that drive the response of consumption through the whole path of borrowing constraints. This is made possible by the decomposition in (2-10), which disaggregates the response into partial equilibrium responses led by each of the path of objects in the economy $\Gamma_t = \{\underline{a}_t, w_t, r_t, T_t\}$. The Partial equilibrium response $\frac{\partial C_t}{\partial \underline{a}_t}$ is the direct channel of credit, and the objects $\frac{\partial C_t}{\partial r_t}$ and $\frac{\partial C_t}{\partial y_t} = \frac{\partial C_t}{\partial w_t} dw_t + \frac{\partial C_t}{\partial T_t} dT_t$ are indirect channels of credit that correspond for the intertemporal substitution and budget effects, respectively.

As demonstrated in Figure 4.2, these different forces shaping the response of consumption change their effects and magnitudes with time. For instance, the direct channel of credit is the main responsible for the short-run “boom” and long-run “bust” cycle. This is explained Figure 4.2 together with the wealth distribution in Figure 3.2 and Proposition 2: The higher share of poor households drive the average Credit MPC up, increasing the strength of the direct channel of credit. On the other hand, indirect channels of credit will reflect distributional forces of the credit deepening affecting consumption. The intertemporal substitution channel dampens the short-run response through the increase in interest rates. However, it has a key role on the consumption rebound on the long-run. Moreover, budget effects are contingent on the aggregate impacts of the household demand shock over the labor market and the resulting fiscal policy through cash-transfers. In the case of the current model, as wages are lower and fiscal policy is pro-cyclical[3], all households

[3]In the Aiyagari flexible-price economy, this household demand shock that follows a credit deepening will lead the economy to a supply-side recession as capital lowers. As a consequence, wages also lower and interest raise increases. This is counterfactual to what is observed in the data – e.g. (MIAN; SUFI; VERNER, 2017), and within a Neo-Keynesian framework may have different effects

Figure 4.2: Decomposition of the Consumption IRFs following a Transitory Credit Shock by Direct and Indirect Effects



Note: All values are log deviations from steady state. The decomposition is given by (2-10)

have lower income and consume less. Thus, budget effects play a lesser role in dampening the effect of consumption all throughout the consumption-credit cycle.

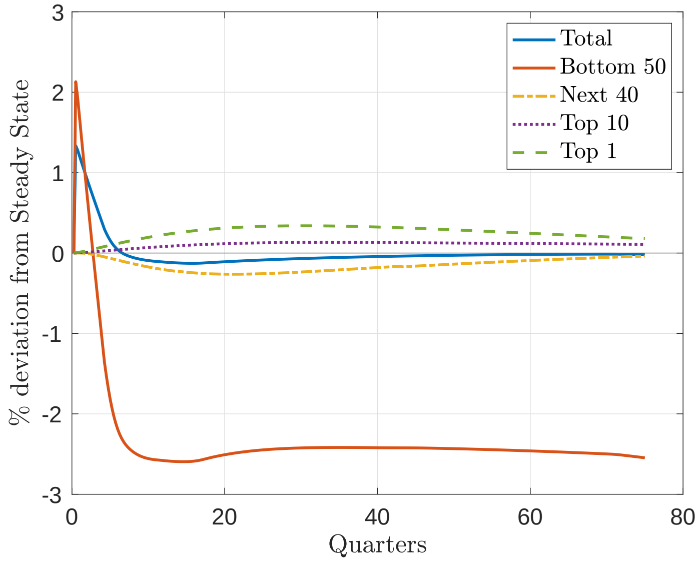
4.3 The Role of Inequality on Credit Deepening

In this subsection, we study how inequality affects the channels of transmission of a credit deepening on aggregate consumption, amplifying or reducing its response. We do this through two types of exercises: First, we rely on the decomposition of the consumption GIRFs by wealth percentiles to see who drives the response of consumption at each point in time. We complement this analysis by also decomposing by wealth percentiles the two main channels of credit to consumption: the direct channel of credit, and the indirect through intertemporal substitution. Second, we do counterfactual credit deepening exercises in which we alter marginally the initial calibration of one of the parameters which affects the initial income and wealth distribution, to then compare the aggregate consumption IRF of the baseline calibration to these alternative scenarios of more or less income and wealth inequality.

Decomposing the Response of Consumption by Wealth Percentiles

The aggregate response GIRF of consumption in Figure 4.1 masks significant heterogeneity in the response of each kind of household throughout the wealth distribution. As suggested by Proposition 3, individuals with

Figure 4.3: Decomposition of the Consumption IRF following a Transitory Credit Shock by Wealth Percentiles

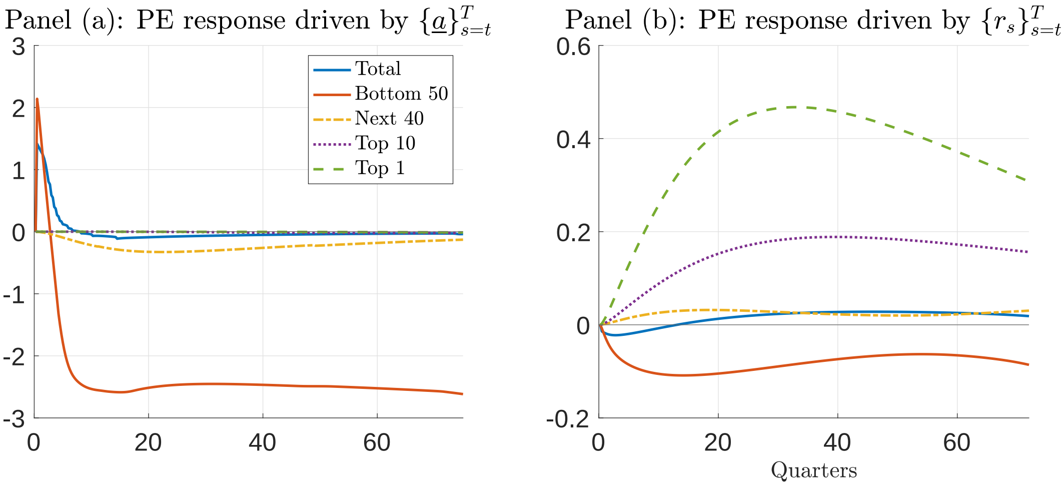


Note: We decompose the total response of of aggregate consumption to the Permanent Credit shock (A-18) into the response by wealth percentile, $C_t = \int_\omega c_t(a, z^P, z^T, t) d\mu_t$ where ω is the subset of liquid wealth holdings corresponding to the wealth percentile. (4-1)

less asset holdings will experience stronger partial equilibrium credit effects driving their response of consumption, as they have sizeble Credit MPCs; whereas wealthier households will be led through budget and intertemporal substitution effects. Therefore, individuals in the very top of the distribution will react positively to the credit deepening, as the shock will increase the real interest rate and generate wealth effects. This is depicted in the responses of consumption of the Top 10 and Top1 percentiles of the wealth distribution, shown at Figure 4.3, who drive upwards the response of consumption in the middle and long-run and are responsible for the rebound of consumption after the "bust".

On the lower-end of the wealth distribution, the short-run consumption boom is driven the individuals on Bottom 50 percentile of the wealth distribution. In the face of the credit deepening, they lever up quickly to increase consumption in the short-run, but at the cost of decreasing their long-run consumption persistently (if not permanently) due to more debt with higher interest rates. Lastly, individuals in the middle of the distribution (From Percentiles 50 to 90) face the situation in which they have low Credit MPCs and are not constrained, thus they do not react to a credit shock by catching up consumption, but have low or negative asset holdings and are more subject to income effects through transfers and wages. Therefore, their response to consumption follows more closely the reaction of wages and government transfers.

Figure 4.4: Sensitivity of the Response of Consumption to Credit Conditional on Income or Wealth Inequality



Note: On Panels (a), we disaggregate the direct channel of credit to transitory credit shock by wealth percentiles. Similarly, on Panel (b) we do the same decomposition but for the indirect channel of credit determined by interest rates (budget and intertemporal substitution). All values are log deviations from steady state. The decomposition is given by (2-10)

Extending our analysis even further, we decompose the heterogeneity in response of consumption looking to the direct channel of credit and the indirect channel of intertemporal savings. We decompose the responses of these channels in Figure 4.2, disaggregating the response of the direct $\underline{a}_t$ and indirect r_t curves in Panel (a) and (b) by wealth percentiles, which are shown at Figure 4.4

Through this exercise, we link the rebound of consumption response by the wealthy households with the indirect intertemporal substitution channel of credit. As shown in Panel (b) of Figure 4.4, the partial equilibrium response to the path of interest rates will lead to an increase in the aggregate consumption which is happens mostly through the Top 1 and Top 10 wealth percentiles. Households in the bottom of the distribution will react negatively as it implies an increase in the cost of debt to them, thus a decrease in their available income for consumption. On the other hand, Panel (a) implies that the short-run boom and long-run bust in consumption through the direct channel of credit is led mostly by the bottom of the wealth distribution (the Bottom 50 wealth percentile). The wealthiest percentiles react little or nothing to the direct channel of credit, only to distributional effects through other channels.

Counterfactual Exercises

In this subsection, we complement the analysis of the Section 4.3 using an alternative approach to study how wealth inequality amplifies the response of consumption following a credit deepening. In the former section, we focused on the heterogeneity of aggregate consumption response by wealth percentile and by each of the channels of credit to consumption. Now, we provide counterfactual exercises in which we alter marginally the initial calibration of one of the parameters which affects the cross-section heterogeneity of households, generating an initial wealth distribution with one point above or below the initial wealth gini benchmark. Then, we simulate a credit deepening exercise in these counterfactual economies and compare the aggregate consumption IRFs.

These exercises support the theory that wealth inequality is the main source of amplification, associating alternative calibrations of higher inequality with more HtM households and higher Average Quarterly MPC out of 500$. Consequently, we know from the previous sections that the higher these are, the wider will the "boom" and "bust" be because of more constrained households and substantial leverage. Concomitantly, we observe the consumption rebound by the wealthy in these exercises, as counterfactual economies more unequal in wealth rebound quicker of a stronger "bust" as households delever faster.

Fiscal Policy Experiment

In our model, fiscal policy has an insurance role within the model by subsidizing those with low income by taxing those with high income. It alters the post-tax income distribution and also the wealth inequality, as it influences precautionary savings. Consequently, post-tax income inequality is lower (higher) in economies with higher (lower) τ and T. On the other hand, wealth inequality is lower (higher) in economies with lower (higher) τ and T.

We set the alternative tax rates to $\tau = 10\%$ and $\tau = 30\%$, targeting wealth gini 0.78 and 0.80, respectively. Lump-sum transfers follow the government budget constraint and increase with the higher tax rate, as there is more insurance for poorer households. In the high tax economy, there are more HtM households and higher household debt, leading to a higher average MPC. On the other hand, in the low tax economy, there are less HtM households and less debt. All of the moments relative to the counterfactual economies can be seen in Table 4.1. Following a credit deepening, Panels (a) and (b) of Figure 4.5 show counterfactual and benchmark responses for the aggregate consumption IRFs and household debt, respectively.

Table 4.1: Calibration of $\{T, \tau\}$ for each of the counterfacutal economies

Variable	Benchmark	High $\{T, \tau\}$	Low $\{T, \tau\}$
Average Quarterly MPC out of 500$	8.97	10.20	7.88
Share of Households with negative wealth (%)	34.90	38.98	29.75
Share of HtM in the economy	41.96	46.00	36.27
Wealth Gini	0.79	0.80	0.78
Household Debt to GDP (%)	12.64	14.98	9.83
Calibration			
Marginal Income Tax τ	19.42%	30%	10%
T as a share of wage	33%	50.6%	16.87%
Panels (c) and (d) Calibration			
Borrowing Limit in US$ Thousands (assets)	17.68 (2.96)	20.28 (3.4)	15.50 (2.6)

Note: Counterfactual simulations are done for the US economy. The mapping assets to Thousand US$ is done by the steady-state US calibration, yielding a ratio of 5.96 Thousand US$ for each asset. In the Panel (c) and (d) calibration, we keep the alternative fiscal policy calibration but change the borrowing limit so as to match the initial Household Debt to GDP of the Benchmark economy.

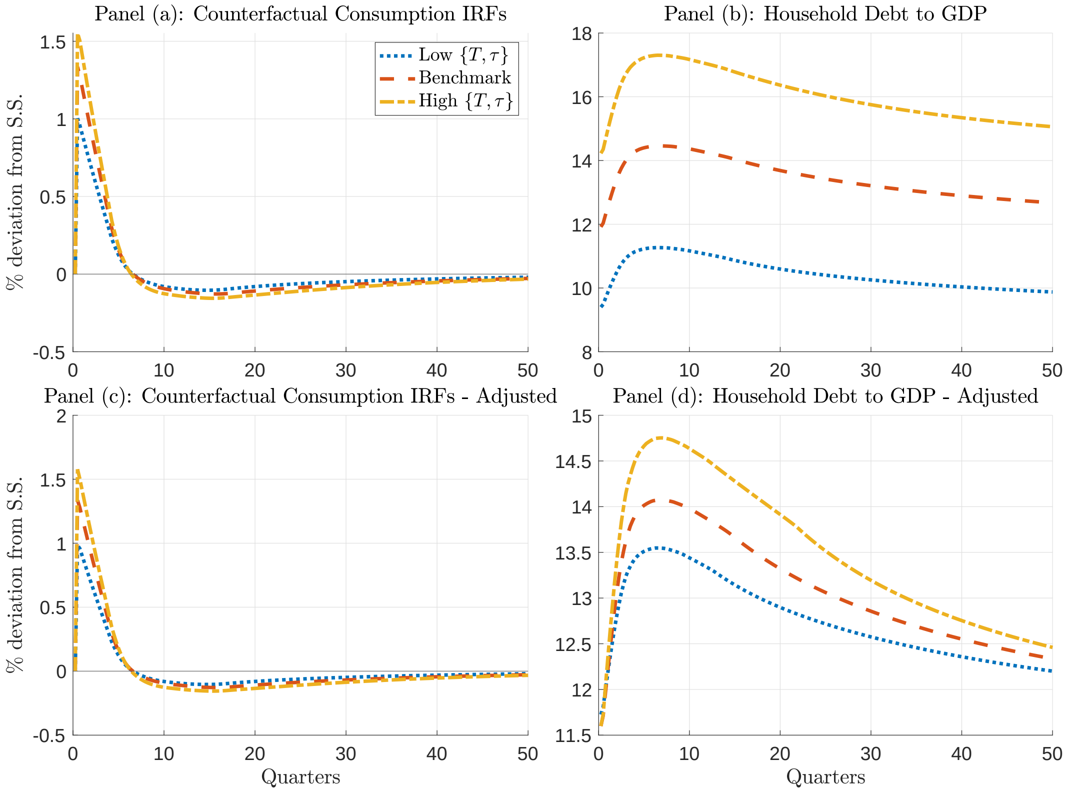
In the economy with higher wealth inequality (higher τ), the consumption expansion in the short-run exceeds that of the benchmark economy, while the lower wealth inequality economy grows less with respect to the benchmark. These dynamics are accompanied by not only a higher initial level on household debt the more wealth inequality in the economy (due to the calibration), but also more leverage from the households following a credit deepening.

In this initial exercise, by altering the calibration marginally, these economies depart from different levels of household debt before the credit deepening. Therefore, we complement the counterfactual exercise by adjusting for the initial household debt via the borrowing constraint, as seen in the last line of Table 4.1, to ensure the level of household debt is the same in these counterfactual economies regardless of the alternative calibration. All results are maintained, with wealth inequality amplifying the consumption “Boom & Bust” cycle following a credit deepening.

Permanent Income Risk Experiment

In this experiment, we alter the arrival rate of permanent income shocks of the household income process (3-3), affecting the level precautionary savings of households in the economy. A marginal increase in the frequency of shocks makes households more exposed to income fluctuations, increasing their precautionary savings. This general increase in precautionary savings is concentrated in the households at the lower end of the wealth distribution, as they have higher potential consumption losses of receiving an income shock. Therefore, the higher λ_P, the lower the wealth inequality in the economy - as well

Figure 4.5: Credit Deepening with More and Less progressive Fiscal Policy



Note: On the Panel (a), we plot the consumption IRFs to the same MIT-Shock to the borrowing constraint, but with different levels of wealth inequality induced by different fiscal policy $\{\tau, T\}$; whereas on Panel (b) we plot the corresponding Debt to GDP IRFs. In Panels (c) and (d), we do the same exercise but control for the initial level of household debt induced by changing fiscal policy.

Table 4.2: Calibration of λ_P for each of the counterfacutal economies

Variable	Benchmark	High λ_P	Low λ_P
Average Quarterly MPC out of 500$	8.97	8.73	9.04
Share of Households with negative wealth (%)	34.90	30.62	42.34
Share of HtM in the economy	41.96	38.17	48.11
Wealth Gini	0.79	0.78	0.80
Household Debt to GDP (%)	12.64	10.39	17.26
Calibration			
Frequency of Permanent Income shocks λ_P	0.197	0.300	0.100
Panels (c) and (d) Calibration			
Borrowing Limit in US$ Thousands (assets)	17.68 (2.96)	19.68 (3.3)	13.78 (2.3)

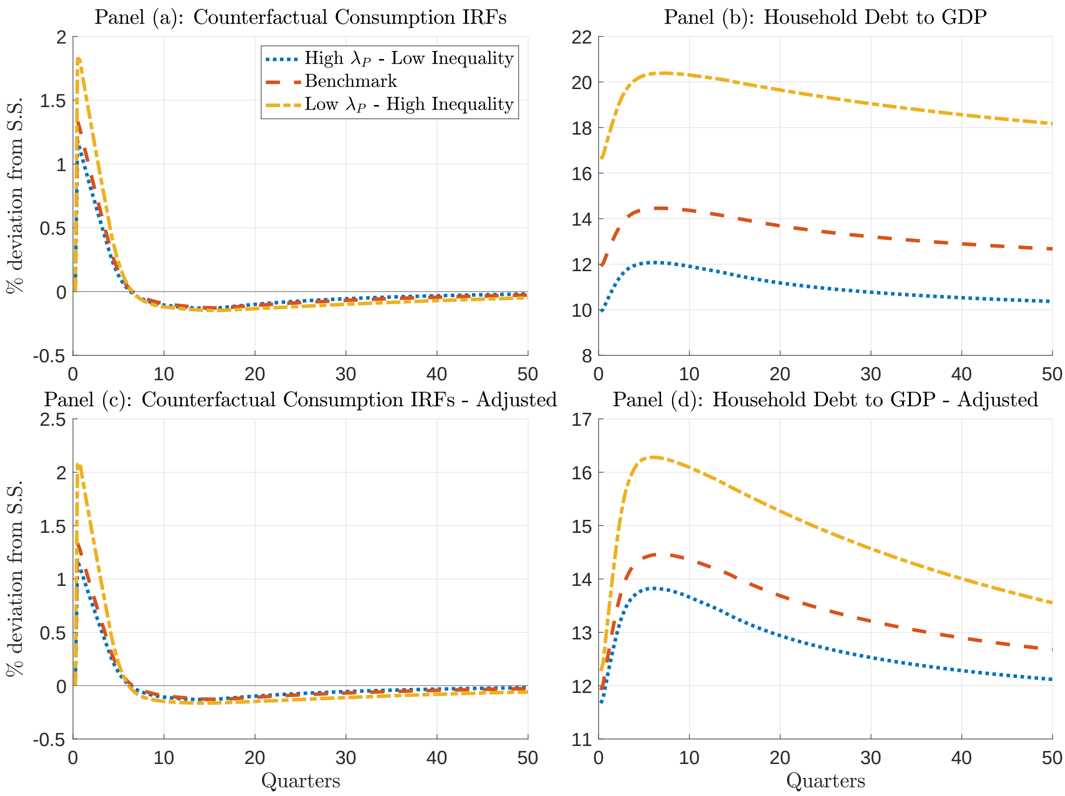
Note: Counterfactual simulations are done for the US economy. The mapping assets to Thousand US$ is done by the steady-state US calibration, yielding a ratio of 5.96 Thousand US$ for each asset. In the Panel (c) and (d) calibration, we keep the alternative fiscal policy calibration but change the borrowing limit so as to match the initial Household Debt to GDP of the Benchmark economy.

as a lower share of Hand-to-Mouth agents.

We set alternative arrival rates $\lambda_P = 0.1$ and $\lambda_P = 0.3$, targeting wealth gini 0.80 and 0.78, respectively. It follows that with less income risk, there are less precautionary savings and a higher share of HtM households. This increases the Average Quarterly MPC out of 500$, increase the Average MPC. On the other hand, with more income risk, there are more precautionary savings and less HtM households, with everything following conversely to the $\lambda_P = 0.1$ case. The moments of the counterfactual economies are shown in Table 4.2. Panels (a) and (b) of Figure 4.6 show and benchmark responses for the aggregate consumption IRFs and household debt, respectively.

In the counterfactual economy with higher wealth inequality (lower λ_P), consumption "Boom & Bust" cycles are wider than the benchmark economy, while the counterfactual economy with lower wealth inequality has smaller "Boom & Bust" cycles.

Figure 4.6: Credit Deepening with More and Less Idiosyncratic Income Risk



Note: On the Panel (a), we plot the consumption IRFs to the same MIT-Shock to the borrowing constraint, but with different levels of wealth inequality induced by distinct idiosyncratic income risk λ_P calibration; whereas on Panel (b) we plot the corresponding Debt to GDP IRFs. In Panels (c) and (d), we do the same exercise but control for the initial level of household debt induced by changing idiosyncratic income risk calibration.

Just as the fiscal policy counterfactual experiment, the expansions in Panel (a) are accompanied by higher household leverage in economies with higher wealth inequality, and they depart from different wealth levels as depicted in Panel (b). We adjust for the initial borrowing constraint the counterfactual calibrations, as seen in the last line of Table 4.2, such that these economies depart from the same level of initial household debt. All results remain, with wealth inequality causing the amplification of consumption cycles following a credit deepening - which can be seen in Panel (c) of Figure 4.6.

5
Empirical Evidence

The aim of this section is two-fold: Firstly, to examine whether the model's observed short and long-term patterns of aggregate consumption in response to credit expansions align with those seen in the data; specifically, whether there are short-term consumption "booms" followed by long-term "busts" and gradual deleveraging, and secondly, to explore whether these cycles are influenced by wealth inequality, mirroring the model's findings. We want to be clear that this analysis does not seek to establish causal relationships but rather to document the association between credit expansions and consumption, considering varying levels of inequality.

5.1
Data and Summary Statistics

To address these questions, we construct a country-level unbalanced panel dataset spanning 82 countries from 1961 to 2021. The dataset includes information on household and nonfinancial corporate credit to GDP, income and wealth Gini coefficients, Top 10^{th} percentile share of wealth and income, and national accounts data. The data is annual, with an average coverage of 25.5 years per country. Additionally, we compute 1 and 3-year differences for these variables, measuring changes in household and firm credit, and consumption from year $t-k$ to year t as $\Delta_k d_{it}^{HH}, \Delta_k d_{it}^{F}, \Delta_k c_{it}$. Summary statistics of the sample are provided in Table 5.1.

Data on household and nonfinancial corporate debt is sourced from the IMF global debt database (MBAYE; BADIA; CHAE, 2018), which aggregates data from national statistical sources. This database offers broad coverage of credit instruments, and separates credit via sector (Households and Non-financial Firms). Credit instruments covered are debt securities (mostly bonds), bank credit (mostly loans) and cross-border bank loans from the Bank for International Settlements. For inequality measures, we use income gini from the SWIID database (SOLT, 2019), which standardizes the World Institute for Development Economics Research (WIDER) data and other sources while minimizing reliance on problematic assumptions by using as much information as possible from proximate years within the same country. Wealth Gini coefficients and top 10 percentile income and wealth shares are sourced from the World Inequality Database (CHANCEL et al., 2021), which relies upon data sources on national accounts, survey data, fiscal data, and wealth rankings. Moreover, for aggregate consumption, we rely on the World Bank's World Development Indicators (WDI) database (BANK, 2016), using deflated time-series.

Table 5.1: Summary Statistics

	Description	Source	N	Mean	St. Dev.	Min	Median	Max
d^{HH}	Household Credit to GDP	IMF Debt Db.	2,185	39.214	29.916	0.183	33.651	137.939
d^{F}	Firm Credit to GDP	IMF Debt Db.	2,185	69.796	52.231	1.094	63.366	566.649
g^{Inc}	Income Gini	SWIID	1,981	0.338	0.078	0.203	0.321	0.635
g^{W}	Wealth Gini	WID	1,816	0.760	0.066	0.577	0.746	1.002
$t10^{w}$	Top 10 Wealth Share	WID	1,841	0.611	0.079	0.420	0.593	0.891
$t10^{Inc}$	Top 10 Income Share	WID	2,049	0.399	0.098	0.228	0.366	0.671
c	Log Real Consumption	World Bank	2,185	2,727.3	281.3	2,101.3	2,722.7	3,634.2
Δc	1yr growth Consumption	World Bank	2,163	3.026	5.092	−61.485	3.073	50.389
Δd^{HH}	1yr diff. HH Credit GDP	IMF Debt Db.	2,117	0.920	2.791	−24.619	0.664	31.203
Δd^{F}	1yr diff. Firm Credit GDP	IMF Debt Db.	2,118	1.030	9.124	−162.367	0.642	128.294
$\Delta_3 c$	3yr growth Cons.	World Bank	2,115	8.851	9.550	−57.178	8.712	83.254
$\Delta_3 d^{HH}$	3yr diff. HH Credit	IMF Debt Db.	1,983	2.815	6.431	−42.335	2.191	34.463
$\Delta_3 d^{F}$	3yr diff. Firm Credit	IMF Debt Db.	1,986	3.212	19.741	−281.134	2.157	292.198

Note: Log changes and ratios are multiplied by 100 to report changes in percentages or percentage points, Δ and Δ_3 denote one-year and three-year changes respectively.

5.2 Evidence on Consumption Boom & Bust Cycles

We first document the relationship between household credit and aggregate consumption without focusing yet on inequality. To do such, we employ two initial methods to assess the full dynamic relationship, to then anaylize their Impulse Response Functions (IRFs).

5.2.1 VAR

We estimate a Vector Autoregression (VAR) with variables in levels $\mathbf{Y_{it}} = [\mathbf{c_{it}}, \mathbf{d^{HH}_{it}}, \mathbf{d^{F}_{it}}]'$, where c_{it} is the log real aggregate consumption, d^{HH}_{it} is the household credit to GDP measure and d^{F}_{it} nonfinancial firm credit to GDP. The inclusion of nonfinancial firm credit is to control for the negative role of this source of sectoral credit on aggregate consumption, as documented by (MIAN; SUFI; VERNER, 2017) in the case of output growth. The Structural VAR in levels with country fixed-effects is determined by:

$$\mathbf{BY_{it}} = \mu_i + \mathbf{\Psi(L)Y_{it}} + \varepsilon_{it} \tag{5-1}$$

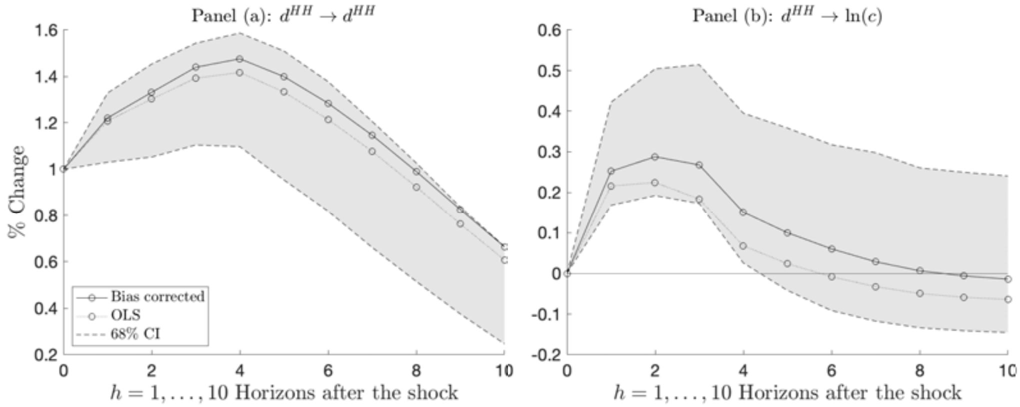
where μ_i is the vector of country fixed effects and ε_{it} is the $n \times 1$ vector of structural shocks with $\mathbb{E}[\varepsilon_{it}\varepsilon'_{it}] = \mathbf{I}$ and $\mathbb{E}[\varepsilon_t\varepsilon'_s] = 0$ for all $s \neq t$. The lag polynomial $\mathbf{\Psi}(\mathbf{L})$ is set to the fifth order, $p = 5$, based on the Akaike criterium - as in (MIAN; SUFI; VERNER, 2017).

We estimate the VAR (5-1) in reduced form as follows:

$$\mathbf{Y_{it}} = \mathbf{a_i} + \mathbf{\Gamma}(\mathbf{L})\mathbf{Y_{it}} + \mathbf{u_{it}} \tag{5-2}$$

where $\mathbf{a_i} = \mathbf{B^{-1}}\mu_i$ and $\mathbf{\Gamma}(\mathbf{L}) = \mathbf{B^{-1}}\mathbf{\Psi}(\mathbf{L})$ are the reduced-form coefficients, and $\mathbf{u_{it}} = \mathbf{B^{-1}}\varepsilon_{it}$ the vector of reduced-form shocks with the covariance matrix $\mathbb{E}[\mathbf{u_{it}}\mathbf{u'_{it}}] = \mathbf{BB'} = \mathbf{\Omega}$. By definition, $\mathbf{B^{-1}}$ is lower triangular by the usual cholesky identification hypothesis. The identification follows (MIAN; SUFI; VERNER, 2017): log real aggregate consumption goes first, followed by nonfinancial firm credit to GDP and household credit to GDP last. On the estimation process of (5-2), we employ an iterative bootstrap procedure to correct for potential Nickell-bias due to inclusion of country fixed-effects μ_i.

Figure 5.1: Impulse Response Functions from the Recursive VAR (5-2) for Real Consumption, Household and Nonfinancial Firm Credit



Note: On the Panel (a), we plot the response of the household credit response to a household credit shock; whereas on Panel (b), we compute the response of log real consumption to a household credit shock. The shocks are identified using a Cholesky decomposition with the ordering $\mathbf{Y_{it}} = [\mathbf{c_{it}}, \mathbf{d^{HH}_{it}}, \mathbf{d^{F}_{it}}]'$, and the VAR is estimated with country fixed effects and corrected for Nickell bias using an iterative bootstrap method. The dashed lines represent 68% confidence intervals that account for contemporaneous cross-country residual correlation are generated via re-sampling cross-sections of residuals using wild bootstrap

In Figure 5.1, we present the Impulse Responses of the estimated VAR (5-2). Panel (a) illustrates the trajectory of household credit after the household credit shock, revealing two key aspects: the strength of the shock, peaking after 4 years and its high persistence, taking around 8 years to revert to its initial level. In Panel (b), we observe the response of aggregate consumption to this positive shock to household credit. Initially, the shock induces a short-lived consumption boom, which starts declining even before household credit does.

Furthermore, the slowdown in aggregate consumption persists, with consumption returning to its previous levels 6-8 years later. Importantly, the long-term effect of the short-lived boom is a level of consumption lower than its starting point.

5.2.2
Local Projections

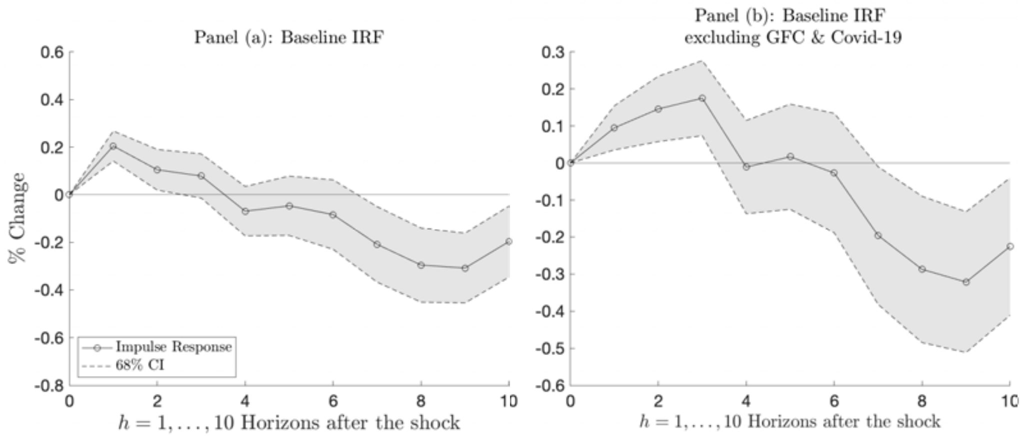
We test the robustness of the dynamic relationship using (JORDà, 2005)'s Local Projections (LPs). According to (JORDà, 2023), LPs offer robustness to misspecification, direct inference of the estimated impulse response function, and flexibility for including control variables and nonlinearities, which is crucial for our study. However, as argued by (LI; PLAGBORG-MöLLER; WOLF, 2021), they come with higher variance compared to their counterpart, the finite-order VAR. The local projection impulse responses to household credit shocks are given by the sequence of coefficients $\{\hat{\beta}^h_{HH,1}\}^H_{h=1}$, estimated from the following specification:

$$c_{it+h-1} = \alpha_i^h + \mu_t^h + \sum_{j=1}^{p} \beta^h_{HH,j} d^{HH}_{it-j} + \sum_{j=1}^{p} X_{it-j}\Gamma^h_j + \varepsilon^h_{it+h-1} \tag{5-3}$$

where α_i^h is to control for country fixed-effects and μ_t^h is a time trend to control for the expansion of private credit over the past four decades or the gradual decline GDP growth in developed economies over the same period (MIAN; SUFI; VERNER, 2017). Moreover, we include a vector of controls which includes nonfinancial firm credit d^F_{it-j} and consumption c_{it-j}, as the former is to control for highly persistent consumption growth while the latter controls for the negative bias of firm credit on consumption. The choice of lags $p = 5$ is based on (OLEA; PLAGBORG-MøLLER, 2021), which display evidence that IRFs estimated from lag-augmented LPs are robust to persistent data, even when accounting for long projection horizons. Concomitantly, the choice of horizon $H = 10$ years is to study the full short and long-run dynamics of credit expansions. Finally, we report dually-clustered standard errors in country and year dimensions to control for heteroskedacity and autocorrelation.

Panel (a) of Figure 5.2 shows the baseline results for the estimation of the aggregate consumption IRF. The dynamics of the response are qualitative and quantitatively similar to those of the estimated VAR (5-2), indicating that aggregate consumption peaks 2 years after the shock and is followed by a decline below its pre-shock levels. However, there are two main differences

Figure 5.2: Impulse Response Functions from the Local Projection (5-3) for Real Consumption, Household and Nonfinancial Firm Credit



Note: Impulse response functions (IRFs) from (JORDà, 2005) estimated in levels. The specification is given by $c_{it+h-1} = \alpha_i^h + \mu_t^h + \sum_{j=1}^{p} \beta_{HH,j}^h d_{it-j}^{HH} + \sum_{j=1}^{p} X_{it-j}\Gamma_j^h + \varepsilon_{it+h-1}^h$ for horizons $h = 1, \ldots, 10$. Panel (a) stands for the baseline regression, while Panel (b) corresponds to the same specification but removing the Great Financial Crisis and Covid-19. Shaded regions correspond for 68% confidence intervals computed using dually clustered standard errors (country and year)

with respect to the VAR (5-2): (i) a quicker consumption slowdown, with the initial consumption boom fading from 4-6 years after the shock in comparison to the VAR's evidence of 6-8 years; (ii) A more intense "bust", with the LP (5-3) estimated response giving a consumption decrease up to 4 times as higher than that of the VAR.

In addition to the baseline estimates, we follow (MIAN; SUFI; VERNER, 2017) and perform robustness tests by excluding extreme events such as the Great Financial Crisis (GFC) and the Covid-19 recession. Excluding these events marginally alters the quantitative results but does not change the "Boom & Bust" pattern of the consumption response, nor the qualitative results regarding the length of the "Boom" period.

In both specifications, the increase in households credit predicts a short-run boom of three years in line with the model-predicted growth. Subsequently, as demonstrated in the model, consumption lowers below the initial household credit growth level four years after the shock, as household leverage reaches its peak. The household-deleveraging process generates a consumption "Bust" that is persistent, lasting up to 6 or more years.

5.3
The Role of Inequality

In this subsection, we empirically investigate a key result from the economic model in this article: Do economies with higher Income or Wealth

Inequality have their "Boom & Bust" cycles amplified following a credit deepening? Consistent with the model, we examine a nonlinear relationship between aggregate consumption, household credit, and the income and wealth distribution as a state variable. To model this relationship, we use linear state-dependent models, treating measures of income and wealth distribution inequality as exogenous state variables.

Assumption 1 (Exogenous state variable) *Define $\mathcal{F}^{t-1} = \{y_{t-1}, y_{t-2}, \ldots\}$ as the history of past endogenous variables $y_t = (c_t, d_t^{HH}, d_t^F)$, $\boldsymbol{Ineq}_t$ the exogenous state-variable and ν_t^{HH} the household credit structural shock. We assume that income and wealth inequality are an exogenous state variable:*

$$\mathbb{E}[\boldsymbol{Ineq}_{t+s}|\boldsymbol{Ineq}_{t-1}, \mathcal{F}^{t-1}, \nu_t^{HH}] = \mathbb{E}[\boldsymbol{Ineq}_{t+s}|\boldsymbol{Ineq}_{t-1}]$$

for all s, t. That is, income and wealth inequality ($\boldsymbol{Ineq}_t$) are unaffected by the household credit shock ν_t^{HH}

Under Assumption 1, (GONCALVES et al., 2023) ensures that state-dependent estimators may be able to recover populational estimates. We base our assumption of exogeneity of inequality to credit shocks on the response of inequality, shown in Figure 4.1, which is negligible under transitory or permanent shocks even when these have a big magnitude - as in the case of Section 4. Given Assumption 1, we employ single-equation estimations to capture the dynamic sensibility of the response of consumption to inequality; and then work with regime-switching models. We focus on the regime-switching local projection models as one does not have to take a stance on how the economy switches from one state to another (TENREYRO; THWAITES, 2016).

5.3.1 Single-Equation Estimations

We build on the evidence of Section 5.2 and use an alternative regression framework which captures in reduced-form the full dynamic relation between household debt and aggregate consumption growth, extending it to capture the sensibility of the "Boom & Bust" cycles to inequality. The specification is as follows:

$$\Delta_3 c_{it+k} = \alpha_i + \beta_{HH,k}\Delta_3 d_{it-1}^{HH} + \beta_{F,k}\Delta_3 d_{it-1}^{F} + \gamma_k(\mathbf{Ineq}_{it-4} \cdot \Delta_3 d_{it-1}^{HH}) + u_{it+k} \tag{5-4}$$

where we estimate (5-4) for different horizons, $k = -1, 0, \ldots, 5$. Therefore, we fix the right-hand side variables to be the change in household debt from 4 years ago to last year, and vary output growth trends from contemporaneous household debt growth from up to 5 years ahead. For instance, when $k = 2$, $(\beta_{HH,k} + \gamma_k \cdot \mathbf{Ineq}_{it-4})$ captures the effect of growth from four to one year ago into 3 years ahead - conditional on the current level of inequality.

Table 5.2 shows the results of the estimations of (5-4). Columns (1), (4) and (7) estimate (5-4) without the additional inequality term for periods $k = -1, 2, 5$, capturing the boom and bust cycle evidence explored in Section 5.2. Columns (2), (5) and (8) include the inequality interaction term for income, the normalized income gini g^{Inc}, and columns (3), (6) and (9) account for the inequality interaction for wealth, the normalized wealth gini g^W. Moreover, we add the estimates for the normalized Top10 Income and Wealth shares in the Table A.5 at the appendix.

Columns (1)-(3) depict a positive correlation between contemporaneous consumption and household credit growth, and in columns (4)-(9) the correlation between past household credit trend and future credit growth turns negative. This is in line with previous empirical and theoretical evidence, and statistically significant at least the 10% level when estimating without the inequality terms for almost all of the coefficients. Relative to Income inequality, a household credit shock is correlated with a contemporaneous increase in the positive response of consumption, a decrease in this response in three years ahead, but a decrease in the consumption below-trend growth up to 6 years ahead. On the other hand, conditional on wealth inequality, a household credit shock is correlated with a simultaneous decrease in the response of consumption which is maintained up to 3 years past of the shock, but decreases the consumption below-trend growth up to 6 years ahead.

With respect to alternate measures of income of income and wealth inequality displayed in Table A.5, conditional on wealth inequality, the relationship of inequality attenuating the depth of contemporaneous consumption growth and long-run (6 years ahead) below trend growth continues. This is accompanied by a amplification of the of the below-trend growth three years ahead of the shock. On the other hand, the coefficients of the interaction term with income inequality suggest that this source of inequality attenuates short-term growth and amplifies long-run below-trend consumption growth. These results contradict the role of wealth inequality in amplifying the consumption "boom & bust" cycles. However, for any of the periods $k = -1, 0, \ldots, 5$ and wealth inequality measures $\{g^W, Top10^W\}$, we cannot reject the hypothesis that any of these coefficients are zero.

Table 5.2: Single Equation Estimation (5-4) of Consumption Growth Trends to Household Credit Growth Accounting for Income and Wealth Inequality

	Dependent variable: $\Delta_3 c_{it+k}, k = -1, 0, \ldots, 5$								
	$\Delta_3 c_{it-1} = c_{it-1} - c_{it-4}$			$\Delta_3 c_{it+2} = c_{it+2} - c_{it-1}$			$\Delta_3 c_{it+5} = c_{it+5} - c_{it+2}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\Delta_3 d^{HH}_{it-1}$	0.252***	0.271***	0.257***	−0.062†	−0.061	−0.064	−0.128**	−0.143**	−0.143**
	(0.059)	(0.059)	(0.065)	(0.039)	(0.040)	(0.041)	(0.048)	(0.043)	(0.047)
$\Delta_3 d^{F}_{it-1}$	−0.010	−0.012	−0.008	−0.039***	−0.040***	−0.038***	−0.021**	−0.018*	−0.022**
	(0.014)	(0.014)	(0.013)	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)	(0.008)
$\Delta_3 d^{HH}_{it-1} \cdot g^{Inc}_{it-4}$		0.004			−0.021			0.008	
		(0.064)			(0.048)			(0.047)	
$\Delta_3 d^{HH}_{it-1} \cdot g^{W}_{it-4}$			−0.037			−0.007			0.038
			(0.048)			(0.027)			(0.036)
Country FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
R^2	0.036	0.048	0.038	0.015	0.017	0.015	0.015	0.019	0.020
Observations	1865	1794	1510	1822	1755	1450	1606	1558	1237

Note: The current table presents estimates for the specification $\Delta_3 c_{it+k} = \alpha_i + \beta_{HH,k}\Delta_3 d^{HH}_{it-1} + \beta_{F,k}\Delta_3 d^{F}_{it-1} + \gamma_k(\mathbf{Ineq}_{it-4} \cdot \Delta_3 d^{HH}_{it-1}) + u_{it+k}$, ran on an unbalanced panel for $k = -1, 2, 5$, each of which ran for normalized inequality variables $\{g^{Inc}, g^{W}, t10^{Inc}, t10^{W}\}$. All specifications include country fixed effects. Reported R^2 values are from within-country variation. Standard errors in parentheses are dually clustered on country and year. The symbols ***, **, *, † indicate significance at 0.1%,1%,5%,10% levels respectively.

5.3.2 Smooth-Transition Between High and Low Inequality Regimes

Another approach common on the literature of state-dependence of the effects of macroeconomic shocks is to work with regime-dependent effects. For instance, (AUERBACH; GORODNICHENKO, 2012) and (AUERBACH; GORODNICHENKO, 2013) focus on the effects of fiscal multipliers in expansions and recessions, while (TENREYRO; THWAITES, 2016) on the effects of monetary policy. Although we are interested in the interaction term to study the amplification of inequality, we can still study the effects of inequality on the consumption credit relationship by defining two regimes: one of lower and another of higher inequality

We then proceed to use the Smooth-Transition Local Projections of (AUERBACH; GORODNICHENKO, 2013) and (TENREYRO; THWAITES, 2016). We opt for this method instead of other markov-switching models to exploit the variation in the degree of being on a particular regime, thus implying that our estimates are based on a larger set of observations. We adapt the specification of (5-3) to account for the smooth transition term:

$$\begin{aligned} c_{it+h-1} = \alpha_i^h + \mu_{t-1}^h + F(\mathbf{Ineq_{it-1}}) \left(\sum_{j=1}^{p} \beta_{HH,j}^{h,-} d_{it-j}^{HH} + \sum_{j=1}^{p} X_{it-j} \Gamma_j^{h,-} \right) \\ + (1 - F(\mathbf{Ineq_{it-1}})) \left(\sum_{j=1}^{p} \beta_{HH,j}^{h,+} d_{it-j}^{HH} + \sum_{j=1}^{p} X_{it-j} \Gamma_j^{h,+} \right) + \varepsilon_{it+h-1}^h \end{aligned} \tag{5-5}$$

where the sequence of coefficients $\left\{\beta_{HH,1}^{h,+}\right\}_{h=1}^{H}$ and $\left\{\beta_{HH,1}^{h,-}\right\}_{h=1}^{H}$ are the High and Low Inequality regime IRFs, respectively. Moreover, μ_t^h accounts for the time trend and we construct the transition function following (AUERBACH; GORODNICHENKO, 2013):

$$F(\mathbf{Ineq_{it-1}}) = \frac{\exp(-\gamma \mathbf{Ineq_{it-1}})}{1 + \exp(-\gamma \mathbf{Ineq_{it-1}})}, \quad \gamma > 0 \tag{5-6}$$

where $\mathbf{Ineq_{it-1}}$ is a transformation of the inequality measures $\{g^W, g^{Inc}, Top10^W, Top10^{Inc}\}$, so as to have zero mean and unit variance[1]. Moreover, we use the Hodrick-Prescott filter[2]on $\mathbf{Ineq_{it-1}}$ and calibrate γ so that half of the observations in the panel are defined as low or a high inequality economy (that is, $\mathbf{Pr}(F(\mathbf{Ineq}) \geq 0.8) = 0.5$). We follow the literature on Smooth-Transition Local Projections and define the low inequality regime if $F(\mathbf{Ineq}) \geq 0.8$ (AUERBACH; GORODNICHENKO, 2012). The results of the

[1]This normalization of the inequality variable is so that the parameter γ is scale invariant
[2]For annual data, we set the smoothing parameter $\lambda = 100$

Table 5.3: Calibration of γ for each of the normalized inequality variables

	Variable	Calibration
g^{Inc}	Income Gini	6.35
g^{W}	Wealth Gini	5.95
$Top10^{Inc}$	Top 10 Income Share	4.25
$Top10^{W}$	Top 10 Wealth Share	5.60

Note: For the logistic calibration function and probability density function of each variables, refer to Figure 16

regime-dependent IRFs estimated in (5-5) are displayed in Figure 5.3, while the Probability density functions and their respective calibrated transition functions are depicted in Figure A.10 in appendix, and the calibrated γ values are shown in Table 5.3

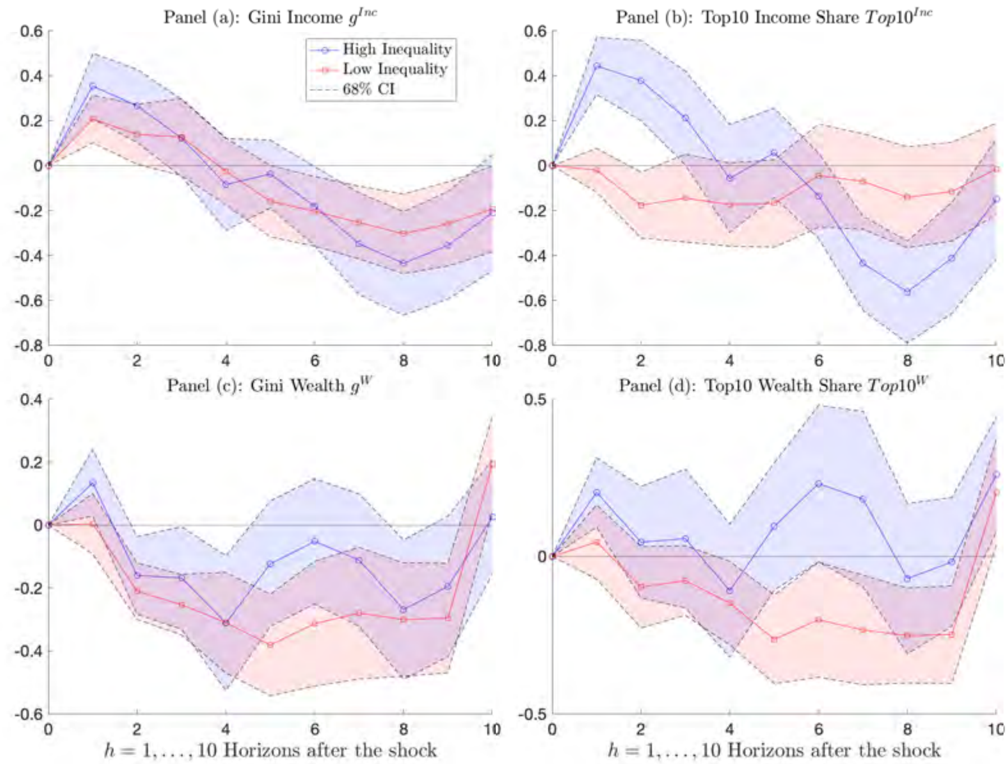
When separating observations into high and low income inequality regimes, the state-dependent IRFs suggests an amplification of boom & bust cycle for the high inequality relative to the low. This follows as in the short-run, the high inequality regime response is greater, and in the long-run consumption falls much below than that of the low inequality regime. On Panel (a), where the state variable is the normalized income gini, this pattern is clear and as depicted in the counterfactual exercises of the model. On Panel (b), where the state variable is the normalized Top10 income share, there is amplification of responses but the estimated low inequality regime IRF fails to account for the predicted “boom” and “bust” cycle.

On the other hand, as shown in Panels (c) and (d), results are similar whether we separate regimes by Wealth Gini or Top10 Wealth shares, but the regime separation fails to account for the predicted “boom” and “bust” cycle. The regime separation indicates that economies with higher wealth inequality have more growth in the short run, and less consumption recession in the long-run. Some potential flaws arise from this exercise, as the regime-switching potentially separates countries or periods of low inequality instead of effective within-country regimes. By clustering regimes by a group of country, these results may be correlated with unobservables, such as financial market development, that drive the difference in the response of consumption. Moreover, we cannot reject the hypothesis that the state-dependent estimates are not different in several cases.

5.3.3 State-Dependent Local Projections

We adapt the approach of the smooth-transition LPs on equation (5-5) to capture the desired sensibility term in equation (5-4) instead of switching

Figure 5.3: State Dependent Impulse Response Functions of the Response of Consumption to Household Credit shock conditional on Inequality regime



Note: State-dependent IRFs estimated for horizons $h = 1, \ldots, 10$ by the smooth-transition local projection (TENREYRO; THWAITES, 2016) with specification (5-5). We calibrate the transition function (5-6) such that for each inequality variable, $\mathbf{Pr}(F(\mathbf{Ineq}) \geq 0.8) = 0.5$. Panel (a) and (b) stands for IRFs with regime conditional on gini income and wealth, respectively. Similarly, Panel (c) and (d) stand for IRFs with regime conditional on $Top10^{Inc}$ and $Top10^{W}$. Shaded regions correspond for 68% confidence intervals computed using dually clustered standard errors

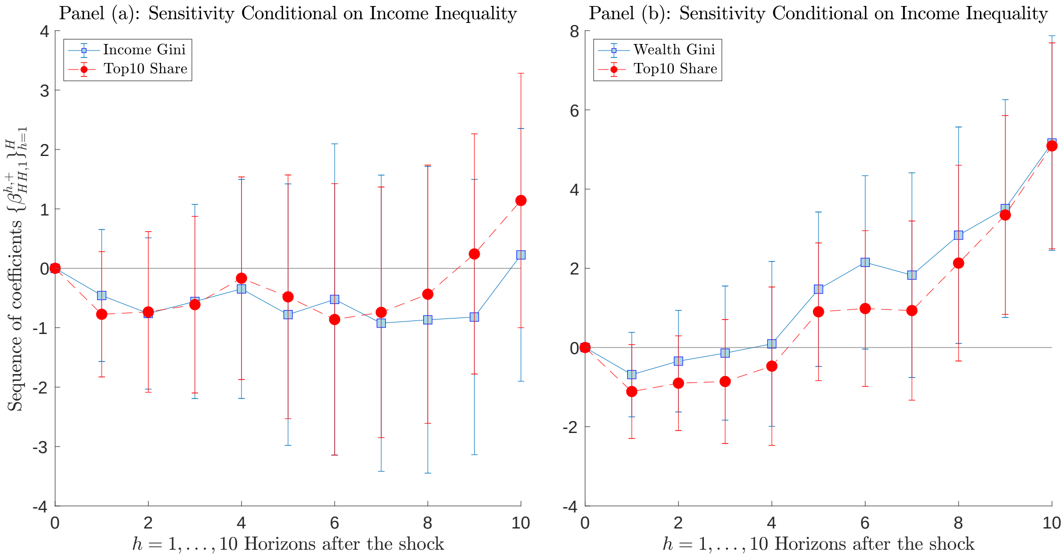
between two regimes of high and low inequality. This approach is done by taking the identity transition function $F(\mathbf{Ineq}) = \mathbf{Ineq}$ and exploiting that $\mathbf{Ineq}$ is already between 0 and 1. This delivers a "continuum" of regimes in which, conditional on a specific value of inequality, $(\beta^{h}_{HH,1} + \beta^{h,+}_{HH,1}\mathbf{Ineq}_{it-1})$ gives the response of aggregate consumption to a credit shock h years after it occurred.

We use the specification in equation (5-5), but with the new transition function, yielding the following state-dependent LP:

$$\begin{aligned} c_{it+h-1} = \alpha^{h}_{i} + \mu^{h}_{t-1} + \sum_{j=1}^{p} \beta^{h}_{HH,j} d^{HH}_{it-j} + \sum_{j=1}^{p} \beta^{h,+}_{HH,j} (\mathbf{Ineq}_{it-1} \cdot d^{HH}_{it-j}) \ldots \\ \ldots + \sum_{j=1}^{p} X_{it-j}\Gamma^{h}_{j} + \sum_{j=1}^{p} (\mathbf{Ineq}_{it-1} \cdot X_{it-j})\Gamma^{h}_{j,+} + \varepsilon^{h}_{it+h-1} \end{aligned} \quad (5\text{-}7)$$

which we proceed likewise as before, estimating (5-7) for $h = 1, \ldots, 10$ periods ahead. We plot on Figure 5.4 the sequence of coefficients $\{\beta^{h,+}_{HH,1}\}_{h=1,\ldots,H}$ to gauge for the sensibility of the response of aggregate consumption to a credit shock h years after it occurred, with the error bars corresponding to the confidence intervals of the estimates.

Figure 5.4: Sensitivity of the Response of Consumption to Credit Conditional on Income or Wealth Inequality



Note: Sequence of coefficients $\{\beta^{h,+}_{HH,1}\}_{h=1,\ldots,H}$ of State-Dependent Local Projections of equation (5-7) estimated for horizons $h = 1, \ldots, 10$ after the shock. Panel (a) refers to the sensibility coefficients estimates for income inequality variables; whereas Panel (b) refers to the sensibility coefficients estimates for wealth inequality variables. Error bars correspond for 95% confidence intervals computer using dually-clustered standard errors

The sequence of sensitivity coefficients $\{\beta^{h,+}_{HH,1}\}_{h=1,\ldots,H}$ with respect to income inequality, shown in Panel (a), suggest that the more income is unequal in the country, the correlation of a household credit shock over consumption growth will be dampened during the initial "boom" phase from 3 to 4 periods ahead, and will be amplified during the "bust" from 5 to 8 periods ahead of the shock. With respect to wealth inequality, depicted in Panel (b), effects are similar to the case of income inequality in the period from 3 to 4 periods ahead of the shock. However, from the 5th period after the shock onward, peers with higher wealth inequality experience a less intense correlation of household credit growth with consumption downturn.

The hypothesis that the correlation of household credit shock with concomitant and future growth consumption is conditional on inequality cannot be rejected at the 5% for almost all of the estimated coefficients, regardless of the inequality variable. Therefore, we cannot find any evidence that goes in favor or against the model.

6
Conclusion

In this paper, we study how inequality affects the response of consumption to a credit deepening. To do such, we relied on an incomplete-market *Heterogeneous Agents* model, relying on preference and idiosyncratic income risk to generate income and wealth inequality. From this framework, we have derived analytic results mapping the household response to the credit deepening in partial and general equilibrium. This characterization is novel in the literature, and explains the heterogeneity of responses to credit across the wealth distribution, as the poor have sizeable Credit MPCs and adjust consumption with respect to credit, while wealthy households react to prices.

Building on the intuition of this result, we estimated the model with high-quality micro and macro data and simulated the aggregate response of consumption to a credit deepening, showing by decomposing the channels of credit to consumption and by wealth percentiles that all the results at the household level also hold for aggregate consumption. Moreover, through counterfactual credit deepening exercises were we alter marginally the level of wealth inequality, we corroborate on the idea that wealth inequality amplifies the aggregate consumption "Boom & Bust" cycles following a credit deepening.

Based on evidence provided by the model, investigated empirically whether consumption "Boom & Bust" cycles occur after a household credit shock; and if there is a nonlinear effect of amplification of these cycles in economies with higher income or wealth inequality. With respect to the former, we find robust evidence of consumption growth in the short-term horizon (up to 3-4 years after the shock), followed by a persistent decrease (of at least 6 years) in consumption below its initial level thereafter. However, when investigating the presence of state-dependent amplification of shocks, we find weak and inconclusive evidence.

7
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A
Appendix

A.1
Proofs of Propositions

A.1.1
Proof of Proposition 1

Proof. We write the proof for a general consumption-savings problem in partial equilibrium, to then adapt specifically to the model at hand. The agent has stochastic income y_t dependent on the state, θ_t, which follows any arbitrary process dictated by an infinitesimal generator $\mathcal{A}$. We write its problem as:

$$\begin{aligned} \max_{\{c_t, a_t\}_{s\geq 0}} \mathbb{E}_t &\left[\int_t^{\infty} e^{-\rho s} u(c_s) ds\right] \\ \text{s.t. } \dot{a}_t &= y_t(\theta_t) + r_t a_t - c_t \\ a_t &\geq -\underline{a} \end{aligned}$$

The proof relies on showing that any path for consumption that is feasible at $(a, y, \theta, t; \underline{a})$, is also feasible at $(a - \varepsilon, y + r_t\varepsilon, \theta, t; \underline{a} + \varepsilon)$. To do such, we show equivalence of consumption-saving decisions given the budget and borrowing constraints. For any $\varepsilon > 0$:

$$\begin{aligned} \dot{a}_t &= y_t(\theta_t) + r_t a_t - c_t \\ &= y_t(\theta_t) + r_t(a_t - \varepsilon) + r_t\varepsilon - c_t \\ &= (y_t(\theta_t) + r_t\varepsilon) + r_t(a_t - \varepsilon) - c_t \end{aligned} \tag{A-1}$$

Moreover, this infinitesimal change will also satisfy the borrowing constraint:

$$a_t \geq -\underline{a} \Leftrightarrow a_t - \varepsilon \geq -(\underline{a} + \varepsilon) \tag{A-2}$$

Therefore, via A-1 and A-2, the consumption policy function that satisfies $(a, y, \theta, t; \underline{a})$, must also satisfy $(a - \varepsilon, y + r_t\varepsilon, \theta, t; \underline{a} + \varepsilon)$, which implies:

$$c(a, y, \theta, t; \underline{a}) = c(a - \varepsilon, y + r_t\varepsilon, \theta, t; \underline{a} + \varepsilon) \tag{A-3}$$

Since this is valid $\varepsilon > 0$, we differentiate A-3 with respect to ε to obtain:

$$\frac{\partial c}{\partial \underline{a}}(a, y, \theta, t; \underline{a}) = \frac{\partial c}{\partial a}(a, y, \theta, t; \underline{a}) - r_t \frac{\partial c}{\partial y}(a, y, \theta, t; \underline{a}) \tag{A-4}$$

In the case of out model, we have that $\theta_t = z_t$ and $y_t = (1-\tau_t)w_t z_t + T_t$. Therefore, infinitesimal changes in y that do not come from $\log z_t$ come from a lump-sum transfer, adapting the result to the formula in 1:

$$\frac{\partial c_t}{\partial \underline{a}}(a,z;\underline{a}) = \frac{\partial c_t}{\partial a}(a,z;\underline{a}) - r_t \frac{\partial c_t}{\partial T}(a,z;\underline{a})$$

■

A.1.2
Proof of Proposition 2

Proof. We begin with the general consumption-savings problem in partial equilibrium, where y_t is a stochastic variable with follows a markov process dictated by the infinitesimal generator $\mathcal{A}$:

$$\begin{aligned} \max_{\{c_t,a_t\}_{t\geq 0}} \mathbb{E}_0 & \left[\int_0^\infty e^{-\rho t} u(c_t) dt\right] \\ \text{s.t. } \dot{a}_t &= y_t + r_t a_t - c_t + T_t \\ a_t &\geq -\underline{a}_t \end{aligned}$$

The first step comprises of making a change of variables, casting the problem in an alternative state space (b,y). Define the variable $b_t \equiv a_t - (-\underline{a}_t) = a_t + \underline{a}_t$ which is equivalent to the quantity of liquid assets with respect to the borrowing constraint, or the distance to the borrowing constraint. In this state space, the optimization constraints are given by

$$\begin{aligned} \text{Budget Constraint : } \dot{b}_t &= y_t + r_t(a_t - \underline{a}_t + \underline{a}_t) - c_t + T_t + \dot{\underline{a}}_t \\ &= y_t + r_t b_t - c_t + T_t - r_t \underline{a}_t + \dot{\underline{a}}_t \\ \text{Borrowing Constraint : } b_t &\geq 0 \end{aligned}$$

From this sequential problem we derive the standard HJB equation in the (b,y) space with value function $\tilde{V}(b,y,t)$

$$\rho \tilde{V}(b,y,t) = \max_{c\geq 0} \left\{u(c) + \partial_b \tilde{V} \tilde{s}(y,b,t) + \mathcal{A}_t \tilde{V}(b,y,t) + \partial_t \tilde{V}(b,y,t)\right\} \quad \text{(A-5)}$$

with state constraints $\partial_b \tilde{V}(b,y,t) \geq u'(y + r\underline{a}_t)$. Differentiating the HJB and using the FOC:

$$(\rho - r_t)\tilde{V}_b(b,y,t) = \partial_b\tilde{V}_b(b,y,t)\tilde{s}(b,y,t) + \mathcal{A}_t\tilde{V}_b(b,y,t) + \partial_t\tilde{V}_b(b,y,t) \quad \text{(A-6)}$$

Now consider a perturbation on the path of borrowing constraints $\{\mathbf{d\underline{a}_s}\}_{\mathbf{s=t}}^{\mathbf{T}}$ with $T > t$. Differentiating at an arbitrary point of the state space (b, y):

$$\begin{aligned}(\rho - r_t)d\tilde{V}_b(b,y,t) &= \partial_b d\tilde{V}_b(b,y,t)\tilde{s}(b,y,t) + \mathcal{A}_t d\tilde{V}_b(b,y,t) + \partial_t d\tilde{V}_b(b,y,t)\\ &+ \partial_b\tilde{V}_b(-r_t d\underline{a}_t - dc)\end{aligned} \quad \text{(A-7)}$$

Using that, by the FOC, $\partial_b\tilde{V}_b dc = \partial_b\tilde{c}d\tilde{V}_b$, on equation (A-7), we have that:

$$\begin{aligned}(\rho - r_t + \partial_b\tilde{c})d\tilde{V}_b(b,y,t) &= \partial_b d\tilde{V}_b(b,y,t)\tilde{s}(b,y,t) + \mathcal{A}_t d\tilde{V}_b(b,y,t) + \partial_t d\tilde{V}_b(b,y,t)\\ &+ \partial_b\tilde{V}_b(-r_t d\underline{a}_t)\end{aligned} \quad \text{(A-8)}$$

It follows that, using the Feynman-Kac theorem on (A-8):

$$d\tilde{V}_b(b,y,t) = \mathbb{E}_t\left[\int_t^\tau e^{-\int_t^s(\rho - r_u + \partial_b\tilde{c})du}\{\partial_b\tilde{V}_b(-r_s d\underline{a}_s)\}ds\right] \quad \text{(A-9)}$$

where τ is the stopping time where the agent hits the borrowing constraint in the (b, y) space, which is $b = 0$. We follow on by using equation (A-9) to find the consumption differential. To do such, first consider the auxiliary random variable $N_t^s\tilde{V}_{bb}(b_s, y_y, s)$ where N_t^s is a predictable process defined as:

$$N_t^s = e^{-\int_t^s(\rho - 2r_u + \partial_b\tilde{c})du}\int_t^s e^{-\int_t^u r_z dz}(-r_u d\underline{a}_u)du$$

we apply Dynkin's formula between t and $T \wedge \tau$ for any arbitrary T

$$\mathbb{E}_t[N_t^{T\wedge\tau}\tilde{V}_{bb}] = \underbrace{N_t^t\tilde{V}_{bb}}_{=0} + \mathbb{E}_t\left[\int_t^{T\wedge\tau}\mathcal{L}N_t^s\tilde{V}_{bb}ds\right] \quad \text{(A-10)}$$

where $\mathcal{L}$ is the infinitesimal generator associated with $N_t^s\tilde{V}_{bb}(b_s, y_y, s)$. Developing the term inside the expression (A-10):

$$
\begin{aligned}
\mathcal{L}N_t^s\tilde{V}_b &= d(N_t^s)\tilde{V}_b + d(\tilde{V}_b)N_t^s \\
&= \left[(\rho - 2r_s + \partial_b\tilde{c})N_t^s + e^{-\int_t^s(\rho - r_u + \partial_b\tilde{c})du}(-r_s d\underline{a}_s)\right]\tilde{V}_{bb} + N_t^s d\tilde{V}_b \\
&= e^{-\int_t^s(\rho - r_u + \partial_b\tilde{c})du}(-r_s d\underline{a}_s)\tilde{V}_{bb} + N_t^s(d\tilde{V}_{bb} - (\rho - 2r_s + \partial_b\tilde{c})\tilde{V}_{bb}) \\
&= e^{-\int_t^s(\rho - r_u + \partial_b\tilde{c})du}(-r_s d\underline{a}_s)\tilde{V}_{bb} + N_t^s\underbrace{(\partial_b\tilde{V}_{bb} + \mathcal{A}_s\tilde{V}_{bb} + \partial_t\tilde{V}_{bb} - (\rho - 2r_s + \partial_b\tilde{c})\tilde{V}_{bb})}_{=0}
\end{aligned}
$$

such that the expression (A-10) becomes:

$$
\mathbb{E}_t\left[N_t^{T\wedge\tau}\tilde{V}_{bb}\right] = \mathbb{E}_t\left[\int_t^{T\wedge\tau} e^{-\int_t^s(\rho - r_u + \partial_b\tilde{c})du}\left\{\partial_b\tilde{V}_b(-r_s d\underline{a}_s)\right\}ds\right]
$$

Using equation (A-9), we obtain:

$$
\begin{aligned}
d\tilde{V}_b(b,y,t) &= \mathbb{E}_t\left[\int_t^{\tau} e^{-\int_t^s(\rho - r_u + \partial_b\tilde{c})du}\{\partial_b\tilde{V}_b(-r_s d\underline{a}_s)\}ds\right] \\
&= \mathbb{E}_t\left[e^{-\int_t^{T\wedge\tau}(\rho - 2r_u + \partial_b\tilde{c})du}\tilde{V}_{bb}\int_t^{T\wedge\tau} e^{-\int_t^s r_u du}(-r_s d\underline{a}_s)ds\right]
\end{aligned}
$$

Finally, we define the stochastic discount factor $M_t^{\tau\wedge T} = e^{-\int_t^{T\wedge\tau}(\rho - 2r_u + \partial_b\tilde{c})du}$ and use that $d\tilde{V}_b = u''(c)dc$ and $\tilde{V}_{bb} = (\partial_b\tilde{c})u''(c)$ to obtain:

$$
d\tilde{c}(b,y,t) = \partial_b\tilde{c}_t\mathbb{E}_t\left[M_t^{\tau\wedge T}\int_t^{T\wedge\tau} e^{-\int_t^s r_u du}(-r_s d\underline{a}_s)ds\right] \tag{A-11}
$$

The last step of the proof consists of changing the result (A-11) back to our original space (a, y). It follows that:

$$
\begin{aligned}
\partial_b\tilde{c}(b,y,t) &= \partial_a c(a,y,t) - r_t\partial_T c(a,y,t) \\
d\tilde{c}(b,y,t) &= dc(a,y,t) - (\partial_a c(a,y,t) - r_t\partial_T c(a,y,t))d\underline{a}_t
\end{aligned}
$$

where T is the lump sum transfer. This is intuitive, as tightening the borrowing constraint by one dollar in a given period is the same as forcing households to save one dollar in a separate account to then consume it next period. substituting these in (A-11):

$$dc(a,y,t) = (\partial_a c_t - r_t \partial_T c)\left(\mathbb{E}_t\left[M_t^{\tau\wedge T}\int_t^{T\wedge\tau} e^{-\int_t^s r_u du}(-r_s d\underline{a}_s)ds\right] + d\underline{a}_t\right) \tag{A-12}$$

■

A.1.3 Proof of Lemma 1

Proof. In partial equilibrium, $r_t = r \ \forall t$ such that the we simplify the stream of changes in the borrowing constraint as follows:

$$\begin{aligned}\int_t^{T\wedge\tau} \exp\left\{-\int_t^s r_u du\right\}(-r_s d\underline{a}_s)ds &= -r\int_t^{T\wedge\tau} e^{-(s-t)r} d\underline{a}_s \\ &= -r\int_t^{T\wedge\tau} e^{-(s-t)r}\nu(a' - a_s)ds \\ &= -r\nu\int_t^{T\wedge\tau} e^{-(s-t)r}(a' - a_s)ds\end{aligned}$$

by plugging it back on the partial equilibrium response of consumption in Proposition 2,

$$\begin{aligned}dc &= \partial_{\underline{a}} c\left(\mathbb{E}_t\left[\exp\left\{-\int_t^{T\wedge\tau}(\rho - 2r + \partial_a c)dt'\right\}(-r\nu)\int_t^{T\wedge\tau} e^{-(s-t)r}(a' - a_s)ds\right] + d\underline{a}_t\right) \\ &= \partial_{\underline{a}} c\left(-r\nu e^{-rt}\mathbb{E}_t\left[e^{-\int_t^{T\wedge\tau}\partial_a c_{t'}dt'}\int_t^{T\wedge\tau} e^{-(\rho-s)s}(\underline{a}' - a_s)ds\right] + d\underline{a}_t\right)\end{aligned}$$

■

A.1.4 Proof of Lemma 2

Proof. We start from the (A-6) in the proof of Proposition 2. We use Dynkin's formula on $e^{-\int_t^s(\rho - r_u + \partial_b\tilde{c})du}(\partial_a c)u''_s$ to show that the discount factor $M_t^{T\wedge\tau}$ makes the stopped process $(\partial_b c_{t\wedge\tau})u''_{t\wedge\tau}$ a martingale.

$$\begin{aligned}&\mathbb{E}_t\left[e^{-\int_t^{T\wedge\tau}(\rho - r_u + \partial_b\tilde{c})du}(\partial_b\tilde{c})u''_{T\wedge\tau}\right] - (\partial_b\tilde{c})u''(\tilde{c}) \\ =&\mathbb{E}_t\left[\int_t^{T\wedge\tau} e^{-\int_t^s(\rho - r_u)du}(\partial_b\tilde{V}_b(b,y,t)\tilde{s}(b,y,t) + \mathcal{A}_t\tilde{V}_b(b,y,t) + \partial_t\tilde{V}_b(b,y,t) - (\rho - r_s)\tilde{V}_b)ds\right] \\ =&0\end{aligned}$$

Therefore, it follows that:

$$(\partial_b \tilde{c})u''(\tilde{c}) = \mathbb{E}_t \left[e^{-\int_t^s (\rho - r_u + \partial_b \tilde{c}) du} (\partial_b \tilde{c}) u''_{T \wedge \tau} \right] \tag{A-13}$$

Lastly, we apply the Radon-Nikodym derivative to change for the prudence-adjusted measure $\mathbb{Q}^I$ with respect to the physical measure:

$$\frac{d\mathbb{Q}^I}{d\mathbb{P}} = \frac{e^{-\int_t^\tau (\rho - r_u + \partial_b \tilde{c}) du} (\partial_a c) u''_{T \wedge \tau}}{\mathbb{E}_t \left[e^{-\int_t^\tau (\rho - r_u + \partial_b \tilde{c}) du} (\partial_a c) u''_{T \wedge \tau} \right]}$$

such that the (A-13) euler equation holds under $\mathbb{Q}^I$:

$$(\partial_b \tilde{c})u''(\tilde{c}) = \mathbb{E}_t^{\mathbb{Q}^I} \left[(\partial_b \tilde{c}) u''_{T \wedge \tau} \right] \tag{A-14}$$

■

A.1.5
Proof of Proposition 3

Proof. Follows from Lemma 2 and (FARHI; OLIVI; WERNING, 2022) ■

A.1.6
HJB and KF equations

To fully characterize the Mean-Field game, composed by the equations (2-4)-(2-7) given the income process (3-3), we need to state the form of the infinitesimal generator $\mathcal{A}$ that encapsulates the risk in the income process.

Lemma 3 *The infinitesimal generator $\mathcal{A}$ of the income process (3-3) for a continuous function $f(x,t)$ is given by*

$$\mathcal{A}f(y,t) = \partial_t f(y,t) + \mu(y)\partial_y f(y,t) + \lambda_y \int_{-\infty}^{\infty} (f(s,t) - f(y,t))\phi(s) ds \tag{A-15}$$

By using lemma 3, it follows that the time-dependent HJB (2-4) is determined as follows:

$$\begin{aligned}
\rho V(a, z^T, z^P, t) = &\max_{c \geq 0} u(c) + \partial_a V(a, z^T, z^P, t) s(a, z^T, z^P) \\
&+ \partial_{z^P} V(a, z^T, z^P, t)(-\beta_P z^P) + \partial_{z^T} V(-\beta_T z^T) \\
&+ \lambda_\eta \int_{-\infty}^{\infty} \left(V(a, z^P, s, t) - V(a, z^P, z^T, t) \right) \phi_\eta(s) ds \\
&+ \lambda_\varepsilon \int_{-\infty}^{\infty} \left(V(a, u, z^T, t) - V(a, z^P, z^T, t) \right) \phi_\varepsilon(u) du + \partial_t V(a, z^P, z^T, t)
\end{aligned} \tag{A-16}$$

whereas, we rely on the adjoint-operator $\mathcal{A}^*$ of the infinitesimal generator (A-15) $\mathcal{A}$ to calculate the Kolmogorov-Forward equation

$$\begin{aligned}\partial_t g(a, z^T, z^P, t) = &- \partial_a(s(a, z^T, z^P, t)g(a, z^T, z^P, t)) - \partial_{z^P}(-\beta_{z^P} z^P g(a, z^T, z^P, t)) \\ &- \partial_{z^T}(-\beta_{z^T} z^T g(a, z^T, z^P, t)) - (\lambda_\eta + \lambda_\varepsilon)g(a, z^T, z^P) \\ &+ \lambda_\eta \phi_\eta(z^T) \int_{-\infty}^{\infty} g(a, z^P, s)ds + \lambda_\varepsilon \phi_\varepsilon(z^P) \int_{-\infty}^{\infty} g(a, u, z^T)du\end{aligned} \tag{A-17}$$

A.2
Calibration Data and Estimates

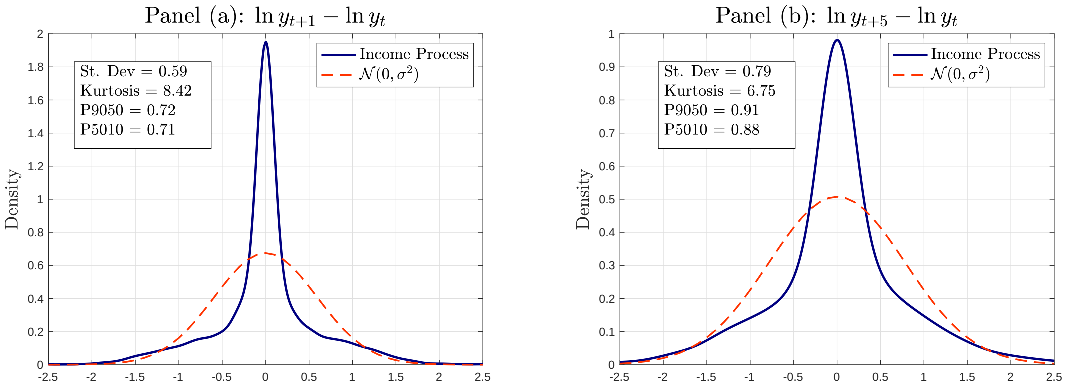
A.2.1
Micro data on Income

Table A.1: Targeted Moments of the distribution of Log Earnings Growth

	Standard Deviation			Kurtosis		P9050		P5010	
	$\Delta t = 0$	$\Delta t = 1$	$\Delta t = 5$	$\Delta t = 1$	$\Delta t = 5$	$\Delta t = 1$	$\Delta t = 5$	$\Delta t = 1$	$\Delta t = 5$
Argentina	0.992	0.622	0.824	10.472	7.262	0.517	0.855	0.512	0.814
Brazil	1.041	0.682	0.825	8.535	6.347	0.614	0.845	0.708	0.943
Canada	0.799	0.503	0.696	14.926	10.044	0.379	0.653	0.352	0.601
Denmark	0.581	0.407	0.561	17.634	11.622	0.255	0.447	0.246	0.430
France	0.671	0.453	0.579	15.863	11.682	0.254	0.422	0.258	0.447
Germany	0.767	0.384	0.533	17.803	11.210	0.241	0.454	0.183	0.382
Italy	0.775	0.448	0.560	16.980	13.323	0.260	0.434	0.233	0.386
Mexico	1.123	0.650	0.902	8.293	6.034	0.617	0.946	0.649	1.066
Norway	0.771	0.517	0.747	17.047	12.574	0.302	0.540	0.317	0.547
Sweden	0.603	0.425	0.604	15.522	10.070	0.300	0.520	0.278	0.496
Spain	0.776	0.482	0.681	14.084	9.152	0.333	0.514	0.329	0.731
US	0.937	0.567	0.785	12.865	8.827	0.437	0.716	0.461	0.771

Note: Sample averages of the Standard Deviation of log residual income (i); (ii)-(iii) Standard Deviation of residual 1 and 5-year log income changes; (iv)-(v) Kurtosis of residual 1 and 5-year log income changes; and (vi)-(ix) 90^{th} to 50^{th} and 50^{th} to 10^{th} Percentile difference of residual 1 and 5-year log income changes. Values expressed are sample averages of the time-series of moments

Figure A.1: Calibrated Distributions of One and Five Year Log Income Changes for Brazil



Note: In Panel (a), distribution of 1 year log income changes and its respective high-order moments; while in Panel (b), distribution of 5 year log income changes. Both correspond to the stationary distributions of the process (3-3) calibrated to match micro moments of RAIS (Brazil) data, available in the Table 8. In the red-dotted lines, we have a gaussian distribution with the same variance as the leptokurtic distribution

A.2.2 Macro Data on Wealth and Fiscal Policy

Table A.2: Targeted Moments of the Wealth Distribution & Fiscal Policy

	Wealth to	Wealth Shares per Quintile					Household	Transfers	Real Interest
	Income Ratio	$P0P20$	$P20P40$	$P40P60$	$P60P80$	$P80P100$	Debt to GDP	to GDP	rate
Argentina	2.115	-0.011	0.024	0.067	0.147	0.773	-	-	-
Brazil	3.613	-0.021	0.012	0.038	0.095	0.877	23.20	14.565	5.461
Canada	5.255	-0.010	0.028	0.078	0.167	0.737	40.15	9.296	-
Denmark	4.676	0.002	0.014	0.060	0.205	0.718	21.16	16.755	-
France	6.090	0.004	0.021	0.078	0.166	0.731	25.91	18.359	-
Germany	5.221	-0.009	0.017	0.073	0.176	0.743	14.87	16.827	-
Italy	6.277	-0.013	0.033	0.082	0.185	0.713	21.72	17.563	-
Mexico	4.210	-0.022	0.012	0.037	0.094	0.879	-	2.085	-
Norway	6.371	-0.041	0.028	0.109	0.218	0.687	-	13.829	-
Spain	7.331	0.000	0.033	0.095	0.161	0.711	19.96	13.550	-
Sweden	3.288	-0.010	0.027	0.078	0.165	0.740	23.11	14.303	-
US	4.883	-0.015	0.004	0.032	0.121	0.859	27.09	12.774	1.803

Note:Targeted moments in the calibration of the steady state of the model. All series besides Household debt to GDP are sample averages of the available data. Household Debt to GDP stands for non-mortgage debt

A.2.3 Parameter Estimates

The cross-country parameters estimated with micro and macro data on income and wealth are depicted below. For those of the first step, which refer to the estimates of the income process, see Table A.3. For those of the second step, referring to the steady-state parameters, see Table A.4

Table A.3: Estimated Set of Paremeters of the Income Process for each Country

	Persistence		Frequency		Variance	
	β_{z^T}	β_{z^P}	λ_{z^P}	λ_{z^T}	σ_{z^P}	σ_{z^T}
Argentina	0.6948	0.0184	0.0101	0.1752	0.9840	1.8053
Brazil	0.7910	0.0019	0.0082	0.1776	1.1716	1.6927
Canada	•	•	•	•	•	•
Denmark	0.0136	0.5415	0.1901	0.0169	0.6973	0.9557
France	0.0245	0.9771	0.0666	0.0147	1.3584	0.8556
Germany	•	•	•	•	•	•
Italy	0.0145	0.2981	0.1685	0.0046	1.8668	0.4494
Mexico	0.7487	0.0018	0.0087	0.1792	1.1874	1.7951
Norway	•	•	•	•	•	•
Sweden	•	•	•	•	•	•
Spain	0.0266	0.4396	0.1076	0.0371	0.6503	0.7762
United States	0.0115	0.5005	0.1978	0.0170	0.8164	1.0257

Note:For the values with •, there has not been estimates yet. To be done in the future

Table A.4: Estimated Set of Paremeters of the Steady-State for each Country

	ρ	∇_1	∇_2	τ	$\underline{a}$
Argentina	•	•	•	•	•
Brazil	0.0743	0.0033	0.0020	0.2170	-2.5414
Canada	•	•	•	•	•
Denmark	•	•	•	•	•
France	•	•	•	•	•
Germany	•	•	•	•	•
Italy	•	•	•	•	•
Mexico	•	•	•	•	•
Norway	•	•	•	•	•
Sweden	•	•	•	•	•
Spain	•	•	•	•	•
United States	0.0224	0.0025	0.0089	0.1748	-2.9765

Note:For the values with •, there has not been estimates yet. To be done in the future

A.2.3.1
Alternative Simulations - Permanent Credit Shock (United States)

In the Permanent credit deepening experiment, there is a shock at $t = 0$ that alters the borrowing constraint from $\underline{a}_0$ to $\underline{a}'$, such that the path of borrowing constraints $\{\underline{a}\}_{t\geq 0}$ follows the trajectory dictated by the Ordinary Differential Equation (ODE) to the new steady state:

$$d\underline{a}_t = \nu(\underline{a}' - \underline{a}_t)dt \tag{A-18}$$

Figure A.2: General IRFs to a Transitory Credit Shock

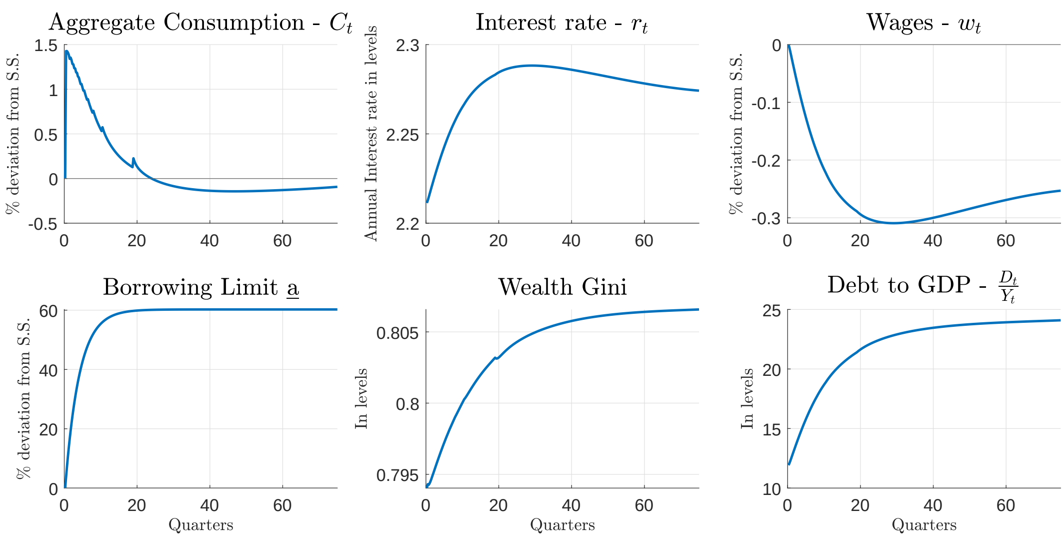
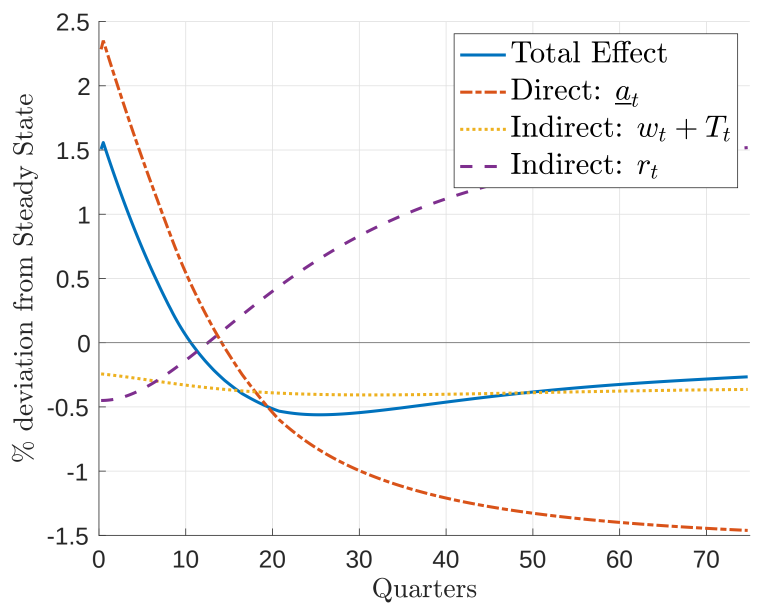
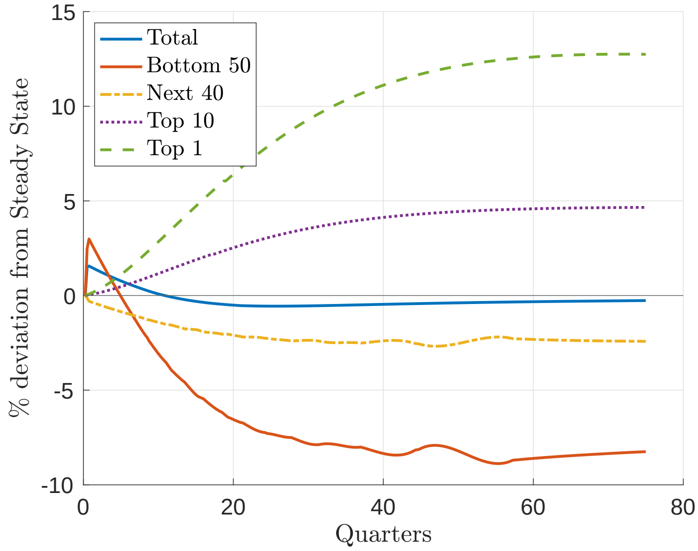


Figure A.3: Decomposition of the Consumption IRFs following a Permanent Credit Shock by Direct and Indirect Effects



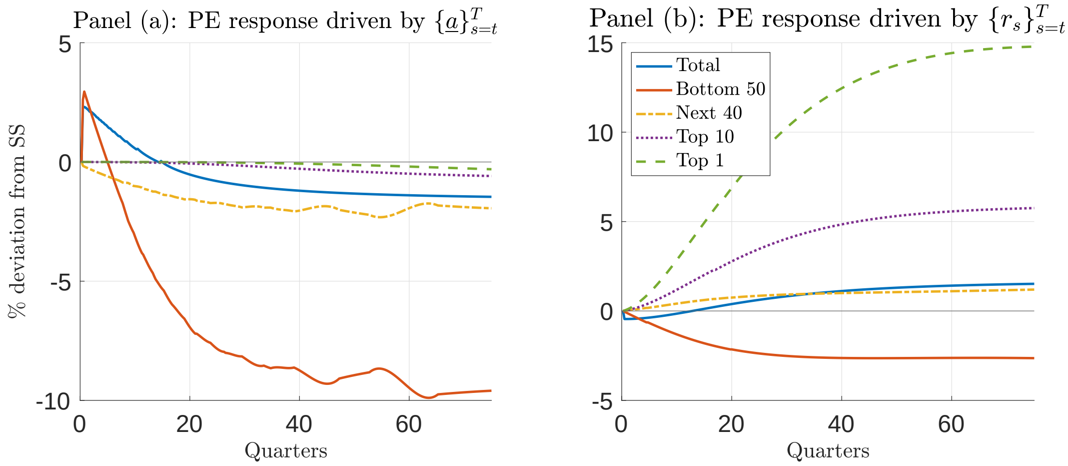
Note: All values are log deviations from steady state. The decomposition is given by (2-10)

Figure A.4: Decomposition of the Consumption IRF following a Permanent Credit Shock by Wealth Percentiles



Note: We decompose the total response of of aggregate consumption to the Permanent Credit shock (A-18) into the response by wealth percentile, $C_t = \int_\omega c_t(a, z^P, z^T, t)d\mu_t$ where ω is the subset of liquid wealth holdings corresponding to the wealth percentile. (4-1)

Figure A.5: Sensitivity of the Response of Consumption to Credit Conditional on Income or Wealth Inequality



Note: On Panels (a), we disaggregate the direct channel of credit to permanent credit shock by wealth percentiles. Similarly, on Panel (b) we do the same decomposition but for the indirect channel of credit determined by interest rates (budget and intertemporal substitution). All values are log deviations from steady state. The decomposition is given by (2-10)

A.2.4
Credit Deepening under Brazilian Calibration

Figure A.6: General IRFs to a Transitory Credit Shock - Brazilian Calibration

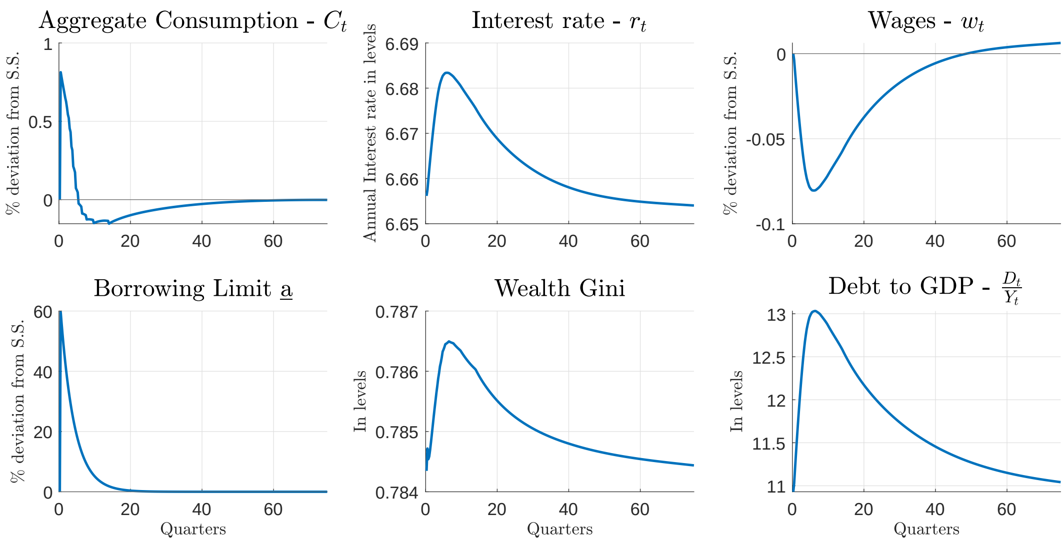
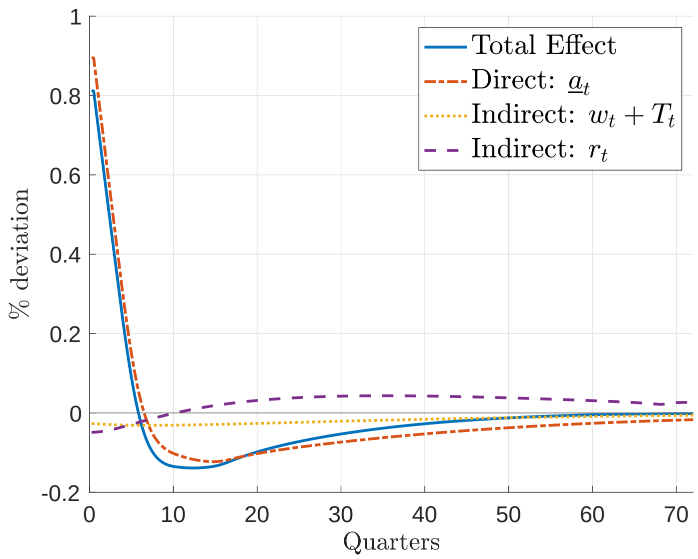
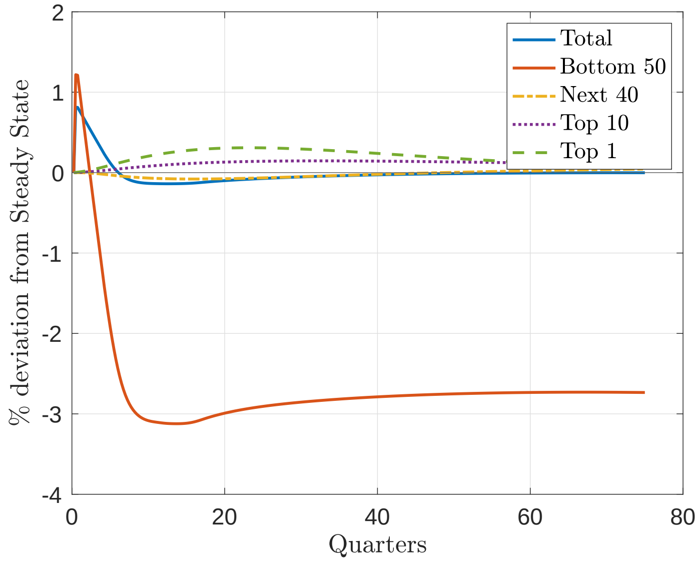


Figure A.7: Decomposition of the Consumption IRFs following a Transitory Credit Shock by Direct and Indirect Effects



Note: All values are log deviations from steady state. The decomposition is given by (2-10)

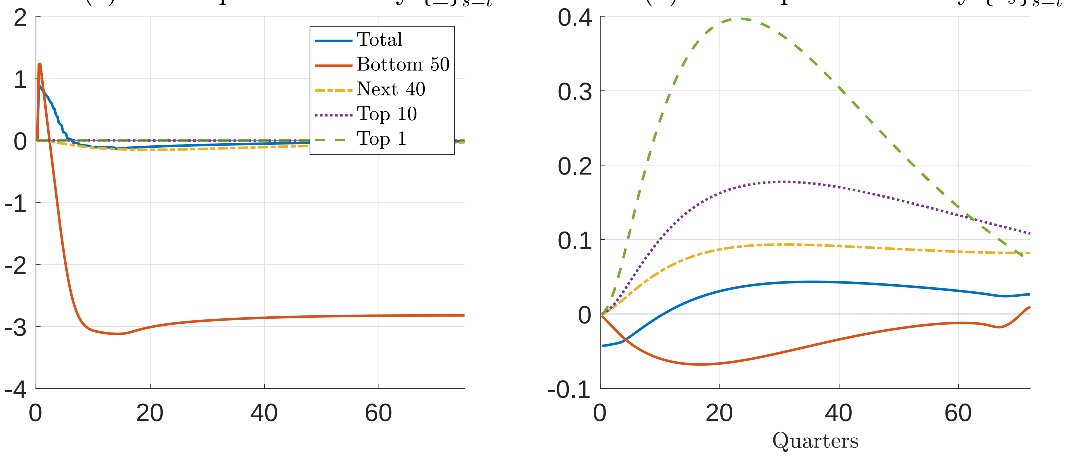
Figure A.8: Decomposition of the Consumption IRF following a Transitory Credit Shock by Wealth Percentiles



Note: We decompose the total response of of aggregate consumption to the Permanent Credit shock (A-18) into the response by wealth percentile, $C_t = \int_{\omega} c_t(a, z^P, z^T, t)d\mu_t$ where ω is the subset of liquid wealth holdings corresponding to the wealth percentile. (4-1)

Figure A.9: Sensitivity of the Response of Consumption to Credit Conditional on Income or Wealth Inequality

Panel (a): PE response driven by $\{\underline{a}\}_{s=t}^{T}$ Panel (b): PE response driven by $\{r_s\}_{s=t}^{T}$



Note: On Panels (a), we disaggregate the direct channel of credit to transitory credit shock by wealth percentiles. Similarly, on Panel (b) we do the same decomposition but for the indirect channel of credit determined by interest rates (budget and intertemporal substitution). All values are log deviations from steady state. The decomposition is given by (2-10)

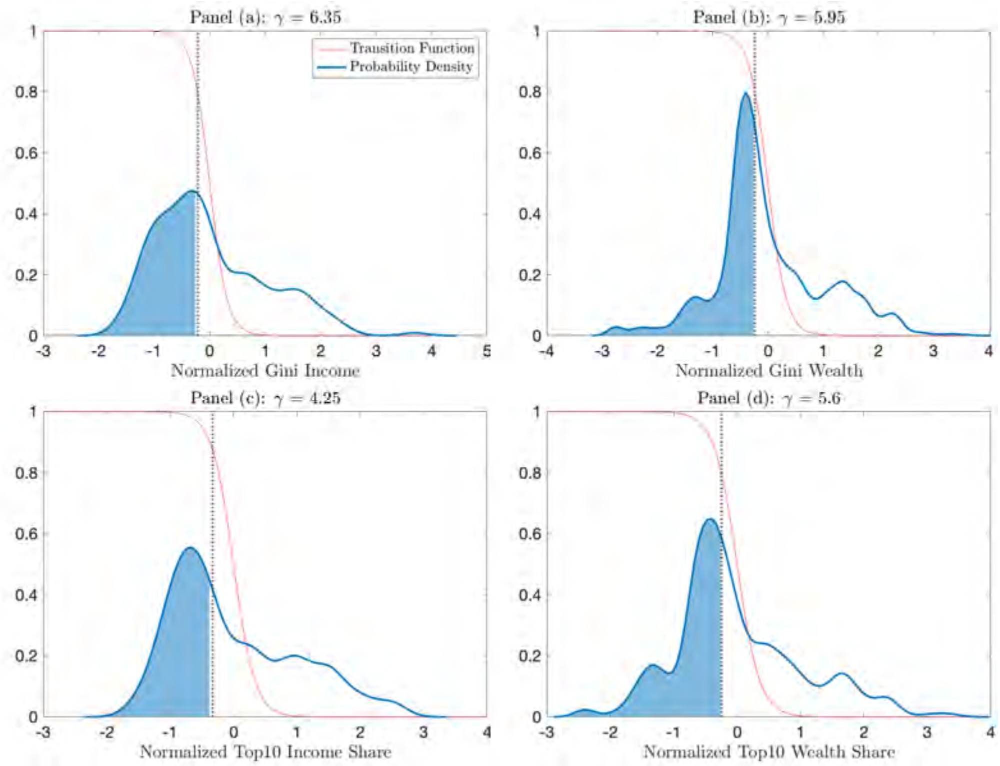
A.2.5
Empirical Robustness Exercises

Table A.5: Single Equation Estimation (5-4) of Consumption Growth Trends to Household Credit Growth Accounting for Income and Wealth Inequality - Top10 share measures

	Dependent variable: $\Delta_3 c_{it+k}, k = -1, 0, \ldots, 5$								
	$\Delta_3 c_{it-1} = c_{it-1} - c_{it-4}$			$\Delta_3 c_{it+2} = c_{it+2} - c_{it-1}$			$\Delta_3 c_{it+5} = c_{it+5} - c_{it+2}$		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$\Delta_3 d^{HH}_{it-1}$	0.252***	0.525†	0.678	−0.062†	0.001	−0.000	−0.128**	0.018	−0.580
	(0.059)	(0.301)	(0.413)	(0.039)	(0.255)	(0.285)	(0.048)	(0.288)	(0.408)
$\Delta_3 d^{F}_{it-1}$	−0.010	−0.012	−0.008	−0.039***	−0.041***	−0.038***	−0.021**	−0.023**	−0.021*
	(0.014)	(0.014)	(0.013)	(0.008)	(0.008)	(0.007)	(0.007)	(0.007)	(0.008)
$\Delta_3 d^{HH}_{it-1} \cdot Top10^{Inc}_{it-4}$		−0.722			−0.171			−0.406	
		(0.804)			(0.711)			(0.772)	
$\Delta_3 d^{HH}_{it-1} \cdot Top10^{W}_{it-4}$			−0.693			−0.105			0.727
			(0.661)			(0.468)			(0.647)
Country FE	✓	✓	✓	✓	✓	✓	✓	✓	✓
R^2	0.036	0.040	0.040	0.015	0.016	0.015	0.015	0.017	0.020
Observations	1865	1741	1535	1822	1688	1478	1606	1477	1266

Note: The current table presents estimates for the specification $\Delta_3 c_{it+k} = \alpha_i + \beta_{HH,k}\Delta_3 d^{HH}_{it-1} + \beta_{F,k}\Delta_3 d^{F}_{it-1} + \gamma_k(\mathbf{Ineq}_{it-4} \cdot \Delta_3 d^{HH}_{it-1}) + u_{it+k}$, ran on an unbalanced panel for $k = -1, 2, 5$, each of which ran for normalized inequality variables $\{Top10^{Inc}, Top10^{W}\}$. All specifications include country fixed effects. Reported R^2 values are from within-country variation. Standard errors in parentheses are dually clustered on country and year. The symbols ***, **, *, † indicate significance at 0.1%,1%,5%,10% levels respectively.

Figure A.10: Transition function calibration for Smooth Transition Local Projections with each of the inequality state variables



Note: Transition Function of specification (5-5) for the estimation with Income Inequality. We estimate the data probability distribution function and calibrate γ such that $\mathbf{Pr}(F(\mathbf{Ineq}) \geq 0.8) = 0.5$

A.3 Numerical Appendix

All the MATLAB codes for solving this model can be found on my Github, `https://github.com/Rlincoln01`

A.3.1 HJB equation - Steady State

For the sake of simplicity, I drop ex-ante heterogeneity $(i, j) \in \mathbb{I} \times \mathbb{J}$ subscripts in the value function:

$$\begin{aligned}
\rho V(a, z^T, z^P) = & \max_{c \geq 0} u(c) + \partial_a V(a, z^T, z^P) s(a, z^T, z^P) \\
& + \partial_{z^P} V(a, z^T, z^P)(-\beta_P z^P) + \partial_{z^T} V(-\beta_T z^T) \\
& + \lambda_\eta \int_{-\infty}^{\infty} \left(V(a, z^P, s) - V(a, z^P, z^T) \right) \phi_\eta(s) ds \\
& + \lambda_\varepsilon \int_{-\infty}^{\infty} \left(V(a, u, z^T) - V(a, z^P, z^T) \right) \phi_\varepsilon(u) du
\end{aligned} \tag{A-19}$$

which can be rewritten in reduced form by using the inifnitesimal generator $\mathcal{L}$ notation

$$\rho V(a, z^T, z^P) = \max_{c \geq 0} \left\{ u(c) + \mathcal{L}(a, z^T, z^P, c)[V] \right\} \quad \text{(A-20)}$$

The problem is initially cast onto a tridimensional grid of individual liquid asset holdings a, log permanent productivity z^P and log transitory productivity z^T. For the income components, we work with a symmetric power grid around zero based on the fact that, as we have a compound poisson process with a normally distributed jump around zero, transitions to regions nearby are more likely. For the asset grid, we work work with a non-symmetric power grid with more points closer to the borrowing constraint as the value and policy functions are strictly concave these regions.

Let the (a, z^P, z^T) gridpoints be represented by, respectively, the indexes i, j, k such that we define $V(a, z^P, z^T) \equiv V_{i,j,k}$. We proceed by using a finite-difference discretization method of the HJB equation (A-19), which is a Partial Differential Equation, ensuring that the solution of the equation converges to the real one - or at least to a weak solution (a viscosity solution). To guarantee this convergence in general terms - that is, when discretizing the PDE as a whole, we resort to Barles & Souganidis (1991):

Definition 3 (Barles & Souganidis (1991) convergence conditions) *A numerical finite difference scheme converges to the viscosity solution as long as it satisfies the three following conditions:*

1. ***Consistency**: when approximating derivatives by finite differences, the approximation will converge to the true value as the grid gets finer*
2. ***Stability**: The numerical scheme doesn't explode*
3. ***Monotonicity**: The values that we are solving for depend positively on those that we have already determined*

To ensure that three conditions are satisfied and that we can use an arbitrarily large step size Δ in the iterative step, we rely on a implicit numerical scheme with the upwind method. The forward and backward approximations of the partial derivatives with respect to assets and log income components are defined as:

$$\partial_{a,F} V_{i,j,k} = \frac{V_{i+1,j,k} - V_{i,j,k}}{\Delta a_{i,+}}; \quad \partial_{z^P,F} V_{i,j,k} = \frac{V_{i,j+1,k} - V_{i,j,k}}{\Delta z^P_{j,+}}; \quad \partial_{z^T,F} V_{i,j,k} = \frac{V_{i,J,k+1} - V_{i,j,k}}{\Delta z^T_{k,+}}$$

$$\partial_{a,B} V_{i,j,k} = \frac{V_{i,j,k} - V_{i-1,j,k}}{\Delta a_{i,-}}; \quad \partial_{z^P,F} V_{i,j,k} = \frac{V_{i,j,k} - V_{i,j-1,k}}{\Delta z^P_{j,-}}; \quad \partial_{z^T,F} V_{i,j,k} = \frac{V_{i,J,k} - V_{i,j,k-1}}{\Delta z^T_{k,-}}$$

where $\Delta a_{i,+} \equiv a_{i+1} - a_i$ and $\Delta a_{i,-} \equiv a_i - a_{i-1}$, such that the upwind approximation is:

$$\begin{aligned}
\partial_a V_{i,j,k} &= \partial_{a,F} V_{i,j,k} \mathbb{1}_{\{s_{i,j,k}>0\}} + \partial_{a,B} V_{i,j,k} \mathbb{1}_{\{s_{i,j,k}<0\}} + \partial_a \bar{V}_{i,j,k} \mathbb{1}_{\{0<s_{i,j,k}<0\}} \\
\partial_{z^P} V_{i,j,k} &= \partial_{z^P,F} V_{i,j,k} \mathbb{1}_{\{\mu(z_j^P)>0\}} + \partial_{z^P,B} V_{i,j,k} \mathbb{1}_{\{\mu(z_j^P)<0\}} \\
\partial_{z^T} V_{i,j,k} &= \partial_{z^T,F} V_{i,j,k} \mathbb{1}_{\{\mu(z_k^T)>0\}} + \partial_{z^T,B} V_{i,j,k} \mathbb{1}_{\{\mu(z_k^T)<0\}}
\end{aligned} \tag{A-21}$$

where the term $\partial_a \bar{V}_{i,j,k} = u'(y_{j,k} + ra_i)$ captures the points where savings are equal to zero. Define the initial step[1] as the value of "staying put"

$$V_{i,j,k}^0 = \frac{u(y_{j,k} + ra_i)}{\rho}$$

such that, given the iteration step Δ and the transition flows $\lambda_\varepsilon(j,j'), \lambda_\eta(k,k')$ calculated in the Income Process section, the Implicit method discretization of equation (A-19) is defined as:

$$\begin{aligned}
\frac{V_{i,j,k}^{n+1} - V_{i,j,k}^n}{\Delta} + \rho V_{i,j,k}^{n+1} &= u(c_{i,j,k}^n) + \partial_a V_{i,j,k}^{n+1}(s_{i,j,k}^n) - \partial_{z^P} V_{i,j,k}^{n+1}(\beta_{z^P} z_j^P) - \partial_{z^T} V_{i,j,k}^{n+1}(\beta_{z^T} z_k^T) \\
&+ \sum_{k' \neq k} \lambda_\eta(k,k')(V_{i,j,k'}^{n+1} - V_{i,j,k}^{n+1}) + \sum_{j' \neq j} \lambda_\varepsilon(j,j')(V_{i,j',k}^{n+1} - V_{i,j,k}^{n+1})
\end{aligned} \tag{A-22}$$

where $n+1$ is the next step of the iteration and n the current step, and the policy functions $c_{i,j,k}^n, s_{i,j,k}^n$ are derived from the current value function:

$$\begin{aligned}
c_{i,j,k}^n &= (u')^{-1}[\partial_a V_{i,j,k}^n] \\
s_{i,j,k}^n &= y_{j,k} + ra_i - c_{i,j,k}^n
\end{aligned} \tag{A-23}$$

Plugging the finite-difference upwind approximations in equation (A-21), we obtain the full form the discretized HJB equation:

$$\begin{aligned}
\frac{V_{i,j,k}^{n+1} - V_{i,j,k}^n}{\Delta} + \rho V_{i,j,k}^{n+1} &= u(c_{i,j,k}^n) + \frac{V_{i+1,j,k}^{n+1} - V_{i,j,k}^{n+1}}{\Delta a_{i,+}}(s_{i,j,k}^{n,F})^+ + \frac{V_{i,j,k}^{n+1} - V_{i-1,j,k}^{n+1}}{\Delta a_{i,-}}(s_{i,j,k}^{n,B})^- \\
&+ \frac{V_{i,j+1,k}^{n+1} - V_{i,j,k}^{n+1}}{\Delta z_{j,+}^P}(-\beta_{z^P} z_j^P)^+ + \frac{V_{i,j,k}^{n} - V_{i,j-1,k}^{n+1}}{\Delta z_{j,-}^P}(-\beta_{z^P} z_j^P)^- \\
&+ \frac{V_{i,j,k+1}^{n} - V_{i,j,k}^{n}}{\Delta z_{k,+}^T}(-\beta_{z^T} z_k^T)^+ + \frac{V_{i,j,k}^{n} - V_{i,j,k-1}^{n+1}}{\Delta z_{k,-}^T}(-\beta_{z^T} z_k^T)^- \\
&+ \sum_{k' \neq k} \lambda_\eta(k,k')(V_{i,j,k'}^{n+1} - V_{i,j,k}^{n+1}) + \sum_{j' \neq j} \lambda_\varepsilon(j,j')(V_{i,j',k}^{n+1} - V_{i,j,k}^{n+1})
\end{aligned} \tag{A-24}$$

[1]Which is equivalent of a boundary condition for computing the PDE

where $x^+ = \max\{x, 0\}$ and $x^- = \min\{x, 0\}$. In matrix notation, the above equation is given by:

$$\frac{1}{\Delta}(V^{n+1} - V^n) + \rho V^{n+1} = u^n + \mathbf{A}^n V^{n+1} + \mathbf{\Lambda}_{z^P} V^{n+1} + \mathbf{\Lambda}_{z^T} V^{n+1} \quad \text{(A-25)}$$

where $\mathbf{\Lambda}_l = \mathbf{\Lambda}_l^D + \mathbf{\Lambda}_l^J$ is the sum of the transition matrices for the drift and jump components of each component of the income process, $l \in \{z^P, z^T\}$. The Vectors V, u^n are of length $I \times J \times K$, while the matrices $\mathbf{A}^n, \mathbf{\Lambda}_{z^P}, \mathbf{\Lambda}_{z^T}$ are of $(I \times J \times K) \times (I \times J \times K)$ dimension. We can simplify further (A-25) by taking a matrix Λ_z, which represents the sum of the persistent and transitory components of the process, and has the same dimension $(I \times J \times K) \times (I \times J \times K)$ but with different structure:

$$\frac{1}{\Delta}(V^{n+1} - V^n) + \rho V^{n+1} = u^n + \mathbf{A}^n V^{n+1} + \mathbf{\Lambda}_z V^{n+1} \quad \text{(A-26)}$$

Lastly, we compute the iteration by defining the infinitesimal generator matrix of the HJB equation (A-19) as $\mathbf{L}^n \equiv \mathbf{A}^n + \mathbf{\Lambda}_z$, such that the next step of the iteration of the value function is given by:

$$V^{n+1} = \left[(1 + \Delta\rho)\mathbf{I} - \Delta\mathbf{L}^n\right]^{-1} (\Delta u^n + V^n) \quad \text{(A-27)}$$

which is monotone given that the matrix $\mathbf{M}^n = [(1 + \Delta\rho)\mathbf{I} - \Delta\mathbf{L}^n]$ is a non-negative matrix. To obtain the stationary value function and policy functions, we follow the algorithm in (ACHDOU et al., 2022): Guess an initial value for the value function $V^0_{i,j,k}$ for all the points in the grid and for $n = 0, 1, 2, \ldots$ follow:

1. Compute the derivatives $\partial_a V_{i,j,k}, \partial_{z^P} V_{i,j,k}, \partial_{z^T} V_{i,j,k}$ using (A-21)
2. Compute c^n from (A-23)
3. Find V^{n+1} from (A-27)
4. If V^{n+1} is close enough to V^n, stop. otherwise, go to step 1

A.3.2 KF equation

The KF equation for the stationary distribution, dropping ex-ante heterogeneity $(i, j) \in \mathbb{I} \times \mathbb{J}$ subscripts in the productivity-wealth distribution, is given by:

$$
\begin{aligned}
0 = & -\partial_a(s(a, z^P, z^T)g(a, z^P, z^T)) - \partial_{z^P}(-\beta_{z^P} z^P g(a, z^P, z^T)) \\
& - \partial_{z^T}(-\beta_{z^T} z^T g(a, z^P, z^T)) - (\lambda_\eta + \lambda_\varepsilon)g(a, z^P, z^T)) \\
& + \lambda_\eta \phi_\eta(z^T) \int_{-\infty}^{\infty} g(a, z^P, s)ds + \lambda_\varepsilon \phi_\varepsilon(z^P) \int_{-\infty}^{\infty} g(a, u, z^T)du
\end{aligned}
\tag{A-28}
$$

which can be reduced when using the infinitesimal generator notation:

$$0 = \mathcal{L}^*(a, z^P, z^T, c(a, z^P, z^T))[V]g(a, z^P, z^T)$$

where $\mathcal{L}^*$ is the adjoint operator of the infinitesimal generator. As we already have the matrix $\mathbf{L}$, solving for the stationary distribution becomes straightforward as finding the kernel of the transpose of matrix $\mathbf{L}$:

$$\mathbf{L}^T g = 0 \tag{A-29}$$

A.3.3 Income Process

We discretize (3-3) drift and compound-jump process components separetely, each of which turns into a different transition matrix. The drift component is straightforward and similar to we has been done on the HJB equation, so we details only the compound drift process. Define $\mathbf{\Lambda}_l^J$ as the transition matrix of the jump component of the income process, $l \in \{z^P, z^T\}$. We obtain it through a transformation of the markov-chain matrix that discretizes the gaussian shock of the brownian motion:

$$\mathbf{\Lambda}_l^J = \lambda_l(\mathbf{\Pi}^l - \mathbf{I}) \tag{A-30}$$

where λ_l is the arrival rate of the transitory or permanent income shock, $\mathbf{I}$ is the identity matrix and $\mathbf{\Pi}^l$ is the markov-chain discretization of the gaussian shocks with variance σ_l. We discretize gaussian shocks using a truncated normal distribution with respect to the income grid. Let (i, j) correspond to the row and column of the markov matrix $\mathbf{\Pi}^l$. Its elements are determined as follows:

$$
\begin{aligned}
p_{i,1} &= F(z_j^l + 1/2\Delta_j^l) \\
p_{i,j} &= F(z_j^l + 1/2\Delta_j^l) - F(z_j^l - 1/2\Delta_{j-1}^l) \\
p_{i,n_l} &= 1 - F(z_j^l - 1/2\Delta_j^l)
\end{aligned}
\tag{A-31}
$$

where $F(\cdot)$ is the cumulative distribution function of the normal distribution with variance σ_l, and Δ_j^l is the grid distance valued at point j.

A.3.4
Feynman-Kac formula

We use the Feynman-Kac formula to compute for the Quarterly MPC of a 500 US$, as in (KAPLAN; MOLL; VIOLANTE, 2018). Let us define the consumption over a period τ as

$$\tilde{C}_{i,j,\tau}(a, z^P, z^T) = \mathbb{E}\left[\int_0^\tau c_{i,j}(a_t, z_t^P, z_t^T)dt \middle| a_0 = a, z_0^P = z^P, z_0^T = z^T\right] \quad \text{(A-32)}$$

such that the fraction consumed out of x additional units of liquid wealth over a period τ is given by:

$$MPC_\tau^x = \frac{\tilde{C}_{j,\tau}(a+x, z^P, z^T) - \tilde{C}_{j,\tau}(a, z^P, z^T)}{x} \quad \text{(A-33)}$$

The conditional expectation in (A-32) can be computed via the Feynman-Kac formula:

$$0 = c_{i,j}(a, z^P, z^T) + \mathcal{L}_{i,j}(a, z^P, z^T, 0)[\Gamma] + \partial_t \Gamma_{i,j}(a, z^P, z^T, 0) \quad \text{(A-34)}$$

where $\tilde{C}_{i,j,\tau}(a, z^P, z^T) = \Gamma_{i,j}(a, z^P, z^T, 0)$, and we set $\Gamma_{i,j}(a, z^P, z^T, \tau) = 0$ as the terminal condition. We solve this PDE numerically, with the matrix $\mathbf{L}$ and the consumption policy function $\mathbf{c}$ obtained in the HJB equation, by iterating backwards

$$\begin{aligned} 0 &= c + \mathbf{L}\Gamma_n + \frac{\Gamma_{n+1} - \Gamma_n}{\Delta t} \\ \Gamma_n &= \left[\frac{1}{\Delta t}\mathbf{I} - \mathbf{L}\right]^{-1}\left(c + \frac{1}{\Delta t}\mathbf{\Gamma_{n+1}}\right) \end{aligned} \quad \text{(A-35)}$$

A.3.5
Stationary Equilibrium

To solve for the stationary equilibrium, we look for the price r which solves the following system of matrix equations:

$$\rho \mathbf{v} = \mathbf{u}(\mathbf{v}) + \mathbf{L}(\mathbf{v}; \mathbf{r})\mathbf{v} \quad \text{(A-36)}$$

$$\mathbf{0} = \mathbf{L}(\mathbf{v}; \mathbf{r})^{\mathbf{T}}\mathbf{g} \quad \text{(A-37)}$$

$$K = A(\mathbf{g}; r) \quad \text{(A-38)}$$

The algorithm used to find r which solves the above system with discount heterogeneity is the following:

1. Guess $r^l \in (-\delta, \bar{\rho} - \nabla_1 - \nabla_2)$
2. Given r^l, solve for w^l
3. Given w^l, solve for equilibrium lump-sum transfers T^l
4. Given Prices and Transfers $\{r^l, w^l, T^l\}$, solve the household problem in stationary equilibrium for each discount rate ρ_j, obtaining consumption policy functions $c_j(a,z)^l$ and income-wealth distributions $g_j(a,z)^l$
5. Use the stationary distributions $g_j(a,z)^l$ to compute total liquid asset holdings

$$A(r^l) = \int_{j\in\mathbb{J}} \int_{a\in\mathcal{A}} \int_{z\in\mathcal{Z}} a g_j(a,z)^l d\mu^l(a,z,j)$$

6. Use firm's FOC to compute capital demand

$$K(r^l) = (f')^{-1}(r^l + \delta)$$

7. Check whether capital market clearing is satisfied, $|A(r^l) - K(r^l)| <$ tol. If convergence, stop. If not, update r^l given the rule below and go back to step 2.

$$r^{l+1} = \phi r^l + (1-\phi)\tilde{r}^l$$

where $\phi \in (0,1)$ is a tuning parameter and $\tilde{r}^l = (f')^{-1}(A(r^l)) - \delta$.

A.3.6 Transition Dynamics

The algorithm for solving transition dynamics with discount heterogeneity after a borrowing-constraint MIT-shock is an adaptation of (ACHDOU et al., 2022). For this, we fix a large $T = 75$ where time is measured in quarters, and solve the following system for the path of interest rates $\{r_1, \ldots, r_N\}$ given terminal condition $\mathbf{v}^N = \mathbf{v}$ and initial condition $\mathbf{g^1} = \mathbf{g_0}$

$$\rho\mathbf{v}^n = \mathbf{u}(\mathbf{v}^{n+1}) + \mathbf{L}(\mathbf{v}^{n+1};\mathbf{r}^n)\mathbf{v}^n + \frac{\mathbf{v}^{n+1} - \mathbf{v}^n}{\Delta t}, \tag{A-39}$$

$$\frac{\mathbf{g}^{n+1} - \mathbf{g}^n}{\Delta t} = \mathbf{L}(\mathbf{v}^n;\mathbf{r}^n)^{\mathbf{T}}\mathbf{g}^{n+1}, \tag{A-40}$$

$$\mathbf{K} = \mathbf{A}(\mathbf{g}^n), \tag{A-41}$$

with the algorithm as follows:

1. Calculate the Stationary Equilibrium before and after the shock. We take the value functions for each ρ_j, $V_j^*(a,z)$, and the initial measure of the economy $\mu_0(a,z,j)$
2. Guess an initial path for capital, $\{K_t^l\}_{t=0}^T$, where $K_0^l = K_0$ and $K_T = K^*$
3. Compute the transition prices and transfers $\{r_t^l, w_t^l, T_t^l\}_{t=0}^T$ from $\{K_t^l\}_{t=0}^T$
4. For each discount rate, solve the Household Problem by backward induction using the stationary value function at the end of the shock as the terminal condition $V_{j,l}^T(a,z) = V_j^*(a,z)$.
5. From the initial distribution and the sequence of infinitesimal generator, obtain the sequence of distributions $\{g_{j,t}^l(a,z)\}_{t=0}^T$ and aggregate as measures $\{\mu_t^l(a,z,j)\}_{t=0}^T$
6. Take the sequence of measures and calculate the path of aggregate savings, $\{A_t^l\}_{t=0}^T$
7. Check convergence of the path of capital and savings, $\max_t |A_t^l - K_t^l| <$ tol. If true, stop. If not, update conjecture path using the following rule and go back to step 3:

$$K_t^{l+1} = \phi K_t^l + (1-\phi)A_t^l$$

A.3.7
Consumption Decomposition

Let $\Theta = (r, w, \underline{a}, T)$. To compute the decomposition (2-10), I employ the numerical algorithm in (KAPLAN; MOLL; VIOLANTE, 2018)

1. Compute the MIT-shock transition path of prices $\{r_t, w_t, \tau_t, T_t\}_{t\geq 0}$ and aggregate consumption $\{C_t\}_{t\geq 0}$ given a path of $\{\underline{a}_t\}_{t\geq 0}$ borrowing constraints or $\{\chi_t\}_{t\geq 0}$ borrowing wedges.
2. Given the path of prices and borrowing constraints $\{\Gamma_t\}_{t\geq 0} = \{r_t, w_t, \tau_t, T_t, \underline{a}_t\}_{t\geq 0}$, we compute the partial equilibrium response of consumption to a time-varying object: Take an input price/allocation path $\theta_t^k \in \Gamma_t$ while leaving the rest held constant at their steady-state values $\theta_t^{-k} = \overline{\theta^{-k}}$ and compute the consumption policy functions $\{c_{i,j}(a, z^P, z^T, t)\}_{T\geq t\geq 0}$ and infinitesimal operators $\{\mathbf{A}_t\}_{T\geq t\geq 0}$ by solving the transition dynamics of the time-dependent HJB 2-4 with a ter-

minal condition $\mathbf{V}_T^k = \lim_{t\to\infty} v_t(\{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$

3. Given the path of infinitesimal operators $\{\mathbf{A}_t\}_{T\geq t\geq 0}$, take the time dependent KF-operators $\{\mathbf{A}^*{}_t\}_{T\geq t\geq 0}$ and compute partial equilibrium distribution objects given $\{\Gamma_t\}_{t\geq 0} = \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0}$, $\mu_t^k = \mu_t(da, dz^P, dz^T; \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$.

4. The aggregate partial-equilibrium consumption response to object θ^k is given by:

$$C_t^{i,j}(\{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0}) = \int c_{i,j}(a, z^P, z^T, t; \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})\mu_t(da, dz^P, dz^T; \{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$$

Aggregate the function $C_t^{i,j}(\{\theta_t^k, \overline{\theta^{-k}}\}_{t\geq 0})$ for fixed effects i and discount rate heterogeneity j to obtain aggregate consumption.

5. The object dC_t from equation (2-10) can be numerically computed as deviations from the steady-state

$$dC_t = \sum_{k=1}^{K} \int_{\tau=t}^{\infty} \frac{\partial C_t}{\partial \Theta_{k\tau}} d\Theta_{k\tau} d\tau \approx \sum_{k=1}^{K} \int_{\tau=t}^{\infty} \left(\frac{C_t - C^{SS}}{\Delta\Theta_{k\tau}} \right) \Delta\Theta_{k\tau} d\tau$$