Pontifícia Universidade Católica
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Monetary Policy and Trade Tariffs: An examination of the optimal policy and the effect of liquidity traps

Dissertação de Mestrado

Thesis presented to the Programa de Pós–graduação em Economia, do Departamento de Economia da PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof. Tiago Couto Berriel

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To Mônica, Rodrigo and Júlia,
for always encouraging me to be better

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Abstract

Lima da Fonseca, Rafael; Couto Berriel, Tiago (Advisor). **Monetary Policy and Trade Tariffs: An examination of the optimal policy and the effect of liquidity traps**. Rio de Janeiro, 2021. 53p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Can trade tariffs be used to help the Central Bank stabilize the economic cycle? To answer that question we build a New Keynesian Open Economy model with two different countries and where firms have enough market power to set prices in both Home and Foreign markets and calculate the optimal monetary and tariff policy under the existence of a Zero Lower Bound on the nominal interest rate. We perform a numerical exercise to analyse two distinct situations: when only one country is restricted by the Zero Lower Bound and when both countries face this constraint. Our results suggest that the Zero Lower Bound creates a situation in which active use of trade tariffs can be optimal, even if countries are cooperating.

Keywords

Optimal Policy; Open Economy; Zero Lower Bound on Nominal Interest Rates; Local Currency Pricing; Trade Tariffs; Global Liquidity Tra.

Resumo

Lima da Fonseca, Rafael; Couto Berriel, Tiago. **Política monetária e tarifas comerciais: uma análise da política ótima e o impacto de armadilhas de liquidez**. Rio de Janeiro, 2021. 53p. Dissertação de Mestrado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Tarifas comerciais podem ser usadas para auxiliar o Banco Central na estabilização da economia? Para responder essa pergunta construímos um modelo Novo Keynesiano de economia aberta com dois países onde as firmas têm poder de mercado suficientemente alto para definir preços diferentes para o mercado local e estrangeiro e obtemos a política monetária e tarifária ótima sob a existência de um limite inferior para a taxa nominal de juros. Fazendo um exercício numérico, analisamos duas situações: quando apenas um país se encontra em uma armadilha de liquidez e quando ambos os países se encontram presos em uma armadilha de liquidez global. Nossos resultados sugerem, que mesmo quando os dois países estão cooperando, a existência do limite inferior da taxa de juros nominal gera uma situação onde o uso ativo de tarifas comerciais pode aumentar o bem-estar da economia.

Palavras-chave

Política Ótima; Economia Aberta; Limite Inferior da Taxa de Juros Nominal; Precificação em Moeda Local; Tarifas Comerciais; Armadilha de Liquidez Global.

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List of Abreviations

LCP – Local Currency Pricing
ZLB – Zero Lower Bound
H – Home
F – Foreign
US – United States
DSGE – Dynamic Stochastic General Equilibrium

1
Introduction

Large economic shocks demand swift action in order to mitigate the negative effects that they might present. In recent times, both the Great Financial Crisis and the Covid-19 pandemic forced central banks all around the world to cut interest rates. Several monetary authorities saw their policy reaction constrained by the Zero Lower Bound (ZLB) on nominal interest rates (or the Effective Lower Bound, since the constraint is not necessarily tied to 0% nominal interest rates). The world faced a liquidity trap and monetary policy alone could not fully stabilize the economy.

Since monetary policy is constrained, policymakers need to find a different instrument to be used in their stabilization effort. One possibility is the active use of fiscal policy[1]. However, fiscal policy is generally a matter that Congress must get involved. Hence, political constraints might make the final package too small[2] or too big[3].

An instrument that could have some similar characteristics to fiscal policy are trade tariffs, since they essentially work as a tax on foreign products. However, unlike fiscal policy, trade tariffs do not require congressional approval to be enacted (at least in the United States)[4]. Therefore, trade tariffs could be used as a stabilization tool without the same domestic political constraints that are present for fiscal policy.

This paper tries to answer the question of what would be the optimal policy in a global liquidity trap when trade tariffs can be used as a policy tool. In order to do it, we build a two country open economy New Keynesian model

[1]The optimal monetary and fiscal policies under a global liquidity trap are further explored in Cook and Devereux (2013)

[2]For example, Stein (2012) notes that Christina Romer had estimated that the fiscal package necessary to deal with the economic impact of the Great Financial Crisis would have been around US$1.8 trillion. However, such a package was seen as politically unfeasible and the Obama administration ended up settling for around US$ 800 billion.

[3]Some economists, like Blanchard (2021) argue that the recently enacted American Rescue Plan is too large and risks overheating the economy. However, it is worth noting that part of the approved packaged are US$1400 stimulus checks, which were a campaign promise of the Democratic party in the Georgia Senate runoffs (Duffy (2021)). Hence, not approving the checks because they were “excesive” was not politically feasible.

[4]Golshan (2018) cites different Acts that allow the US President to set trade tariffs without congressional approval under very broad conditions

with Local Currency Pricing[5], very similar to Engel (2011), and add trade tariffs to it. Then we derive the optimal policy under credible commitment and cooperation between the two countries and perform a numerical exercise to better understand the characteristics of the optimal policy when one country is hit by a strong preference shock.

Our four main findings are as follows. First, the optimal policy still presents the characteristics of history dependence[6] and international dependence[7], as it is commonly found in the literature[8]. Second, the optimal monetary policy in the country that has been directly hit by the preference shock is unaffected by the presence of tariffs. Third, when trade tariffs are available, the optimal policy under cooperation involves using them to shift the shock's burden from the country with constrained monetary policy towards the country with unconstrained monetary policy. Finally, our fourth finding is that when the preference shock hits locally, a global liquidity trap does not alter the optimal policy (at least qualitatively).

This work relates to two different types of literature. The most direct contribution is to the literature of optimal monetary policy in the Zero Lower Bound. Initial results such as Eggertsson and Woodford (2003) focused on closed economies and emphasized the history dependence characteristic of the optimal monetary policy. Some other papers that analyse the closed economy case include Jung et al. (2005), Adam and Billi (2006) and Adam and Billi (2007). Later, papers such as Nakajima (2008) and Fujiwara et al. (2013) focused on the optimal policy in the Zero Lower Bound in a two country open economy model. These papers show that the optimal monetary policy in such a situation involves not only history dependence but also international dependence. Cook and Devereux (2013) also studies the optimal policy in the Zero Lower Bound in an open economy, but also include an analysis on how fiscal policy should behave in such a situation.

The literature on Local Currency Pricing is also closely related to our work. The closest connection is with Engel (2011), which our model is heavily inspired. Other papers that analyse the optimal policy in a two country environment with Local Currency Pricing are Devereux and Engel (2003) and Corsetti and Pesenti (2005). However, it is important to note that our model is significantly different from the set up presented in these two papers. For

[5]When firms have enough market power to differentiate between the prices of one good in one country and the price of the same good in a different country, i.e. the law of one price does not always hold.

[6]When the monetary authority commits to generating higher future inflation in order to further stimulate the economy in the present.

[7]When the optimal policy of one country is affected by the situation in the other country

[8]Nakajima (2008) and Fujiwara et al. (2013) are a couple of examples

example, our paper follows Engel (2011) and allow for a staggered price setting environment.

There are two main contributions in our work. First, we analyse the optimal monetary policy in the Zero Lower Bound under a Local Currency Pricing regime. This is one the extensions suggested in Fujiwara et al. (2013). The second contribution that our paper brings is the analysis of the effects of tariffs as policy tools in the Zero Lower Bound.

There is one additional paper that merits attention. Among other things, Caballero et al. (2015) studies the effect tariffs can have on the Zero Lower Bound. However, their set up is significantly different from what we present in this paper. Their focus is on studying the impact of ZLB and tariffs on capital flows. In order to do it, they create an overlapping generations model with permanently rigid local prices of local products. Our focus is to study the challenges related to macroeconomic stabilization under a liquidity trap. Hence, our setup follows more closely the tradition of Clarida et al. (2002), where we have an open New Keynesian model with staggered price settings and an infinitely lived representative agent in each of the two countries.

Finally, before we continue, it is important to make a disclaimer. Our model abstracts from any political economy considerations and is mute regarding the distributional impact from the use of trade tariffs. This is not an accurate representation of reality. For example, Autor et al. (2013) show that increased trade with China had affected the US labor market in an unequal way, generating profound distributional effects, while Fetzer and Schwarz (2019) show that China's reaction to Trump's trade war seem to involve a large dose of political motivation. Even though these issues are very much worth diving deeper, our model is poorly suited for this endeavor. Therefore, these considerations remain outside the scope of this paper.

The rest of the paper is organized as follows. First, chapter 2 describes our two country model. Then, chapter 3 describes the social welfare function and define the Ramsey problem our Central Planner will solve. After that, chapter 4 explains the numerical exercise we perform and analyse its results. Finally, chapter 5 concludes.

2
The Model

The model is very similar to the one presented in Engel (2011). There are two countries of equal size, called *home* (H) and *foreign* (F). Households have utility over consumption of goods and disutility from provision of labor. There is a continuum of monopolistic firms in each country, each one producing a differentiated good. Households supply labor for the firms in their own country and consume goods produced in both countries. Monopolistic firms produce output using only labor and are subject to technology shocks. We assume a complete international market of state contingent claims. While optimizing, firms can set different values for the price of its good in the home and foreign markets.

Our model has two main differences from the one in Engel (2011). First, we allow for preference shocks, modeled as in Galí (2015). Second, each country's government can charge a trade tariff on goods produced by the other country and sold in their own country.

2.1
Households

A representative household in country H has preferences given by

$$U_t = \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\sigma} C_{t+j}^{1-\sigma} - \frac{1}{1+\phi} N_{t+j}^{1+\phi} \right] Z_{t+j} \right\} \tag{2-1}$$

where $0 < \beta < 1$, $\sigma > 0$ and $\phi \geq 0$. N_t is the labor supplied by representative household[1]. Z_t is a preference shifter [2]. C_t is a consumption index and can be defined as:

$$C_t \equiv C_{H,t}^{\frac{\nu}{2}} C_{F,t}^{1-\frac{\nu}{2}}$$

where $0 < \nu \leq 2$. When $\nu > 1$, the household presents a home-bias in their preferences. In turn, $C_{H,t}$ and $C_{F,t}$ are consumption indexes for goods produced

[1] We assume the representative household supplies labor for each firm i in country H. N_t is an aggregate index for the labor supplied by the household and can be defined as $N_t \equiv \int_0^1 N_t(i) \mathrm{d}i$

[2] As stated in Galí (2015), because of the way this preference shifter enters in the utility function, we can interpret it as a shock in the effective discount factor, which becomes $Z_t \beta^t \quad \forall t \geq 0$

in country H and F, respectively, and can be defined as:

$$C_{H,t} \equiv \left[\int_0^1 C_{H,t}(h)^{\frac{\xi-1}{\xi}}\mathrm{d}h\right]^{\frac{\xi}{\xi-1}} \quad \text{and} \quad C_{F,t} \equiv \left[\int_0^1 C_{F,t}(f)^{\frac{\xi-1}{\xi}}\mathrm{d}f\right]^{\frac{\xi}{\xi-1}}$$

where $\xi > 1$. $C_{H,t}(h)$ is the home household's consumption of good h produced in country H at the time period t and, analogously, $C_{F,t}(f)$ is the home household's consumption of good f produced in country F at the time period t.

At each period t, the representative household in country H faces a budget constraint with complete international financial markets[3] given by:

$$P_{H,t}C_{H,t}+(1+\tau_t)P_{F,t}C_{F,t}+\sum_{\nabla^{t+1}\in\Omega_{t+1}} Q(\nabla^{t+1}|\nabla^t)D(\nabla^{t+1}) = W_tN_t+\Gamma_t-T_t+D(\nabla^t)$$

where τ_t is a tariff imposed on goods produced on country F. $D(\nabla^t)$ represents the payoff the household has on state-contingent claims for state ∇^t and $Q(\nabla^{t+1}|\nabla^t)$ is the price of a claim that pays one unit of currency of country H in state ∇^{t+1}, conditional on state ∇^t occurring at time t. Γ_t represents the aggregate profits of home firms. T_t is a lump-sum tax. W_t is the wage the representative household receives for its labor. $P_{H,t}$ and $P_{F,t}$ are price indexes, which can be defined as:

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(h)^{1-\xi}\mathrm{d}h\right]^{\frac{1}{1-\xi}} \qquad P_{F,t} \equiv \left[\int_0^1 P_{F,t}(f)^{1-\xi}\mathrm{d}f\right]^{\frac{1}{1-\xi}}$$

where $P_{H,t}(h)$ is the price of consumption good h produced in the country H and sold in the country H, while analogously $P_{F,t}(f)$ is the price of consumption good f produced in the country F and sold in the country H.

We also impose a No Ponzi Scheme condition on the household. As in Galí (2015), we use:

$$\lim_{T\to\infty} \mathbb{E}_t\left[\Lambda_{t,T}\frac{D(\nabla^{T+1})}{P_T}\right] \geq 0$$

where $\Lambda_{t,T}$ is the stochastic discount factor, equal to:

$$\Lambda_{t,T} = \beta^{T-t}\left(\frac{C_T}{C_t}\right)^{\sigma}$$

and P_t is a price index defined as:

[3] Claims are arbitrarily denominated in Home country currency

$$P_t \equiv k^{-1} P_{H,t}{}^{\frac{\nu}{2}} P_{F,t}{}^{1-\frac{\nu}{2}} \quad \text{where} \quad k \equiv \left(1-\frac{\nu}{2}\right)^{1-\frac{\nu}{2}} \left(\frac{\nu}{2}\right)^{\frac{\nu}{2}}$$

Solving the household's maximization problem we get[4] :

$$\frac{Q(\nabla^{t+1}|\nabla^t)}{\Pi(\nabla^{t+1}|\nabla^t)} = \beta \left[\left(\frac{C_{t+1}(\nabla^{t+1})}{C_t(\nabla^t)}\right)^{-\sigma} \left(\frac{1+\tau_t}{1+\tau_{t+1}}\right)^{1-\frac{\nu}{2}} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{Z_{t+1}}{Z_t}\right) \right] \tag{2-2}$$

where $\Pi(\nabla^{t+1}|\nabla^t)$ is the probability of state ∇^{t+1} occurring conditional on state ∇^t occurring at time t. We also find:

$$\frac{W_t}{P_{H,t}} = k^{-1}(1+\tau_t)^{1-\frac{\nu}{2}} S_t^{1-\frac{\nu}{2}} C_t^{\sigma} N_t^{\phi} \quad \text{where } S_t \equiv \frac{P_{F,t}}{P_{H,t}}$$

The foreign household's problem is analogous to the one faced by the household in country H. Variables in the foreign household's problem are indexed by a *. Hence, by solving the problem faced by the household in country F, we find:

$$\mathbb{E}_t \left[\frac{E_{t+1}}{E_t}\right] \frac{Q(\nabla^{t+1}|\nabla^t)}{\Pi(\nabla^{t+1}|\nabla^t)} = \beta \left[\left(\frac{C^*_{t+1}(\nabla^{t+1})}{C^*_t(\nabla^t)}\right)^{-\sigma} \left(\frac{1+\tau^*_t}{1+\tau^*_{t+1}}\right)^{1-\frac{\nu}{2}} \left(\frac{P^*_t}{P^*_{t+1}}\right) \left(\frac{Z^*_{t+1}}{Z^*_t}\right) \right] \tag{2-3}$$

where E_t is the nominal exchange rate between country H and country F, defined as currency in country H divided by currency in country F. We also find:

$$\frac{W^*_t}{P^*_{F,t}} = k^{-1}(1+\tau^*_t)^{1-\frac{\nu}{2}} S_t^{*\,1-\frac{\nu}{2}} C_t^{*\sigma} N_t^{*\phi} \quad \text{where } S^*_t \equiv \frac{P^*_{H,t}}{P^*_{F,t}}$$

Finally, we also find a risk sharing condition between both households:

$$\left(\frac{C_t}{C^*_t}\right)^{\sigma} = \frac{E_t P^*_{H,t}}{P_{H,t}} \left(\frac{1+\tau^*_t}{1+\tau_t}\right)^{1-\frac{\nu}{2}} S_t^{*-\frac{\nu}{2}} S_t{}^{-\left(1-\frac{\nu}{2}\right)} \tag{2-4}$$

2.2 Firms

We assume each firm in the home country is a monopolist of good h and produces output $Y_t(h)$ using a linear technology:

$$Y_t(h) = A_t N_t(h)$$

where A_t is a productivity shock, common to all firms in country H.

The profit function of each firm in the home country is given by:

[4]Complete calculations for the household problem can be found in the Appendix

$$\Gamma_t(h) = P_{H,t}(h)C_{H,t}(h) + E_t P^*_{H,t}(h)C^*_{H,t}(h) - (1-\tau_S)W_t N_t(h)$$

where τ_S is an employment subsidy from the government[5].

By hypothesis, firms operate under a Local Currency Pricing (LCP) framework. Firms will have market power to differentiate between the price it charges for good h in country H and the price it charges for good h in country F. Hence, when maximizing its profit, the firm will choose both $P_{H,t}(h)$ and $P^*_{H,t}(h)$. However, we assume prices can only be readjusted at random intervals as in Calvo (1983). When the firm h is selected to adjust prices in period t, it will adjust both $P_{H,t}(h)$ and $P^*_{H,t}(h)$.

Solving firm's h maximization problem we can derive the following conditions[6]:

$$\mathbb{E}_t\left\{\sum_{j=0}^{\infty}(\theta\beta)^j\left(\frac{C_{t+j}}{C_t}\right)^{-\sigma}\left(\frac{1+\tau_t}{1+\tau_{t+j}}\right)^{1-\frac{\nu}{2}}\frac{P_t}{P_{t+j}}\left(\frac{P^o_{H,t}}{P_{H,t+j}}\right)^{-\xi}C_{H,t+j}\left[P^o_{H,t}-\frac{\xi}{\xi-1}(1-\tau_S)\frac{W_{t+j}}{A_{t+j}}\right]\right\}=0 \tag{2-5}$$

where $(1-\theta)$ is the probability of the firm being called to adjust its price at time t and $P^o_{H,t}$ is the optimal price chosen by the firm for good h, sold in country H, when the firm is called to adjust its prices at time t. We can also derive:

$$\mathbb{E}_t\left\{\sum_{j=0}^{\infty}(\theta\beta)^j\left(\frac{C_{t+j}}{C_t}\right)^{-\sigma}\left(\frac{1+\tau_t}{1+\tau_{t+j}}\right)^{1-\frac{\nu}{2}}\frac{P_t}{P_{t+j}}\left(\frac{{P^*_{H,t}}^o}{P^*_{H,t+j}}\right)^{-\xi}C^*_{H,t+j}\left[E_t{P^*_{H,t}}^o-\frac{\xi}{\xi-1}(1-\tau_S)\frac{W_{t+j}}{A_{t+j}}\right]\right\}=0 \tag{2-6}$$

where ${P^*_{H,t}}^o$ is the optimal price chosen by the firm for good h, sold in country F, when the firm is called to adjust its prices at time t.

The problem firm f faces in the foreign country is analogous. Variables in foreign firms' problem are indexed by a *. Hence, when solving the foreign firm's problem we find the following condition:

$$\mathbb{E}_t\left\{\sum_{j=0}^{\infty}(\theta\beta)^j\left(\frac{C^*_{t+j}}{C^*_t}\right)^{-\sigma}\left(\frac{1+\tau^*_t}{1+\tau^*_{t+j}}\right)^{1-\frac{\nu}{2}}\frac{E_t P^*_t}{E_{t+j}P^*_{t+j}}\left(\frac{P^o_{F,t}}{P_{F,t+j}}\right)^{-\xi}C_{F,t+j}\left[P^o_{F,t}-\frac{\xi}{\xi-1}(1-\tau^*_S)\frac{W^*_{t+j}}{A^*_{t+j}}\right]\right\}=0 \tag{2-7}$$

[5]This subsidy is a way for the government to eliminate inefficiencies related to market power. Hence, we assume $\tau_S = \tau^*_S = \frac{1}{\xi}$ throughout the entire work

[6]Complete calculations for firm's h maximization problem can be found in the Appendix

where $P^o_{F,t}$ is the optimal price chosen by the firm for good f, sold in country H, when the firm is called to adjust its price at time t. We also get:

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty} (\theta\beta)^j \left(\frac{C^*_{t+j}}{C^*_t}\right)^{-\sigma} \left(\frac{1+\tau^*_t}{1+\tau^*_{t+j}}\right)^{1-\frac{\nu}{2}} \frac{E_t P^*_t}{E_{t+j} P^*_{t+j}} \left(\frac{{P^*_{F,t}}^o}{P^*_{F,t+j}}\right)^{-\xi} C^*_{F,t+j} \left[{P^*_{F,t}}^o - \frac{\xi}{\xi-1}(1-\tau^*_S)\frac{W^*_{t+j}}{A^*_{t+j}} \right] \right\} = 0 \tag{2-8}$$

where ${P^*_{F,t}}^o$ is the optimal price chosen by the firm for good f, sold in country F, when the firm is called to adjust its price at time t.

2.3 Market Clearing

For markets to clear, aggregate supply of a given good must be equal to the aggregate demand of said good. Hence, we have:

$$Y_t(h) = C_{H,t}(h) + C^*_{H,t}(h) \quad \text{and} \quad Y_t(f) = C_{F,t}(h) + C^*_{F,t}(f)$$

This implies[7]:

$$Y_t = k^{-1}\left[\frac{\nu}{2}(1+\tau_t)^{1-\frac{\nu}{2}} {S_t}^{1-\frac{\nu}{2}} C_t + \left(1-\frac{\nu}{2}\right)(1+\tau^*_t)^{-\frac{\nu}{2}} {S^*_t}^{-\frac{\nu}{2}} C^*_t\right] \tag{2-9}$$

and

$$Y^*_t = k^{-1}\left[\left(1-\frac{\nu}{2}\right)(1+\tau_t)^{-\frac{\nu}{2}} {S_t}^{-\frac{\nu}{2}} C_t + \frac{\nu}{2}(1+\tau^*_t)^{1-\frac{\nu}{2}} {S^*_t}^{1-\frac{\nu}{2}} C^*_t\right] \tag{2-10}$$

2.4 Log-linearized Model

We now present log-linearized versions of the equations from the model above[8]. In our notation, lower case letters represent the deviation from the log of the corresponding upper case from steady state.

Before we proceed, we define m_t, the *currency misalignment*, as:

$$m_t \equiv \frac{1}{2}(e_t + p^*_{H,t} - p_{H,t} + e_t + p^*_{F,t} - p_{F,t})$$

As in Engel (2011), m_t is the average deviation of consumer prices in the foreign country from consumer prices in the home currency.

[7]Complete calculations can be found in the Appendix

[8]Complete calculations can be found in the Appendix

We start by log-linearizing equations (2-2) and (2-3). We find:

$$i_t - \mathbb{E}_t[\pi_{t+1}] = \sigma(\mathbb{E}_t[c_{t+1}] - c_t) + \left(1 + \frac{\nu}{2}\right)(\mathbb{E}_t[\tau_{t+1}] - \tau_t) - (\mathbb{E}_t[z_{t+1}] - z_t) \quad (2\text{-}11)$$

and

$$i_t^* - \mathbb{E}_t[\pi_{t+1}^*] = \sigma(\mathbb{E}_t[c_{t+1}^*] - c_t^*) + \left(1 + \frac{\nu}{2}\right)(\mathbb{E}_t[\tau_{t+1}^*] - \tau_t^*) - (\mathbb{E}_t[z_{t+1}^*] - z_t^*) \quad (2\text{-}12)$$

It will also be useful to log-linearize the definitions of π_t and π_t^*. We get:

$$\pi_t = \frac{\nu}{2}\pi_{H,t} + \frac{2-\nu}{2}\pi_{F,t} \quad (2\text{-}13)$$

and

$$\pi_t^* = \frac{\nu}{2}\pi_{F,t}^* + \frac{2-\nu}{2}\pi_{H,t}^* \quad (2\text{-}14)$$

Next, by combining log-linearized versions of the risk sharing condition (2-4) and the market clearing conditions (2-9) and (2-10) to get:

$$c_t^R = \frac{\nu-1}{D}y_t^R + \frac{\nu(2-\nu)}{2D}m_t - \frac{\nu(2-\nu)}{4D}\tau_t + \frac{\nu(2-\nu)}{4D}\tau_t^* \quad (2\text{-}15)$$

$$c_t^W = y_t^W \quad (2\text{-}16)$$

$$s_t = \frac{2\sigma}{D}y_t^R - \frac{\nu-1}{D}m_t + \frac{(2-\nu)[(1-\sigma)\nu-1]}{2D}\tau_t - \frac{(2-\nu)[(1-\sigma)\nu-1]}{2D}\tau_t^* \quad (2\text{-}17)$$

where we are using the following definitions:

$$c_t^R \equiv \frac{1}{2}(c_t - c_t^*) \quad (2\text{-}18)$$

$$c_t^W \equiv \frac{1}{2}(c_t + c_t^*) \quad (2\text{-}19)$$

$$y_t^R \equiv \frac{1}{2}(y_t - y_t^*) \quad (2\text{-}20)$$

$$y_t^W \equiv \frac{1}{2}(y_t + y_t^*) \quad (2\text{-}21)$$

$$D \equiv (\nu-1)^2 + \sigma\nu(2-\nu)$$

It will also be useful to log-linearize the definition for s_t. We get:

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t} \quad (2\text{-}22)$$

Now, we move to the firms' equations. By log-linearizing the first order conditions (2-5) and (2-6) we found solving the Home firm's optimization problem, we have:

$$\pi_{H,t} = \delta\left[\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W + \frac{D-(\nu-1)}{2D}m_t + \Xi_1\tau_t + \Xi_2\tau_t^*\right] + \beta\mathbb{E}_t[\pi_{H,t+1}] + u_t \tag{2-23}$$

and

$$\pi_{H,t}^* = \delta\left[\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W - \frac{D+\nu-1}{2D}m_t + \Xi_1\tau_t + \Xi_2\tau_t^*\right] + \beta\mathbb{E}_t[\pi_{H,t+1}^*] + u_t \tag{2-24}$$

where we define:

$$\tilde{y}_t^R \equiv y_t^R - \overline{y}_t^R \tag{2-25}$$

$$\tilde{y}_t^W \equiv y_t^W - \overline{y}_t^W \tag{2-26}$$

$$\delta \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$$

$$\Xi_1 \equiv -[\sigma+\phi(\nu-1)]\left(\frac{\nu(2-\nu)}{4D}\right) + (1+\phi\nu)\left(\frac{2-\nu}{2}\right)\frac{(2-\nu)[(1-\sigma)\nu-1]}{2D} + \left(1+\frac{\phi\nu}{2}\right)\left(\frac{2-\nu}{2}\right)$$

$$\Xi_2 \equiv [\sigma+\phi(\nu-1)]\left(\frac{\nu(2-\nu)}{4D}\right) - (1+\phi\nu)\left(\frac{2-\nu}{2}\right)\frac{(2-\nu)[(1-\sigma)\nu-1]}{2D} - \frac{\phi\nu}{2}\left(\frac{2-\nu}{2}\right)$$

We call $\tilde{y}_t^R$ the *relative output gap* and $\tilde{y}_t^W$ the *world output gap*. Note that we have also used $\overline{y}_t^R$, the *relative natural output*, and $\overline{y}_t^W$, the *world natural output*. We can find the natural output values by solving the following system of equations:

$$a_t = \left(\frac{\sigma}{D}+\phi\right)\overline{y}_t^R + (\sigma+\phi)\overline{y}_t^W - \phi a_t \tag{2-27}$$

$$a_t^* = -\left(\frac{\sigma}{D}+\phi\right)\overline{y}_t^R + (\sigma+\phi)\overline{y}_t^W - \phi a_t^* \tag{2-28}$$

Similarly, when we log-linearize the first-order conditions (2-7) and (2-8) related to the foreign firm optimization problem, we get:

$$\pi_{F,t} = \delta\left[-\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W + \frac{D+\nu-1}{2D}m_t + \Xi_2\tau_t + \Xi_1\tau_t^*\right] + \beta\mathbb{E}_t[\pi_{F,t+1}] + u_t^* \tag{2-29}$$

and

$$\pi^*_{F,t} = \delta\left[-\left(\frac{\sigma}{D}+\phi\right)\tilde{y}^R_t + (\sigma+\phi)\tilde{y}^W_t - \frac{D-(\nu-1)}{2D}m_t + \Xi_2\tau_t + \Xi_1\tau^*_t\right] + \beta\mathbb{E}_t[\pi^*_{F,t+1}] + u^*_t \tag{2-30}$$

Finally, we assume that the productivity shock a_t, the cost-push shock u_t and the preference shock z_t all follow an auto-regressive structure. Hence,

$$a_t = \rho_a a_{t-1} + \epsilon^a_t \tag{2-31}$$

$$u_t = \rho_u u_{t-1} + \epsilon^u_t \tag{2-32}$$

$$z_t = \rho_z z_{t-1} + \epsilon^z_t \tag{2-33}$$

The corresponding shocks a^*_t, U^*_t and z^*_t in the foreign country have an analogous structure:

$$a^*_t = \rho_a a^*_{t-1} + \epsilon^{a^*}_t \tag{2-34}$$

$$u^*_t = \rho_u u^*_{t-1} + \epsilon^{u^*}_t \tag{2-35}$$

$$z^*_t = \rho_z z^*_{t-1} + \epsilon^{z^*}_t \tag{2-36}$$

3
The Ramsey Problem

To close the model presented in the previous chapter, we need a policy block with equations for instruments i_t, i_t^*, τ_t and τ_t^*. However, since we are interested in the optimal policies, we do not present preset policy functions. Instead, we will solve a Ramsey problem to find the optimal policy responses.

In order to solve the Ramsey problem, we must first derive a loss function. To do it, we perform second order approximations to the households' utility functions (equation (2-1) and its foreign household counterpart). To avoid complications related to non-cooperative strategic behavior from each country, we will only analyse a situation in which both (credibly) cooperate[1]. Therefore, we derive a joint welfare function.

The Central Planner seeks to minimize a loss function given by:

$$\text{Loss} = \mathbb{E}_t \left[\sum_{j=0}^{\infty} \beta^j X_{t+j} \right]$$

where X_t is the period utility loss, i.e. the difference between the maximum utility achievable given an efficient allocation and the utility achieved by a given allocation. We can show that X_t is equal to[2]:

$$\begin{aligned} X_t = & \left[\frac{\sigma}{D} + \phi \right] \left(\tilde{y}_t^R \right)^2 + (\sigma + \phi) \left(\tilde{y}_t^W \right)^2 + \frac{\nu(2-\nu)}{4D}(m_t)^2 + \chi_1(\tau_t)^2 + \chi_1(\tau_t^*)^2 + \\ & + \frac{\sigma\nu(2-\nu)}{4D} y_t^R(\tau_t - \tau_t^*) + \chi_2 m_t(\tau_t - \tau_t^*) + \chi_3 \tau_t \tau_t^* + \\ & + \frac{\xi}{2\delta} \left[\frac{\nu}{2}(\pi_{H,t})^2 + \frac{2-\nu}{2}(\pi_{F,t})^2 + \frac{\nu}{2}(\pi_{F,t}^*)^2 + \frac{2-\nu}{2}(\pi_{H,t}^*)^2 \right] \end{aligned}$$

where

$$\chi_1 \equiv \frac{\nu(2-\nu)}{8D^2} \left\{ \frac{(1+\sigma)\nu(2-\nu)}{2} + 1 + D^2 + D\frac{(2-\nu)[(1-\sigma)\nu - 1]}{2} + \frac{(2-\nu)^2[(1-\sigma)\nu-1]^2}{2} \right\}$$

[1] We abstract from the question of how this commitment to cooperation is credible

[2] Complete calculations can be found in the Appendix

$$\chi_2 \equiv \frac{\nu(2-\nu)}{8D^2}\left\{2(2-\nu)[1-(1-\sigma)\nu(2-\nu)]+D(\nu-1)\right\}$$

$$\chi_3 \equiv \frac{\nu(2-\nu)}{8D^2}\left\{\sigma\nu(2-\nu)+D(2-\nu)[(1-\sigma)\nu-1]+\frac{(2-\nu)^2[(1-\sigma)\nu-1]^2}{2}\right\}$$

Note that this loss function is different from the one presented in Engel (2011). Tariffs not only generate a direct loss of efficiency but tariffs misalignment $(\tau_t - \tau_t^*)$ can also be an important factor by increasing (or decreasing) the losses related with relative output y_t^R and currency misalignment m_t.

What is the intuition behind the presence of tariffs in the loss function? There are two parts to it. The first one is related to the change in households cost. By altering how much households have to pay for a good, tariffs alter the intertemporal consumption-saving decision, distorting the equilibrium. Note that this effect is independent of the other variables in the model.

The second channel by which tariffs directly influence households' welfare is by distorting the optimal allocation between consumption of goods produced in country H and goods produced in country F. This is the component related with tariff misalignment. When $(\tau_t - \tau_t^*) > 0$ there is an aggregate increase in the relative cost of Foreign products, leading to an increase in the aggregate demand for Home goods. An analogous effect happen when $(\tau_t - \tau_t^*) < 0$, leading to an increase in the aggregate demand for Foreign goods. Note that the welfare loss related to tariff misalignment interacts with the relative output y_t^R and with currency misalignment m_t. Both relative output y_t^R and the currency misalignment m_t also affect the allocation between consumption of Home and Foreign goods. Tariff misalignment can either increase the welfare loss associated with the relative output and currency misalignment by distorting the equilibrium in the same direction as these variables or decrease this welfare cost if it distorts the economy in the opposite direction of them.

For example, if $y_t^R > 0$, there is pressure towards increasing the consumption share of goods produced in country H (since there is an increase in the relative quantity of goods produced in country H when compared to goods produced in country F). When $(\tau_t - \tau_t^*) > 0$, tariff misalignment is also adding pressure towards a higher share of Home goods consumption. Therefore, it is increasing the welfare loss associated with $y_t^R > 0$. On the other hand, if $(\tau_t - \tau_t^*) < 0$, tariff misalignment is distorting the equilibrium in favor of a higher share of Foreign goods consumption. Hence, it is partially correcting the distortion associated with $y_t^R > 0$, decreasing the welfare loss associated with it.

In solving the Ramsey Problem, the Central Planner will choose the sequence of $\{\pi_{H,t}, \pi^*_{F,t}, \pi^*_{H,t}, \tilde{y}^R_t, \tilde{y}^W_t, m_t, i_t, \pi_t, c_t, i^*_t, \pi^*_t, c^*_t, c^R_t, c^W_t, y^W_t, y^R_t, y^*_t, y_t, s_t, \tau_t, \tau^*_t\}_{t=0}^{\infty}$ that minimizes the loss function, subject to equations (2-11) to (2-36) as constraints. It is worth noting that we are solving the Ramsey problem under credible commitment from both countries[3].

Since we are also interested in how the optimal policy changes when it is subject to the Zero Lower Bound, we also include two other constraints:

$$i_t \geq -\frac{1}{\beta} \tag{3-1}$$

$$i^*_t \geq -\frac{1}{\beta} \tag{3-2}$$

Note that $\frac{1}{\beta}$ is the steady state value of the nominal interest rate[4]. Our model is expressed in log deviations of the steady state. Therefore, for a given variable to be below zero, its negative log deviation must be larger, in absolute value, than its steady state value. Hence, constraints (3-1) and (3-2) are equivalent to a Zero Lower Bound constraint on the level of the nominal interest rate.

[3]We abstract from the question of which technology is used to ensure the credibility of the commitment

[4]This can be derived by evaluating (2-2) and (2-3) with steady state values for all variables

4
Some Numerical Examples

In order to understand the impact of tariffs and the Zero Lower Bound in the optimal policies chosen by the Central Planner, we will consider some numerical examples.

4.1
Model calibration

The first step in these numerical exercises will be to calibrate the model's parameters. Most of our calibration is similar to Engel (2011)[1]. Table 4.1 summarizes our assumptions:

Parameter	Value	Explanation
β	0.99	Subjective discount factor
θ	0.75	Probability of a firm not being called to readjust prices in t
ν	1.5	Home bias
σ	2	Inverse of the intertemporal elasticity of substitution
ϕ	0	Elasticity of the disutility of labor supply
ξ	7.88	Elasticity of substitution among differentiated goods
$\rho_a = \rho_u = \rho_z$	0.95	Shocks' autoregressive coefficients

Table 4.1: Parameter Values

4.2
Characteristics of the numerical exercise

Now, we will describe the characteristics of our numerical exercise. They are as follows:

- On $t = 0$, both economies are in the non-inflationary steady state equilibrium
- On $t = 5$, a large negative preference shock (ϵ_t^z, from equation (2-33)) hits the Home economy, forcing the natural interest rate to be lower than zero[2]. Since the Home country is constrained by the Zero Lower Bound,

[1]There are two exceptions. Since values for σ and ξ are not provided by Engel (2011), we adopt the calibration in Fujiwara et al. (2013) for these parameters.

[2]As noted in chapter 2, the way we model the preference shifter Z_t means it can be considered a shock to the effective discount rate. Since the steady state value of the interest

it cannot set the interest rate equal to the natural interest rate as it normally would.

– On $t = 11$, a large positive preference shock (the same ϵ_t^z as in $t = 5$) hits the Home economy. This shock is equal, in absolute value, to the shock that hit the economy on $t = 5$. Hence, the natural interest rate is back to a positive value.

Note that we will restrict our attention to the case in which there are no productivity shocks ($a_t = a_t^* = 0$ for all t) and no cost push shocks ($u_t = u_t^* = 0$ for all t).

4.3 Results

We use the OccBin algorithm[3] to calculate the optimal policy given the first order conditions of the Ramsey Problem described in chapter 3 and the shock described in section 4.2. There are four different versions of our exercise. First, we analyse a situation where only the Home country is constrained by the Zero Lower Bound. We start by considering the case where countries do not have access to trade tariffs and then move to a situation in which the countries can actively use trade tariffs as a policy tool. After that, we analyse the optimal responses when both countries are constrained by the Zero Lower Bound. Once again, we start in a situation in which tariffs are unavailable and then move to the case where they can be use as policy tools.

Finally, we have also considered the possibility that the preference shock hits both countries simultaneously. However, the results do not contribute meaningfully to the discussions presented below. Hence, we have relegated these results and our discussion on it for the Appendix.

4.3.1 Local Liquidity Trap

First, we start by considering a situation in which only the Home country is constrained by the Zero Lower Bound. This is similar to the situation analysed in Nakajima (2008). However, there are two main differences. First, our model uses the Local Currency Pricing hypothesis (instead of Producer Currency Pricing). Second, we allow the use of tariffs as a policy tool for both countries.

rate, the natural interest rate, is a function of the effective discount rate, a shock to the effective discount rate can be interpreted as a shock to the natural interest rate.

[3]Described in Guerrieri and Iacoviello (2015)

4.3.1.1
No Tariffs

Figure 4.1 presents the impulse response functions for the Central Planner's optimal policy when only the Home country is constrained by the Zero Lower Bound and countries do not have access to trade tariffs.

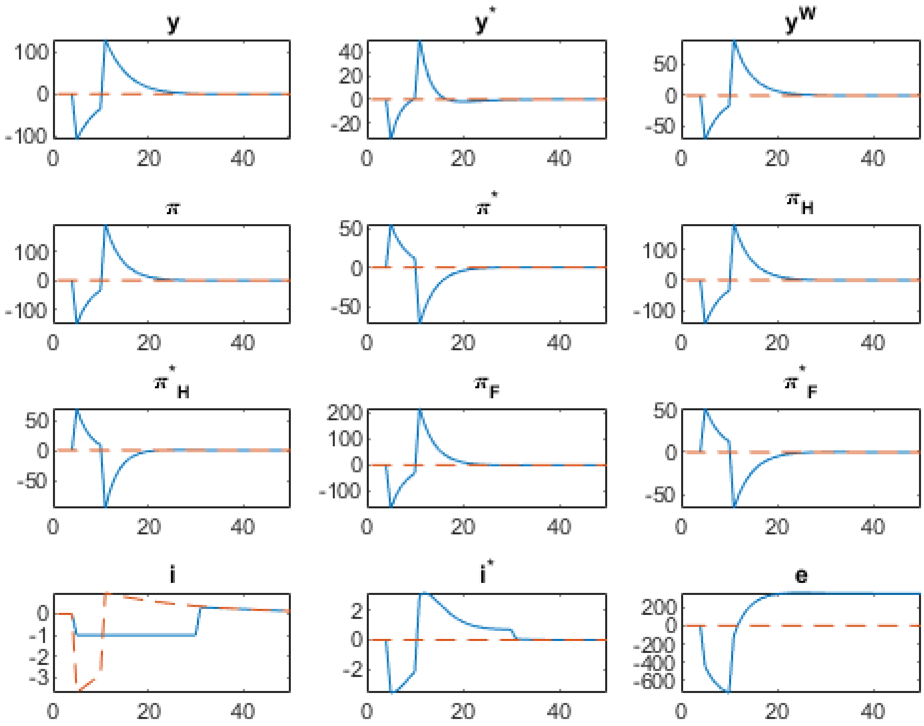

Figure 4.1: Optimal policy without tariffs in a local liquidity trap. Solid blue line represents the impulse response functions when the Home country is constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.

First, as in Nakajima (2008) (and initially noted in Eggertsson and Woodford (2003) in a closed economy environment), the optimal policy implies *history dependence*. That means that a country in a liquidity trap can reduce the losses it sustains by committing itself to generating inflation and a positive output gap in the future.

Another characteristic that is similar to Nakajima (2008) is *international dependence*. When the Zero Lower Bound binds, the optimal monetary policy of the foreign country (which did not suffer from the preference shock) depends on the situation the home country. That is, the optimal policy path requires coordination between both monetary authorities. The exact form of the cooperation varies with the parameter calibration. As further explored in

Fujiwara et al. (2013), the value of σ will determine if goods produced in the two countries are Edgeworth complements or substitutes[4], which in turn will determine the optimal policy reaction in the Foreign country.

Note that both history dependence and international dependence are a feature of the optimal policy in the Zero Lower Bound. When the ZLB does not bind, the Home country can fully stabilize the output gap using monetary policy[5].

4.3.1.2
Tariffs

Figure 4.2 presents the impulse response functions for the Central Planner's optimal policy when only the Home country is constrained by the Zero Lower Bound and countries can use trade tariffs as a policy tool.

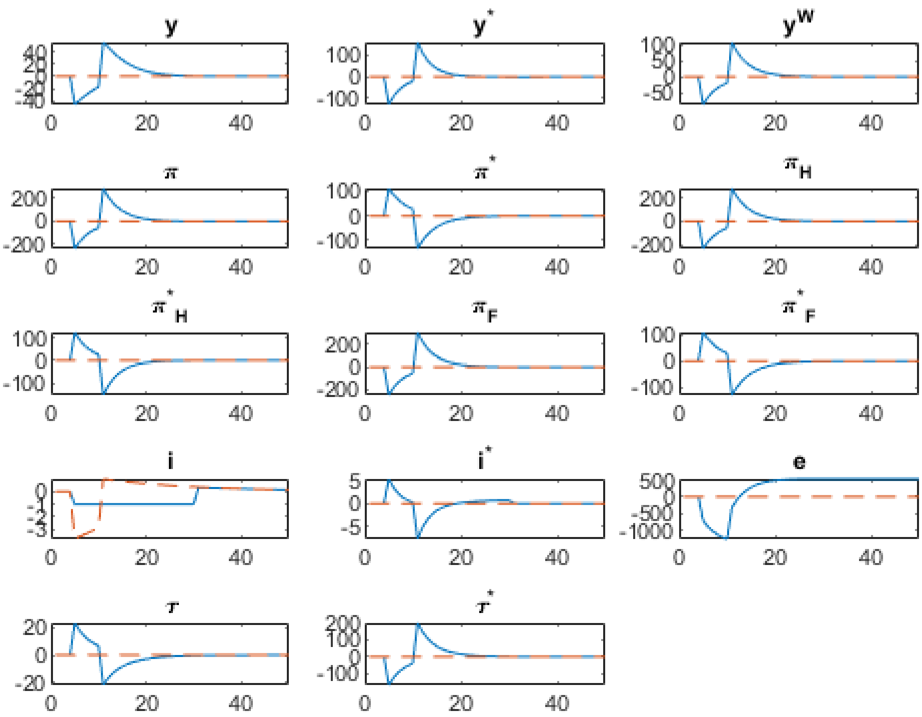

Figure 4.2: Optimal policy with tariffs in a local liquidity trap. Solid blue line represents the impulse response functions when the Home country is constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.

[4]If $\sigma > 1$, goods are Edgeworth substitutes, if $\sigma < 1$, goods are Edgeworth complements. Hence, since $\sigma = 2$ in our calibration, goods produced in the two countries are substitutes

[5]Note that in Engel (2011) the optimal monetary policy in a model with Local Currency Pricing is not efficient. However, the shock analysed in that paper is a productivity shock, instead of a preference shock as we are analysing here

The first result we have is that the optimal monetary policy of the Home country, which was hit by the preference shock and is constrained by the Zero Lower Bound, doesn't change when we introduce the possibility of using tariffs as a policy tool. The optimal monetary policy still implies history dependence. It is worth noting that the timing of the liftoff is also unchanged. Also note that the optimal policy still exhibits international dependence, i.e. the optimal policy in the Foreign country is affected by the situation in Home country.

However, even though the optimal monetary policy in the Home country is similar to the scenario with no tariffs, there are some notable differences when comparing the optimal path for other variables and the impulse response functions presented in subsubsection 4.3.1.1.

Figure 4.3 compares the optimal paths of output gaps between the solution with no tariffs and the solution with tariffs. Note that, when tariffs are available, the negative output gap in the Home country (which was hit by the preference shock) is smaller (in absolute value) when compared to the solution with no tariffs. On the other hand, the output gap in the Foreign country (which was not hit a preference shock) is more negative in the optimal policy with tariffs.

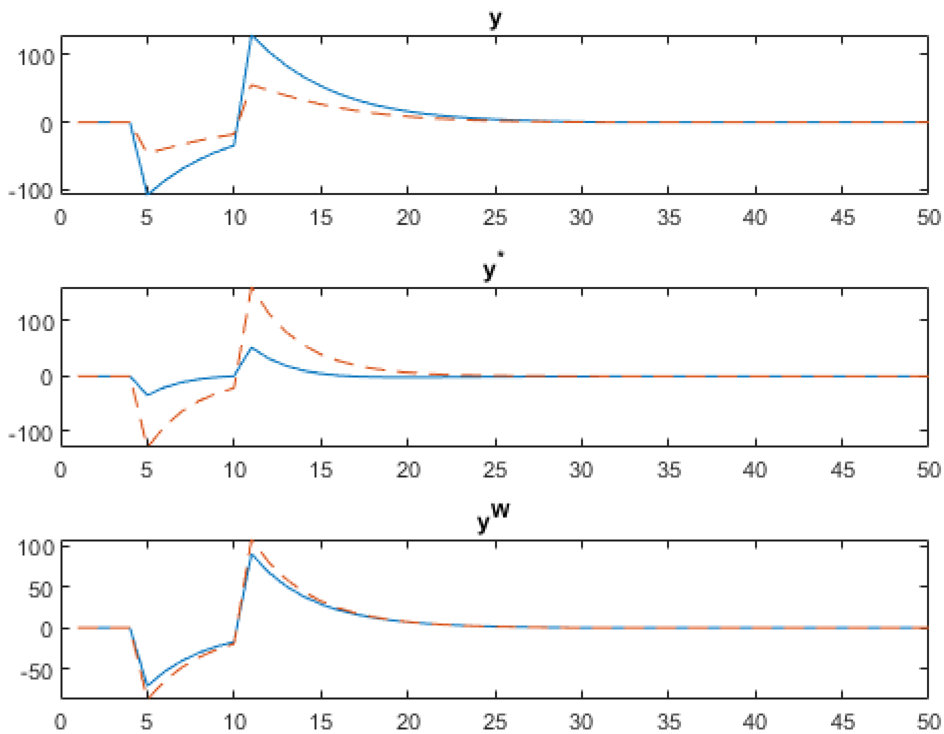

Figure 4.3: Optimal behavior of output gaps. Solid blue line represents the optimal response function when tariffs are not available. Red dashed line represents the optimal response function when tariffs are a policy tool

Why is the Central Planner adopting such a policy? Tariffs allow the Central Planner to transfer part of the shock from the country that has a constrained monetary policy (country H) towards the country which has unconstrained monetary policy (country F). Since the Foreign country is not subject to the Zero Lower Bound, it has better tools to handle the negative effect of the shock. Therefore, overall welfare in the economy improves, even if country F is in a worse situation than when compared to the equilibrium without tariffs. This intuition also highlights the importance of cooperation for achieving our results. The Foreign country has to willingly accept a worse domestic situation in order to improve the world's welfare. If each country were to optimize only considering domestic conditions, results would probably look very different.

How does the Central Planner shifts the burden of the shock from the Home country to the Foreign country? By increasing the share of Home goods consumed through the use of tariffs. In particular, between $t = 5$ and $t = 11$, the Central Planner chooses to increase tariffs in country H and decrease them in country F. Since the economy works under Local Currency Pricing, the exchange rate won't automatically fully offset the effect of an increase in tariffs. Hence, higher tariffs in country H can (temporarily) make Foreign products more expensive and generate a shift in consumption towards local products, boosting domestic production. Similarly, lower tariffs in country F can (temporarily) make products from the Home country cheaper and generate a shift in consumption towards them, boosting production in country H. Figure 4.4 illustrates how tariffs alter the ratio between consumption of goods produced in the Home country and consumption of goods produced in the Foreign country. Note that, while the Home country has a negative natural interest rate (between $t = 5$ and $t = 11$), the share of Home goods consumed increases in both countries. Hence, the Home country must increase its domestic production in order to meet this increase in (relative) demand. On the other hand, the Foreign country lowers its production, since there is a (relative) decrease in demand for its products. This generates the change in the output gaps we can observe in figure 4.3.

Finally, it should also be mentioned how tariffs change the international dependence aspect of the optimal policy. Note that the optimal interest rate path is significantly different than what we had in subsubsection 4.3.1.1. This is especially notable since we did not change the calibration of the model (in particular, the fact that goods produced in both countries are substitutes). When tariffs are unavailable, the optimal policy involves added stimulus in the Foreign economy through a lower interest rate i_t^*. However, when active

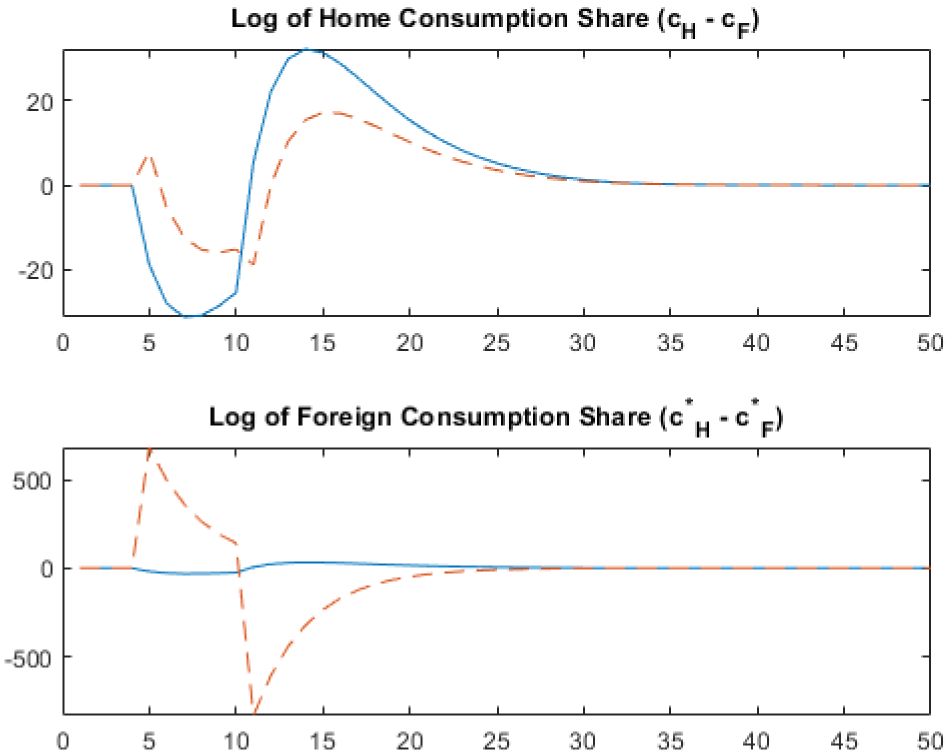

Figure 4.4: Evolution of the (log) ratio between consumption goods produced in the Home country and consumption of goods produced in the Foreign country. Solid blue line represents the optimal response function when tariffs are not available. Red dashed line represents the optimal response function when tariffs are a policy tool

use of tariffs is an option, the optimal policy path involves reducing τ_t^*, which already provides some stimulus to country F (through lower prices). In fact, instead of adding stimulus through lower interest rates, as in the previous scenario, the optimal policy now involves a higher interest rate i_t^*, reducing part of the stimulus provided by lower τ_t^*

4.3.2
Global Liquity Trap

Now, we will consider a scenario in which both countries are subject to the Zero Lower Bound. This situation is similar to the one analysed in Fujiwara et al. (2013). As with subsection 4.3.1, the two main differences from the previous work are the use of Local Currency Pricing (instead of Producer Currency Pricing) and the possibility of using trade tariffs as a policy tool.

4.3.2.1
No Tariffs

Figure 4.5 presents the impulse response functions for the Central Planner's optimal policy when both countries are constrained by the Zero Lower Bound and countries do not have access to trade tariffs.

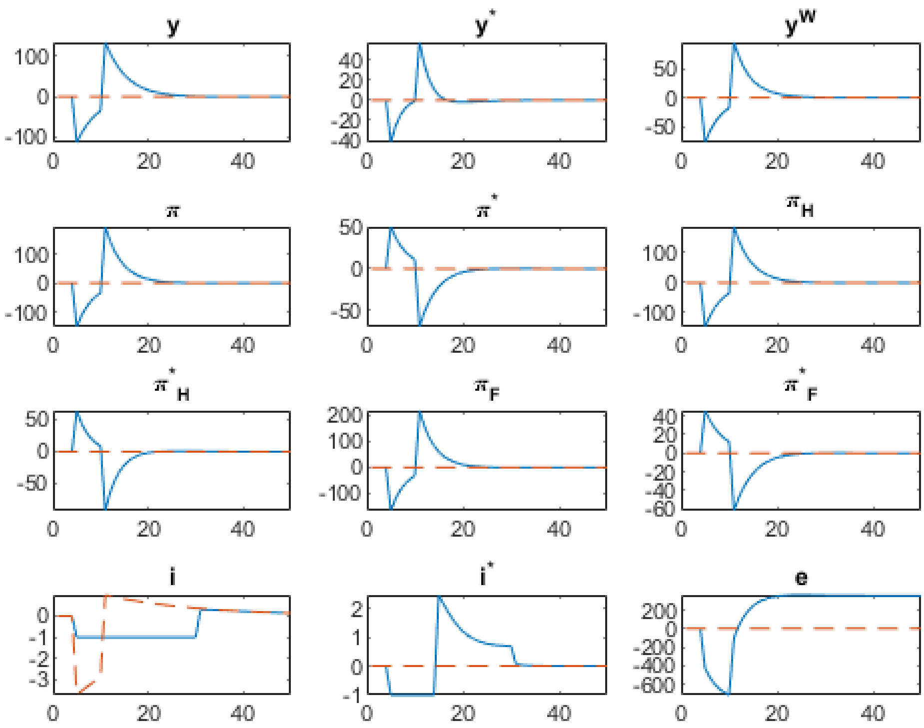

Figure 4.5: Optimal policy without tariffs in a global liquidity trap. Solid blue line represents the impulse response functions when the both countries are constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.

Results are very similar to the ones presented in subsubsection 4.3.1.1. Even though the Zero Lower Bound is also a binding constraint for the Foreign country, the optimal policy paths are, qualitatively, similar to the case where only the Home country is constrained.

4.3.2.2
Tariffs

Figure 4.6 presents the impulse response functions for the Central Planner's optimal policy when both countries are constrained by the Zero Lower Bound and countries can use trade tariffs as a policy tool.

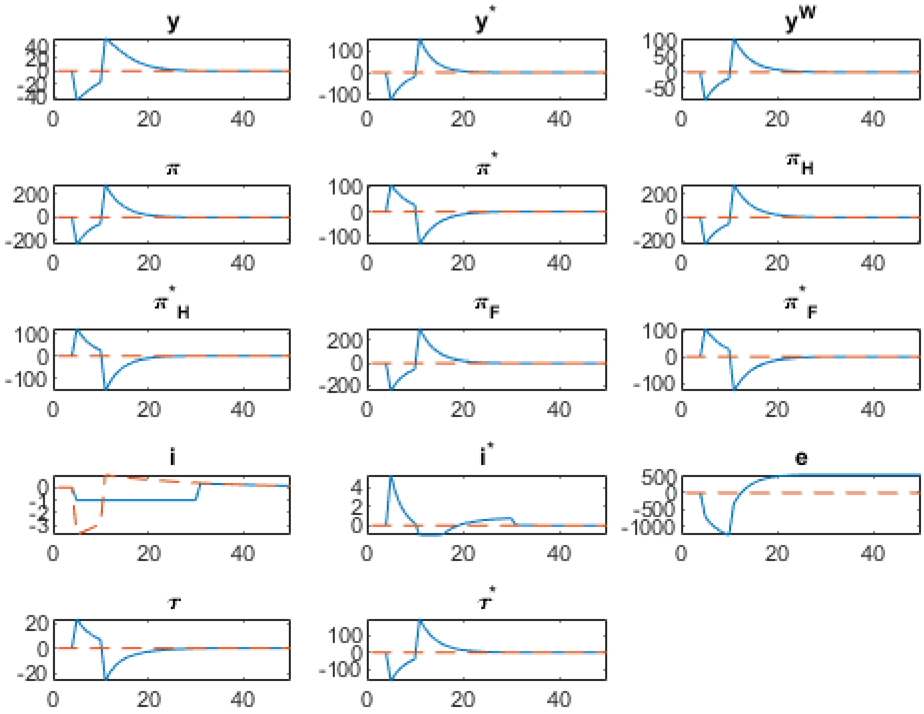

Figure 4.6: Optimal policy with tariffs in a global liquidity trap. Solid blue line represents the impulse response functions when the both countries are constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.

Once again, the presence of the Zero Lower Bound in the Foreign country as well does not significantly change the optimal policy when compared to what was found in subsubsection 4.3.1.2.

5
Conclusion

We have studied the effect that trade tariffs can have on the optimal policy choices under a liquidity trap. Under a cooperative scenario, the optimal policy still exhibits traditional characteristics already seen in other works, such as history dependence and international dependence. However, the introduction of tariffs in the model changes the optimal policy path for several key variables. In particular, tariffs allow for the shock to be softened in the country that has a constrained policy, by partially shifting the burden towards the country that has an unconstrained policy.

There are several possible extensions to our work. One possibility is allowing for countries to be of different sizes and analysing its effect on the optimal policy path. Another venue for future research is comparing our findings with a non-cooperative scenario in which there would be a strategic interaction between each country and how it would change the optimal policies. These questions are left as future research.

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A
Detailed Model Calculations

A.1
Households

Household in country H solves:

$$\max_{\{C_t, N_t, D(\nabla^{t+1})\}_{t=0}^{\infty}} U_t = \mathbb{E}_t \left\{ \sum_{j=0}^{\infty} \beta^j \left[\frac{1}{1-\sigma} C_{t+j}^{1-\sigma} - \frac{1}{1+\phi} N_{t+j}^{1+\phi} \right] Z_{t+j} \right\}$$

subject to the budget constraint:

$$\int_0^1 P_{H,t}(h) C_{H,t}(h) + \int_0^1 (1+\tau_t) P_{F,t}(f) C_{F,t}(f) + \\ + \sum_{\nabla^{t+1} \in \Omega_{t+1}} Q(\nabla^{t+1} | \nabla^t) D(\nabla^{t+1}) = W_t N_t + \Gamma_t - T_t + D(\nabla^t)$$

We start by optimizing the bundles for $C_{H,t}$ and $C_{F,t}$. We solve:

$$\max_{\{C_{H,t}(h)\}_{h=0}^{1}} \left[\int_0^1 C_{H,t}(h)^{\frac{\xi-1}{\xi}} \mathrm{d}i \right]^{\frac{\xi}{\xi-1}}$$

subject to

$$\int_0^1 P_{H,t}(h) C_{H,t}(h) = X_{H,t}$$

Solving the optimization problem, we get the familiar expression

$$C_{H,t}(h) = \left(\frac{P_{H,t}(h)}{P_{H,t}} \right)^{-\xi} C_{H,t}$$

We can solve an analogous problem for $C_{F,t}(f)$ and get:

$$C_{F,t}(f) = \left(\frac{P_{F,t}(f)}{P_{F,t}} \right)^{-\xi} C_{F,t}$$

Now, we must find the optimal ratio between $C_{H,t}$ and $C_{F,t}$. In order to do it, we solve:

$$\max_{C_{H,t},C_{F,t}} {C_{H,t}}^{\frac{\nu}{2}} {C_{F,t}}^{(1-\frac{\nu}{2})}$$

subject to

$$P_{H,t}C_{H,t} + (1+\tau_t)P_{F,t}C_{F,t} = X_t$$

We get:

$$P_{H,t}C_{H,t} = \frac{\nu}{2}(1+\tau)^{(1-\frac{\nu}{2})}P_tC_t \quad \text{and} \quad P_{F,t}C_{F,t} = \left(1-\frac{\nu}{2}\right)(1+\tau_t)^{-\frac{\nu}{2}}P_tC_t$$

We can rewrite the budget constraint using the results above. The new budget constraint will be:

$$(1+\tau_t)^{1-\frac{\nu}{2}}P_tC_t + \sum_{\nabla^{t+1}\in\Omega_{t+1}} Q(\nabla^{t+1}|\nabla^t)D(\nabla^{t+1}) = W_tN_t + \Gamma_t - T_t + D(\nabla^t)$$

Using the new budget constrain, we can compute the first order conditions for the household's problem:

$$\mathbf{C_t} : \beta^t C_t(\nabla^t)^{-\sigma} Z_t \Pi(\nabla^t) - \beta^t \mu_t(\nabla^t)(1+\tau_t)^{1-\frac{\nu}{2}} P_t(\nabla^t) = 0 \tag{A-1}$$

$$\mathbf{N_t} : -\beta^t N_t(\nabla^t)^{\phi} Z_t \Pi(\nabla^t) + \beta^t \mu_t(\nabla^t) W_t(\nabla^t) = 0 \tag{A-2}$$

$$\mathbf{D_{t+1}} : -\beta^t \mu_t(\nabla^t) Q(\nabla^{t+1}|\nabla^t) + \beta^{t+1}\mathbb{E}_t\left[\mu_{t+1}(\nabla^{t+1})\right] \tag{A-3}$$

Combining (A-1) and (A-2), we get:

$${C_t}^{\sigma}{N_t}^{\phi} = \frac{W_t}{(1+\tau_t)^{1-\frac{\nu}{2}}P_t}$$

Substituting the definition for P_t in the equation above, we get:

$$\frac{W_t}{P_{H,t}} = k^{-1}(1+\tau_t)^{1-\frac{\nu}{2}}{S_t}^{1-\frac{\nu}{2}}{C_t}^{\sigma}{N_t}^{\phi}$$

Combining (A-1) and (A-3), we get:

$$\frac{Q(\nabla^{t+1}|\nabla^t)}{\Pi(\nabla^{t+1}|\nabla^t)} = \beta\left[\left(\frac{C_{t+1}(\nabla^{t+1})}{C_t(\nabla^t)}\right)^{-\sigma}\left(\frac{1+\tau_t}{1+\tau_{t+1}}\right)^{1-\frac{\nu}{2}}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{Z_{t+1}}{Z_t}\right)\right]$$

The problem faced by the household in country F is analogous.

Finally, we will combine the equation above ((2-2) in the main text) with its foreign country analogous ((2-3) in the main text). The resulting equation is:

$$\left(\frac{C_t}{C_t^*}\right)^{\sigma} = \frac{E_t P_{H,t}^*}{P_{H,t}} \left(\frac{1+\tau_t^*}{1+\tau_t}\right)^{1-\frac{\nu}{2}} S_t^{*-\frac{\nu}{2}} S_t^{-\left(1-\frac{\nu}{2}\right)}$$

A.2 Firms

We will solve the problem of a firm in country H

First, note that the production function is linear in labor

$$Y_t(h) = A_t N_t(h)$$

Also, since we are assuming Calvo pricing, aggregate price levels follow

$$P_{H,t} = \left[\theta P_{H,t-1}{}^{1-\xi} + (1-\theta) P_{H,t}^{o}{}^{1-\xi}\right]^{\frac{1}{1-\xi}} \quad \text{and} \quad P_{H,t}^* = \left[\theta P_{H,t-1}^*{}^{1-\xi} + (1-\theta) P_{H,t}^{*o}{}^{1-\xi}\right]^{\frac{1}{1-\xi}} \tag{A-4}$$

where the "o" superscript denote the optimal price choice in t

The firm's profit function is given by

$$\Gamma_t(h) = P_{H,t}(h) C_{H,t}(h) + E_t P_{H,t}^*(h) C_{H,t}^*(h) - (1-\tau_S) W_t N_t(h)$$

We must find the optimal prices a firm will charge when it is selected to adjust its price at time t. They are the solution to the profit maximization problem:

$$\max_{P_{H,t}^{o}, P_{H,t}^{*o}} \mathbb{E}_t \left[\sum_{j=0}^{\infty} \theta^j Q_{t,t+j} (P_{H,t}^{o} C_{H,t+j}(h) + E_t P_{H,t}^{*o} C_{H,t+j}^*(h) - W_{t+j} N_{t+j}(h))\right]$$

We substitute the expression for $Q_{t,t+j}$, the production function, the market clearing condition and the expressions for $C_{H,t+j}(h)$ and $C_{H,t+j}^*(h)$ in the equation above. We get:

$$\max_{P^o_{H,t},P^{*o}_{H,t}} \mathbb{E}_t \sum_{j=0}^{\infty}(\theta\beta)^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} \left(\frac{1+\tau_t}{1+\tau_{t+j}}\right)^{1-\frac{\nu}{2}} \left(\frac{P_t}{P_{t+j}}\right) [P^o_{H,t} \left(\frac{P^o_{H,t}}{P_{H,t+j}}\right)^{-\xi} +$$

$$+ E_t P^{*o}_{H,t} \left(\frac{P^{o*}_{H,t}}{P^*_{H,t+j}}\right)^{-\xi} C^*_{H,t+j} - W_{t+j} \frac{\left(\frac{P^o_{H,t}}{P_{H,t+j}}\right)^{-\xi} C_{H,t+j} + \left(\frac{P^{*o}_{H,t+j}}{P^*_{H,t+j}}\right) C^*_{H,t+j}}{A_{t+j}}]$$

Calculating the first order conditions for this problem, we get:

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty}(\theta\beta)^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} \left(\frac{1+\tau_t}{1+\tau_{t+j}}\right)^{1-\frac{\nu}{2}} \frac{P_t}{P_{t+j}} \left(\frac{P^o_{H,t}}{P_{H,t+j}}\right)^{-\xi} C_{H,t+j} \left[P^o_{H,t} - \frac{\xi}{\xi-1}\frac{W_{t+j}}{A_{t+j}}\right]\right\} = 0 \tag{A-5}$$

and

$$\mathbb{E}_t \left\{ \sum_{j=0}^{\infty}(\theta\beta)^j \left(\frac{C_{t+j}}{C_t}\right)^{-\sigma} \left(\frac{1+\tau_t}{1+\tau_{t+j}}\right)^{1-\frac{\nu}{2}} \frac{P_t}{P_{t+j}} \left(\frac{{P^*_{H,t}}^o}{P^*_{H,t+j}}\right)^{-\xi} C^*_{H,t+j} \left[E_t {P^*_{H,t}}^o - \frac{\xi}{\xi-1}\frac{W_{t+j}}{A_{t+j}}\right]\right\} = 0$$

The problem for firms in the Foreign country is analogous

A.3 Equilibrium

Market clearing implies

$$Y_t = C_{H,t} + C^*_{H,t}$$

Substituting $C_{H,t}$ and $C^*_{H,t}$, we get:

$$Y_t = \frac{\nu}{2}(1+\tau_t)^{1-\frac{\nu}{2}}\frac{P_t C_t}{P_{H,t}} + (1-\frac{\nu}{2})(1+\tau^*_t)^{-\frac{\nu}{2}}\frac{P^*_t C^*_t}{P^*_{H,t}}$$

Substituting the definitions for P_t and P^*_t, we get:

$$Y_t = k^{-1}\left[\frac{\nu}{2}(1+\tau_t)^{1-\frac{\nu}{2}} {S_t}^{1-\frac{\nu}{2}} C_t + \left(1-\frac{\nu}{2}\right)(1+\tau^*_t)^{-\frac{\nu}{2}} {S^*_t}^{-\frac{\nu}{2}} C^*_t\right]$$

Market clearing for output in the Foreign country is analogous

A.4 Log-Linearized Model

We start by log-linearizing the market clearing conditions

$$y_t = \frac{\nu}{2}c_t + \left(1-\frac{\nu}{2}\right)c_t^* + \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)s_t - \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)s_t^* + \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)\tau_t - \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)\tau_t^* \quad \text{(A-6)}$$

$$y_t^* = \left(1+\frac{\nu}{2}\right)c_t + \frac{\nu}{2}c_t^* - \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)s_t + \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)s_t^* - \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)\tau_t + \frac{\nu}{2}\left(1-\frac{\nu}{2}\right)\tau_t^* \quad \text{(A-7)}$$

Now, we log-linearize the risk sharing condition

$$\sigma c_t - \sigma c_t^* = e_t + p_{H,t}^* - p_{H,t} + \left(1-\frac{\nu}{2}\right)\tau_t^* - \left(1-\frac{\nu}{2}\right)\tau_t - \frac{\nu}{2}s_t^* - \left(1-\frac{\nu}{2}\right)s_t \quad \text{(A-8)}$$

Combining (A-6), (A-7) and (A-8), we can get three new equations:

$$c_t^W = y_t^W$$

$$c_t^R = \frac{\nu-1}{D}y_t^R + \frac{\nu(2-\nu)}{2D}m_t - \frac{\nu(2-\nu)}{4D}\tau_t + \frac{\nu(2-\nu)}{4D}\tau_t^*$$

$$s_t = \frac{2\sigma}{D}y_t^R - \frac{\nu-1}{D}m_t + \frac{(2-\nu)[(1-\sigma)\nu-1]}{2D}\tau_t - \frac{(2-\nu)[(1-\sigma)\nu-1]}{2D}\tau_t^*$$

Now, we move to log-linearize the wage equation:

$$w_t - p_{H,t} = \left(\frac{2-\nu}{2}\right)\tau_t + \left(\frac{2-\nu}{2}\right)s_t + \sigma c_t + \phi n_t \quad \text{(A-9)}$$

Note that the production function can be approximated by:

$$y_t = a_t + n_t$$

Now, we will substitute the production function, the market clearing condition and the terms of trade equation into the wage equation. With some algebra, we get:

$$w_t - p_{H,t} = \left(\frac{\sigma}{D}+\phi\right)y_t^R + (\sigma+\phi)y_t^W + \frac{D-(\nu-1)}{2D}m_t + \Xi_1\tau_t + \Xi_2\tau_t^* - \phi a_t$$

where

$$\Xi_1 \equiv -[\sigma+\phi(\nu-1)]\left(\frac{\nu(2-\nu)}{4D}\right) + (1+\phi\nu)\left(\frac{2-\nu}{2}\right)\frac{(2-\nu)[(1-\sigma)\nu-1]}{2D} + \left(1+\frac{\phi\nu}{2}\right)\left(\frac{2-\nu}{2}\right)$$

$$\Xi_2 \equiv [\sigma+\phi(\nu-1)]\left(\frac{\nu(2-\nu)}{4D}\right)-(1+\phi\nu)\left(\frac{2-\nu}{2}\right)\frac{(2-\nu)[(1-\sigma)\nu-1]}{2D}-\frac{\phi\nu}{2}\left(\frac{2-\nu}{2}\right)$$

Next, we follow the traditional steps towards log-linearizing equations (A-4) and (A-5) and find

$$\pi_{H,t} = \delta(w_t - p_{H,t} - a_t) + \beta\mathbb{E}_t[\pi_{H,t+1}]$$

Substituting the wage equation we had previously found in the equation above gives us the Phillips Curve we present in the main text

$$\pi_{H,t} = \delta\left[\left(\frac{\sigma}{D}+\phi\right)\tilde{y}_t^R + (\sigma+\phi)\tilde{y}_t^W + \frac{D-(\nu-1)}{2D}m_t + \Xi_1\tau_t + \Xi_2\tau_t^*\right]+\beta\mathbb{E}_t[\pi_{H,t+1}]+u_t$$

The processes involved in finding the other three Phillips curves of our model is analogous.

B
Deriving the Welfare Function

We follow the calculations for the derivation of the welfare function in Appendix C for Engel (2011).

Since both representative consumers in our model have the exact same utility function as in Engel (2011), the derivation can proceed similarly up to

$$\begin{aligned}\upsilon - \upsilon_{\max} = 2\tilde{c}_t^W - 2\tilde{n}_t^W + (1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right) + \\ + 2(1-\sigma)\left(\overline{c}_t^R\tilde{c}_t^R + \overline{c}_t^W\tilde{c}_t^W\right) - 2(1+\phi)\left(\overline{n}_t^R\tilde{n}_t^R + \overline{n}_t^W\tilde{n}_t^W\right) \quad \text{(B-1)}\end{aligned}$$

where $\upsilon - \upsilon_{\max}$ is the welfare loss for a given allocation compared with the efficient outcome.

As in Engel (2011), we can divide $\upsilon - \upsilon_{\max}$ in two parts. $2\tilde{c}_t^W - 2\tilde{n}_t^W$ will require a second order approximation. However

$$\begin{aligned}(1-\sigma)\left((\tilde{c}_t^R)^2 + (\tilde{c}_t^W)^2\right) - (1+\phi)\left((\tilde{n}_t^R)^2 + (\tilde{n}_t^W)^2\right) + \\ + 2(1-\sigma)\left(\overline{c}_t^R\tilde{c}_t^R + \overline{c}_t^W\tilde{c}_t^W\right) - 2(1+\phi)\left(\overline{n}_t^R\tilde{n}_t^R + \overline{n}_t^W\tilde{n}_t^W\right)\end{aligned}$$

can be derived using only the first order approximations we already calculated for our log-linearized model. Substituting the values for $\tilde{c}_t^R$, $\overline{c}_t^R$, $\tilde{c}_t^W$, $\overline{c}_t^W$, $\tilde{n}_t^R$, $\overline{n}_t^R$, $\tilde{n}_t^W$ and $\overline{n}_t^W$ and doing some algebra we get:

$$\begin{aligned}&\left[(1-\sigma)\left(\frac{\nu-1}{D}\right)^2 - (1+\phi)\right]\left(\tilde{y}_t^R\right)^2 - (\sigma+\phi)\left(\tilde{y}_t^W\right)^2 + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}(m_t)^2 + \\ &+ \frac{(1-\sigma)\nu^2(2-\nu)^2}{16D^2}(\tau_t)^2 + \frac{(1-\sigma)\nu^2(2-\nu)^2}{16D^2}(\tau_t^*)^2 + 2\nu(2-\nu)\left(\frac{(1-\sigma)(\nu-1)}{D}\right)^2\overline{y}_t^R\tilde{y}_t^R + \\ &+ \frac{(1-\sigma)(\nu-1)\nu(2-\nu)}{D^2}m_t y_t^R - \frac{(1-\sigma)(\nu-1)\nu(2-\nu)}{2D^2}\tau_t y_t^R + \frac{(1-\sigma)(\nu-1)\nu(2-\nu)}{2D^2}\tau_t^* y_t^R - \\ &- \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}m_t\tau_t + \frac{(1-\sigma)\nu^2(2-\nu)^2}{4D^2}m_t\tau_t^* - \frac{(1-\sigma)\nu^2(2-\nu)^2}{8D^2}\tau_t\tau_t^*\end{aligned} \quad \text{(B-2)}$$

Now we move to find an approximation to $2\tilde{c}_t^W$. We start with equations (2-9) and (2-10)

$$Y_t = k^{-1}\left[\frac{\nu}{2}(1+\tau_t)^{1-\frac{\nu}{2}}S_t^{1-\frac{\nu}{2}}C_t + \left(1-\frac{\nu}{2}\right)(1+\tau_t^*)^{-\frac{\nu}{2}}S_t^{*-\frac{\nu}{2}}C_t^*\right]$$

$$Y_t^* = k^{-1}\left[\left(1-\frac{\nu}{2}\right)(1+\tau_t)^{-\frac{\nu}{2}}S_t^{-\frac{\nu}{2}}C_t + \frac{\nu}{2}(1+\tau_t^*)^{1-\frac{\nu}{2}}S_t^{*1-\frac{\nu}{2}}C_t^*\right]$$

We perform second-order approximations around the steady state on both equations. We get:

$$\begin{aligned} y_t+\frac{1}{2}{y_t}^2 &= \frac{\nu}{2}c_t+\left(\frac{2-\nu}{2}\right)c_t^*+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_t-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_t^*+ \\ &+\frac{1}{2}\{\frac{\nu}{2}{c_t}^2+\left(\frac{2-\nu}{2}\right)c_t^{*2}+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2{s_t}^2+\left(\frac{\nu}{2}\right)^2\left(\frac{2-\nu}{2}\right)s_t^{*2}+ \\ &\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2{\tau_t}^2++\left(\frac{\nu}{2}\right)^2\left(\frac{2-\nu}{2}\right)\tau_t^{*2}\}+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_tc_t-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*c_t^*+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_tc_t- \\ &-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_t^*c_t^*+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2s_t\tau_t+\left(\frac{\nu}{2}\right)^2\left(\frac{2-\nu}{2}\right)s_t^*\tau_t^* \quad \text{(B-3)} \end{aligned}$$

$$\begin{aligned} y_t^*+\frac{1}{2}y_t^{*2} &= \frac{\nu}{2}c_t^*+\left(\frac{2-\nu}{2}\right)c_t+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_t^*-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_t+ \\ &+\frac{1}{2}\{\frac{\nu}{2}c_t^{*2}+\left(\frac{2-\nu}{2}\right){c_t}^2+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2s_t^{*2}+\left(\frac{\nu}{2}\right)^2\left(\frac{2-\nu}{2}\right){s_t}^2+ \\ &\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2\tau_t^{*2}++\left(\frac{\nu}{2}\right)^2\left(\frac{2-\nu}{2}\right){\tau_t}^2\}+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_t^*c_t^*-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)s_tc_t+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_t^*c_t^*- \\ &-\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)\tau_tc_t+\frac{\nu}{2}\left(\frac{2-\nu}{2}\right)^2s_t^*\tau_t^*+\left(\frac{\nu}{2}\right)^2\left(\frac{2-\nu}{2}\right)s_t\tau_t \quad \text{(B-4)} \end{aligned}$$

We average (B-3) and (B-4) and get:

$$\begin{aligned} y_t^W+\frac{1}{2}\left[\left(y_t^R\right)^2+\left(y_t^W\right)^2\right] &= c_t^W+\frac{1}{2}\left[\left(c_t^R\right)^2+\left(c_t^W\right)^2\right]+\frac{\nu(2-\nu)}{16}({s_t}^2+s_t^{*2})+ \\ &+\frac{\nu(2-\nu)}{16}({\tau_t}^2+\tau_t^{*2})+\frac{\nu(2-\nu)}{16}s_t(\tau_t-\tau_t^*) \quad \text{(B-5)} \end{aligned}$$

We substitute c_t^R, c_t^W and s_t in (B-5). With some algebra, we get:

$$
\begin{aligned}
\tilde{c}_t^W &= \tilde{y}_t^W - \frac{\nu(2-\nu)}{2}\left[\frac{(\nu-1)(1-\sigma)}{D}\right]^2\left[\left(\tilde{y}_t^R\right)^2 + 2\overline{y}_t^R\tilde{y}_t^R\right] - \frac{\nu(2-\nu)}{8D^2}{m_t}^2 - \\
&-\frac{\nu(2-\nu)}{16D^2}\left\{\nu(2-\nu) + \frac{(2-\nu)^2[(1-\sigma)\nu-1]^2}{2} + D^2 + 1 + D\frac{(2-\nu)[(1-\sigma)\nu-1]}{2}\right\}{\tau_t}^2 - \\
&-\frac{\nu(2-\nu)}{16D^2}\left\{\nu(2-\nu) + \frac{(2-\nu)^2[(1-\sigma)\nu-1]^2}{2} + D^2 + 1 + D\frac{(2-\nu)[(1-\sigma)\nu-1]}{2}\right\}{\tau_t^*}^2 - \\
&- \frac{(1-\sigma)\nu(2-\nu)(\nu-1)}{2D^2}y_t^R m_t + \frac{\nu(2-\nu)}{8D^2}[\sigma D + 2(1-\sigma)(\nu-1)]y_t^R\tau_t - \\
&- \frac{\nu(2-\nu)}{8D^2}[\sigma D + 2(1-\sigma)(\nu-1)]y_t^R\tau_t^* + \\
&+ \frac{\nu(2-\nu)}{16D^2}\{2\nu(2-\nu) + 2(\nu-1)(2-\nu)[(1-\sigma)\nu-1] + D(\nu-1)\}m_t\tau_t - \\
&- \frac{\nu(2-\nu)}{16D^2}\{2\nu(2-\nu) + 2(\nu-1)(2-\nu)[(1-\sigma)\nu-1] + D(\nu-1)\}m_t\tau_t^* + \\
&+\frac{\nu(2-\nu)}{16D^2}\left\{\nu(2-\nu) + \frac{(2-\nu)^2[(1-\sigma)\nu-1]^2}{2} + D(2-\nu)[(1-\sigma)\nu-1]\right\}\tau_t\tau_t^*
\end{aligned} \tag{B-6}
$$

In order to find an approximation for $2\tilde{n}_t^W$ we proceed with the same calculation presented in the appendix in Engel (2011). Note that, in these calculations, our equations are exactly the same as in Engel (2011). Hence, the result will also be the same and equal to:

$$
2\tilde{n}_t^W = 2\tilde{y}_t^W + \frac{\xi}{2}\left[\frac{\nu}{2}\left(\pi_{H,t}\right)^2 + \frac{2-\nu}{2}\left(\pi_{H,t}^*\right)^2 + \frac{\nu}{2}\left(\pi_{F,t}^*\right)^2 + \frac{2-\nu}{2}\left(\pi_{F,t}\right)^2\right] \tag{B-7}
$$

Finally, we substitute (B-2), (B-6) and (B-7) in (B-1). With some algebra, we get the loss function we present in the main text:

$$
\begin{aligned}
\upsilon - \upsilon_{\max} = &\left[\frac{\sigma}{D} + \phi\right]\left(\tilde{y}_t^R\right)^2 + (\sigma+\phi)\left(\tilde{y}_t^W\right)^2 + \frac{\nu(2-\nu)}{4D}(m_t)^2 + \chi_1(\tau_t)^2 + \chi_1(\tau_t^*)^2 + \\
&+ \frac{\sigma\nu(2-\nu)}{4D}y_t^R(\tau_t - \tau_t^*) + \chi_2 m_t(\tau_t - \tau_t^*) + \chi_3\tau_t\tau_t^* + \\
&+ \frac{\xi}{2\delta}\left[\frac{\nu}{2}(\pi_{H,t})^2 + \frac{2-\nu}{2}(\pi_{F,t})^2 + \frac{\nu}{2}(\pi_{F,t}^*)^2 + \frac{2-\nu}{2}(\pi_{H,t}^*)^2\right]
\end{aligned}
$$

where

$$
\chi_1 \equiv \frac{\nu(2-\nu)}{8D^2}\left\{\frac{(1+\sigma)\nu(2-\nu)}{2} + 1 + D^2 + D\frac{(2-\nu)[(1-\sigma)\nu-1]}{2} + \frac{(2-\nu)^2[(1-\sigma)\nu-1]^2}{2}\right\}
$$

$$\chi_2 \equiv \frac{\nu(2-\nu)}{8D^2}\left\{2(2-\nu)[1-(1-\sigma)\nu(2-\nu)] + D(\nu-1)\right\}$$

$$\chi_3 \equiv \frac{\nu(2-\nu)}{8D^2}\left\{\sigma\nu(2-\nu) + D(2-\nu)[(1-\sigma)\nu - 1] + \frac{(2-\nu)^2[(1-\sigma)\nu-1]^2}{2}\right\}$$

C
Preference Shock on Both Countries

We present the results for when the preference shock hits both countries instead of only directly affecting the Home country, as is the case in the main text.

Figures C.1 and C.2 present the results for the case in which only the local country is constrained by the Zero Lower Bound. Conclusions are quite similar to the situation in which the preference shock only hits the Home country. The biggest difference is that now, monetary policy in the Foreign country must be even more reactive, since it must also take into account the preference shock that has hit locally.

Figures C.3 and C.4 illustrate the optimal trajectories when the world faces a global liquidity trap and both countries are hit by the same preference shock. Now both countries exhibit the exact same policy response. Note that the economies in our model are symmetric. Hence, the preference shock affects both economies in the exact same way. Consequently, the optimal response to the shock is also the same in both countries.

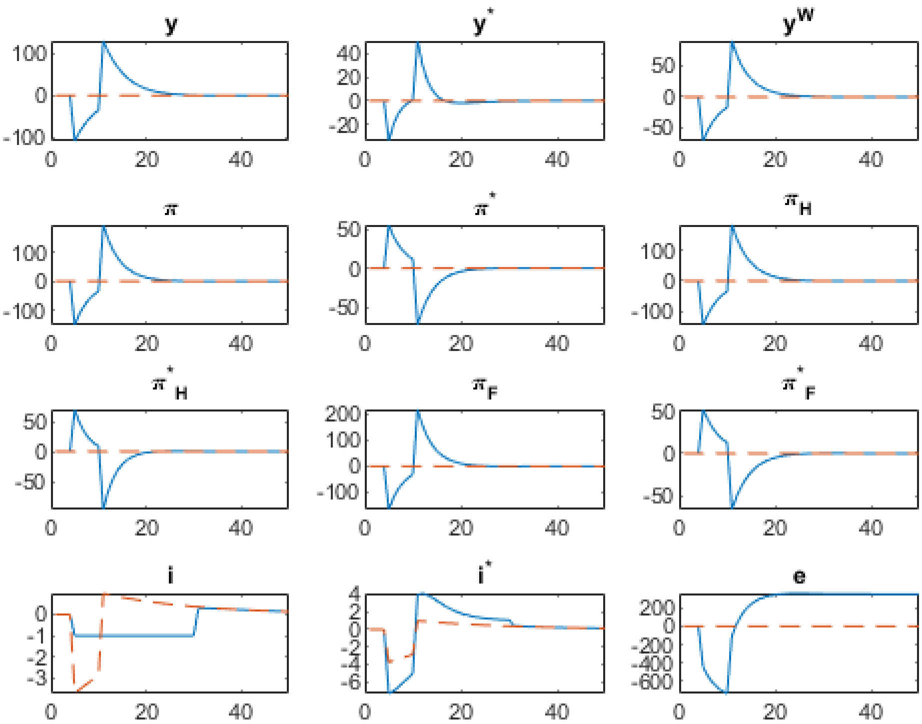

Figure C.1: Optimal policy without tariffs in a local liquidity trap when both countries are hit by a simultaneous preference shock. Solid blue line represents the impulse response functions when the both countries are constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.

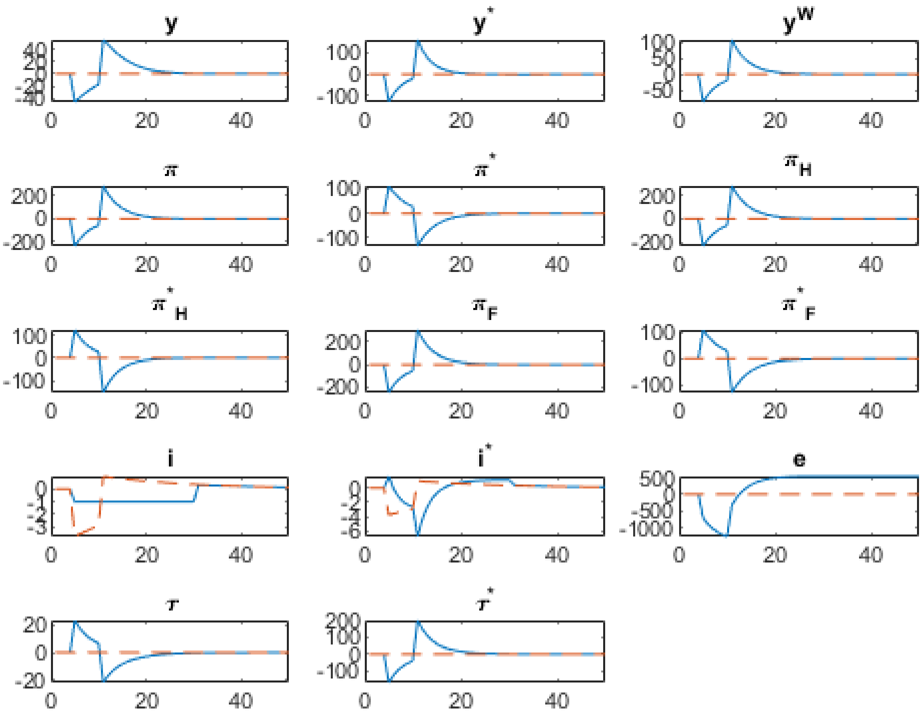

Figure C.2: Optimal policy with tariffs in a local liquidity trap when both countries are hit by a simultaneous preference shock. Solid blue line represents the impulse response functions when the both countries are constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.

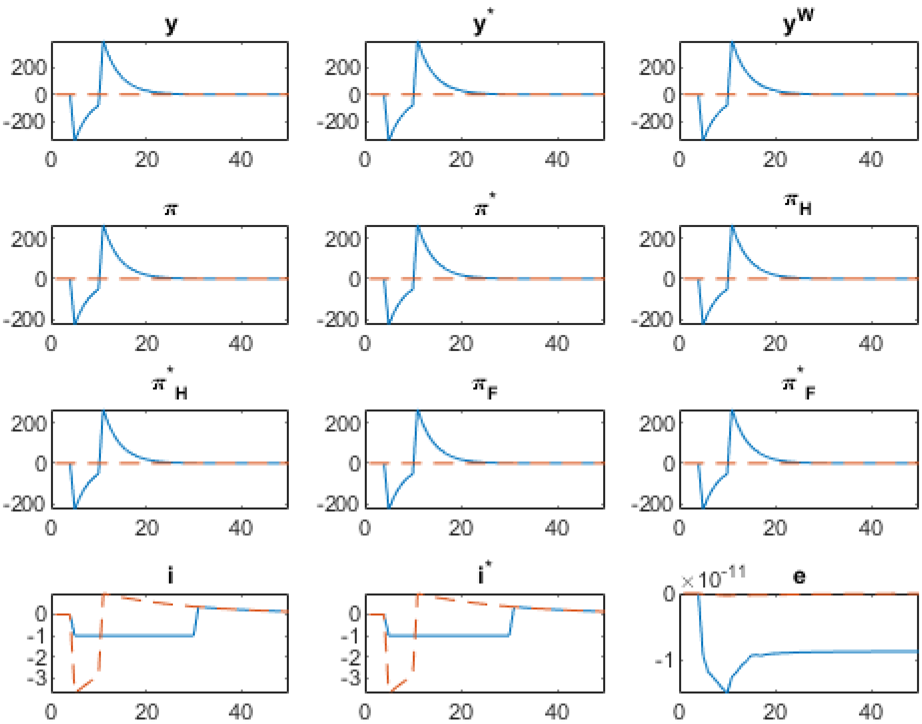

Figure C.3: Optimal policy without tariffs in a global liquidity trap when both countries are hit by a simultaneous preference shock. Solid blue line represents the impulse response functions when the both countries are constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.

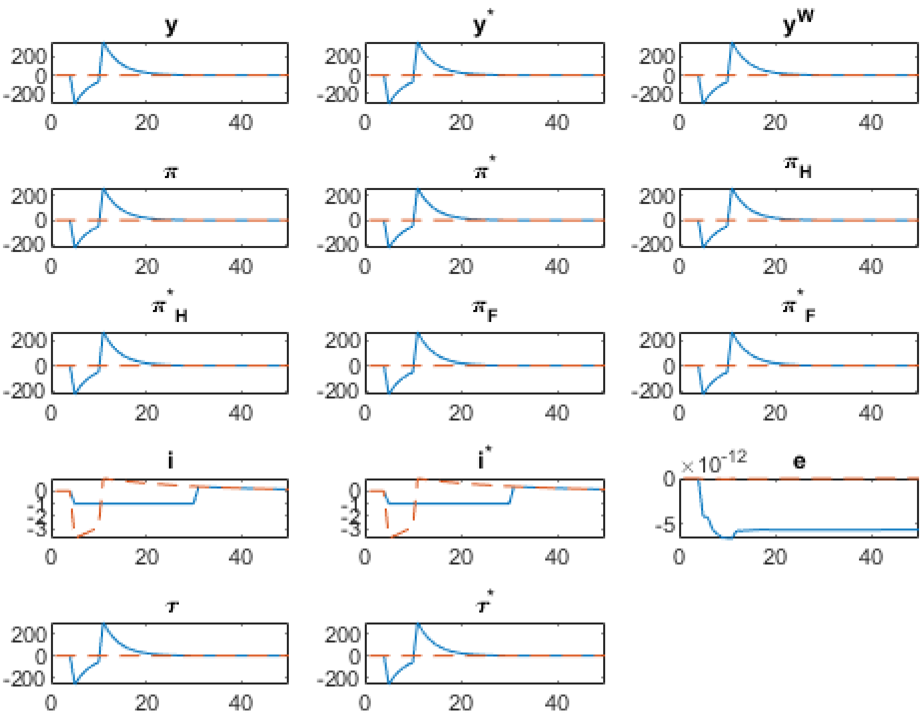

Figure C.4: Optimal policy with tariffs in a global liquidity trap when both countries are hit by a simultaneous preference shock. Solid blue line represents the impulse response functions when the both countries are constrained by the Zero Lower Bound. Red dashed line represents the impulse response functions when there is no Zero Lower Bound constraint.