PONTIFÍCIA UNIVERSIDADE CATÓLICA DO RIO DE JANEIRO

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# Structural and Transitory Changes in the Commodity Futures Risk Premium

**Dissertação de Mestrado**

Dissertation presented to the Programa de Pós–Graduação em Macroeconomia e Finanças of PUC-Rio in partial fulfillment of the requirements for the degree of Mestre em Economia.

Advisor: Prof Marcelo Cunha Medeiros
Co-Advisor: Ruy Monteiro Ribeiro

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## Abstract

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Commodity index investing (CII) became a major trend among portfolio managers by the early 2000s causing a large influx of non-commercial investors to the commodity futures market. By improving the integration of commodity futures market to the broad financial market, CII is expected to have affected the risk premium in commodity futures. A new methodology is proposed to investigate both structural and transitory changes of risk premium behavior in the term structure of crude oil futures. The methodology consists of introducing Markov switching to the framework of affine term structure models while avoiding over-parametrization and unrealistic regime-switching in the cross-section relations of the term structure. Overall, results are in agreement with the previous literature by indicating the existence of a structural break coinciding with the popularization of CII followed by a period of lower and more volatile risk premium.





## Keywords

Commodities Index Traders; Risk Premium; Affine Term Structure Model; Markov Switching; Regime Shifts

# Resumo

Soares, Fernando Saint-Martin de Abreu; Medeiros, Marcelo Cunha; Ribeiro, Ruy Monteiro. **Mudanças Estruturais e Transitórias no Prêmio de Risco de Futuros de Commodity.** Rio de Janeiro, 2020. 49p. Dissertação de Mestrado - Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Investimento em Índices de Commodity (IIC) tornou-se uma forte tendência entre gestores de portfólio por volta do começo da década de 2000, o que acarretou a entrada de um grande número de investidores não-comerciais no mercado de futuros de commodity. Por gerar maior integração entre o mercado de futuros de commodity e o mercado financeiro em geral, IIC traz possíveis consequências para o prêmio de risco em futuros de commodity. Uma nova metodologia é proposta para investigar mudanças tanto estruturais quanto transitórias no comportamento do prêmio de risco na estrutura a termo de futuros de petróleo. Essa metodologia consiste em introduzir Markov-switching à formulação de modelos afim de estrutura a termo enquanto evitando parâmetros em excesso e mudanças de regime irreais nas relações ao longo da seção transversal da estrutura a termo. De uma forma geral, os resultados estão de acordo com a literatura prévia por indicar a existência de uma quebra estrutural coincidindo com a popularização de IIC seguida de um período de prêmio de risco mais baixo e mais volátil.



# Palavras-chave

Traders de Índice de Commodity; Prêmio de Risco; Modelos Afim de Estrutura a Termo; Markov-Switching; Mudanças de Regime.

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# 1
# Introduction

The recent decades were marked by profound changes in the commodity futures market due to the surge of a new kind of investor, known as commodity index trader or commodity index investor (CII). Unlike traditional investors in commodity futures, these new market participants regard commodities as an asset class, playing a role analogous to stocks or bonds in a portfolio diversification strategy. The increased importance of CIIs to the trading of commodity futures is commonly referred to as the financialization of the commodity futures market.

While most works on financialization have focused on investigating wellfare reducing consequences of unwarranted speculation by CIIs, recent literature underscores the importance of understanding the transformations taking place in the mechanisms of information discovery and risk sharing in the commodity futures market (Cheng and Xiong, 2014). The issue of risk sharing, in particular, deserves close attention, as the emergence of market participants with distinct risk sharing incentives likely affects the behavior of the risk premium in commodity futures.

The conventional view on the risk premium in commodity futures is rooted in the classic Hedging Pressure Theory by Keynes (1923) and Hicks (1939), which postulates that commodity producers are generally seeking to reduce their exposure to price fluctuations and thus are naturally drawn to the short side of the futures market. In order to attract investors to the opposite side, producers accept futures prices lower than the expected spot price. Therefore, risk premium emerges as consequence of the "hedging pressures" induced by commodity producer[1].

A modern approach to the Hedging Pressure Theory was introduced by Hirschleifer (1988 and 1990) based on insights from Stoll (1979). These works outline two specific conditions for the validity of the theory: the revenues earned by commodity producers must be nonmarketable and there must be some fixed cost to the investors taking long positions. The former justifies the use of the futures market for hedging, while the latter creates barriers that prevent the producers hedging demands from being fully absorbed by financial investors and the commodity consumers. Owing to the presence of these frictions, the risk premium in commodity futures should be dependent on a systematic component (i.e. non-diversifiable risk) and a commodity-specific component related to producers' aversion to revenue variability. This marks a departure from the classic capital asset pricing model (CAPM), in which only the correlation with the systemic risk is rewarded.

Early empirical works found the systematic risk component to be statistically insignificant (Dusak, 1973; Jagannathan, 1985). On the other hand, in support to the hedging pressure view, the net positions of commercial investors were found to be inversely related to excess returns (Bessembinder, 1992; De Roon et. al, 2000). Assuming commercials are mostly commodity producers, their short positions can be translated as hedging pressures. The prevalence of commodity-specific hedging pressures over systematic risk factors in determining risk premium serves as evidence that the commodity futures market was mostly segmented from the broad

[1] Follows from this theory that risk premia should be necessarily positive. Empirical data on excess returns suggest that risk premia during the second half of 20$^{th}$ century were indeed positive for most commodity futures (Gorton and Rouwenhorst 2006).

financial market before financialization took place[2].

Commodity index investing can be seen as technology that improves risk sharing in the futures market by lowering the participation costs for financial investors. Therefore, financialization should ease the hedging-pressures-inducing frictions and ultimately lead to smaller risk premia.

However, the popularization of CIIs coincided with a persistent increase in correlations across different commodities as well as between commodities and financial assets (Tang and Xiong, 2012). Empirical results[3] suggest that the trading of commodity futures by hedge funds a have a direct impact on the intensification of cross-market correlations (Büyükşahin and Robe, 2014). Interestingly, this effect undermines the main reason for financial investors engagement with commodity futures, namely their portfolio diversification benefits (Stoll and Whaley, 2009). If commodity futures are no longer negatively correlated with stocks, their risk premia should be increased by a systematic risk component.

By diminishing the importance of producers' hedging pressures while enhancing the exposure of commodity futures to systematic risk, the impact of financialization on the risk premium can be described as ambiguous. Determining which of the two opposing effects prevails requires a thorough empirical investigation.

Pioneering this kind of investigation, Hamilton and Wu (2014) modelled the term structure of crude oil futures under an affine framework. In order to detect structural changes arising from financialization, they divided the estimating sample into two intervals using January 2005 as breaking point. Results suggest that the risk premium for holding a two-month futures contract was mostly positive and low-volatility until 2005. In the subsequent sample, which roughly corresponds to the period of CIIs popularization, risk premium became very volatile and averaged around zero.

In spite of making considerable strides, the work of Hamilton and Wu (HW) has also some limitation. First, the popularization of CIIs was a gradual process, rendering the choice of January 2005 as breaking point completely arbitrary. In fact, alternative interval divisions can equally reject the parameter-constancy hypothesis in the likelihood ratio test employed by the authors. Furthermore, the second interval is much shorter than the first and it largely coincides with the financial crisis, when extreme volatility potentially caused abnormal risk premium behavior.

Building upon the work of HW, the model presented as part of this dissertation introduces Markov regime-switching to the classic framework of discrete-time Gaussian affine term structure models. Markov-switching can crucially improve the study of commodity futures financialization by avoiding arbitrary choices of regime change. Under appropriate specification, Markov-switching also offers the advantage of distinguishing structural changes associated with financialization from transitory regime changes taking place during market turmoil. Lastly, the model retains the tractability of affine term structure models thus being able to represent risk premia across the entire term structure with few

[2] Some works (e.g. Acharya et. al 2013, Etula 2013) suggest that the degree of segmentation varies over time depending chiefly on the capital constraints of financial investors acting on the long side of the market.

[3] From a theoretical standpoint, the association between the financialization of commodity futures and cross-market correlations received very little attention. A noteworthy exception is the model proposed by Basak and Pavlova (2016).

parameters.

Unlike previous affine term structure models with Markov-switching, the pricing kernel in the model presented is defined in a way that preserves flexibility and yet avoids unrealistic regime-dependence in the cross-section relations of the term strucutre. By confining regime-switching to the dynamics of the latent factors, the model benefits from both easier estimation and better agreement with the data. Hence, this dissertation also makes a methodological contribution.

Results indicate the existence of two persistent low-volatility regimes interrupted by brief occurences of a high-volatility third regime. Consistent with the notion of financialization, one of the persistent regimes becomes the predominant regime around the mid-2000s while the other disappears. Surprinsingly, the late persistent regime is associated with higher risk premia. This appearent contradiction with HW happens because the last instances of the high-volatility regime present much lower risk premium than the early instances, causing a decrease in the overall risk premium average for the post-financialization period.

The content of this dissertation is organized in a total of seven chapters. The next chapter offers a comprehensive review of commodity index investing, highlighting its relevance to policy-making and summarizing the early literature on the subject. The third chapter describes how affine term structure models for bonds can be adapted to commodity futures. Subsequently, the fourth chapter introduces an innovative affine term structure model with Markov-switching. Results obtained from this model are shown in the fifth chapter. The last two chapters present the conclusions and references, respectively.

# 2 Commodity Index Investing

Traders of physical commodities in the spot market deal with considerable logistical hurdles, particularly when it comes to transportation and storage. CIIs, on the other hand, capitalize on the availability of low-cost financial products designed to mimic the returns of commodity indexes. For institutional investors, these products are presented in the form of commodity index funds and commodity return swaps, both of which being typically long-only and fully-collateralized (Stoll and Whaley, 2010). There are also similar products available to smaller investors, such as commodity index exchange-traded funds.

Regardless of the way it is structured, every commodity index investment ultimately consists of establishing long positions in a group of exchange-traded commodity futures and/or over-the-counter commodity forwards with some small duration. When these contracts are close to expiration, the original positions are closed while new position are established in contracts with the same duration as before. The repetition of this methodology, known as contract rolling, aims to replicate the returns from owning the physical commodities. Provided that the markets for commodity derivatives remain liquid and deep, this replication should be close to the actual returns.

Aside from being a convenient approach to tracking commodity returns, commodity index investing also benefits from the use of benchmarks based on well-diversified and publicly available commodity indexes, such as the Standard & Poor's–Goldman Sachs Commodity Index (S&P–GSCI) and the Bloomberg Commodity Index[4] (BCOM). By adopting passive benchmark strategies, CIIs avoid having to select commodities individually based on specific fundamentals, which would entail domain expertise.

These unique features combined with the popularization of portfolio allocation strategies involving commodities resulted in a quick and widespread adoption of commodity index investing around the 2000s. According to an assessment by the Commodity Futures Trading Commission (CFTC), the total position held by CIIs grew from $15 billion to $200 billion between 2003 and 2008 (CFTC, 2008).

One of the most visible consequences of this rapid investment expansion was a massive increase in open interest across several commodities. Figure 1 displays open interest data for a group of four commonly traded commodities based on the Commitment of Traders (COT) report[5] published by the CFTC. Overall, open interest in these commodities grew by a factor of four in the last two decades. Except for a brief period following the 2008 financial crisis, the data indicate a fairly consistent growth trend.

[4]Previously known as the Dow Jones-UBS Commodity Index (DJ-UBSCI).
[5]The COT reports contain weekly open interest data breakdown by trader category for futures markets in which 20 or more traders hold positions surpassing the reporting threshold. Traders are classified into categories based on their self-reported predominant business activity.

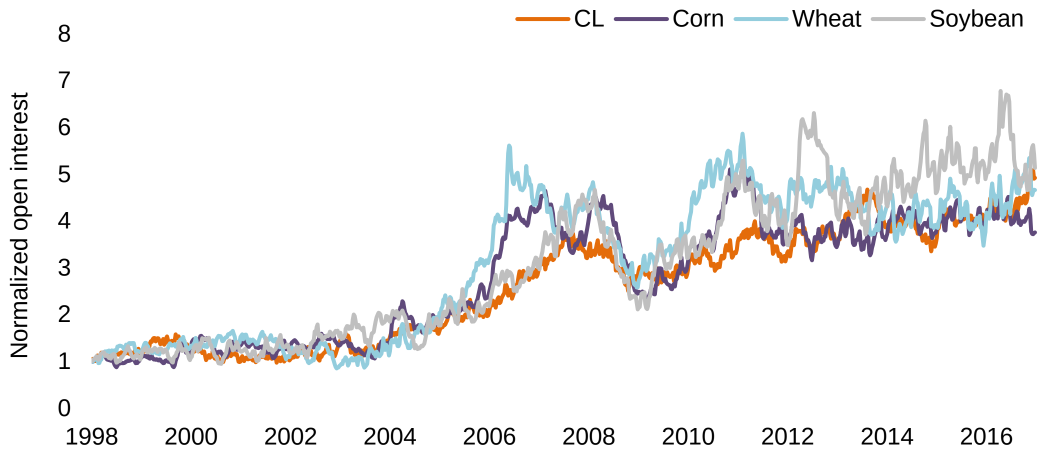


Figure 1: Evolution of the open interest in crude oil, corn, wheat and soybean futures normalized to 1998 levels

Given its novelty, commodity index investing is a natural candidate for explaining the remarkable similarity in the open interest patterns displayed by a group of apparently unrelated commodities. Specific data on CIIs activity are released through the COT supplemental reports for years starting from 2006. Unfortunately, the data cover only a group of 13 agricultural commodities[6], leaving out some of the largest components of commodity indexes. Nonetheless, the conclusions drawn from the COT supplemental reports can likely be extrapolated to other commodities.

Figure 2 compares the net positions of CIIs with those of other market participants in an aggregate of all the 13 commodities reported. The other market participants are divided into two groups: commercials and non-commercials. The former consists of investors who deal with the physical commodity, while the latter are mostly non-CII financial investors. As expected, CIIs are heavily concentrated on the long side of the futures market, being a natural counterparty to the commercial investors. Non-commercials investors are also predominantly on the long side, but, in contrast to CIIs, their net position is highly variable. The steadiness of the CIIs net position since the end of the financial crisis reflects the passive nature of commodity index investment.

In addition to the sheer size of the positions held by CIIs, the data from the COT supplemental report sheds light on the particularities of this new kind of investor. Given the persistency of their positions as well as their strong preference for the long side, CIIs behave in a strikingly different way compared to the traditional investors in commodity futures. Hence, the impact of financialization on the commodity futures market possibly goes beyond simply raising open interests.

Although the debate around the consequences of financialization is not new, its focus shifted considerably over time. When commodity index investing was still a recent development, the alleged risks of greater speculation on commodities prompted concern among policymakers and calls for stricter regulations on derivatives trading (US Senate Perm. Subcomm. Investig., 2009). Their concern echoed fears that speculation was the main factor driving up the costs of food and energy during the 2000s.

[6]Soft red winter (SRW) wheat, hard red winter (HRW) wheat, corn, soybeans, soybean oil, soybean meal, cotton no. 2, lean hogs, live cattle, feeder cattle, cocoa, sugar no. 11 and coffee.

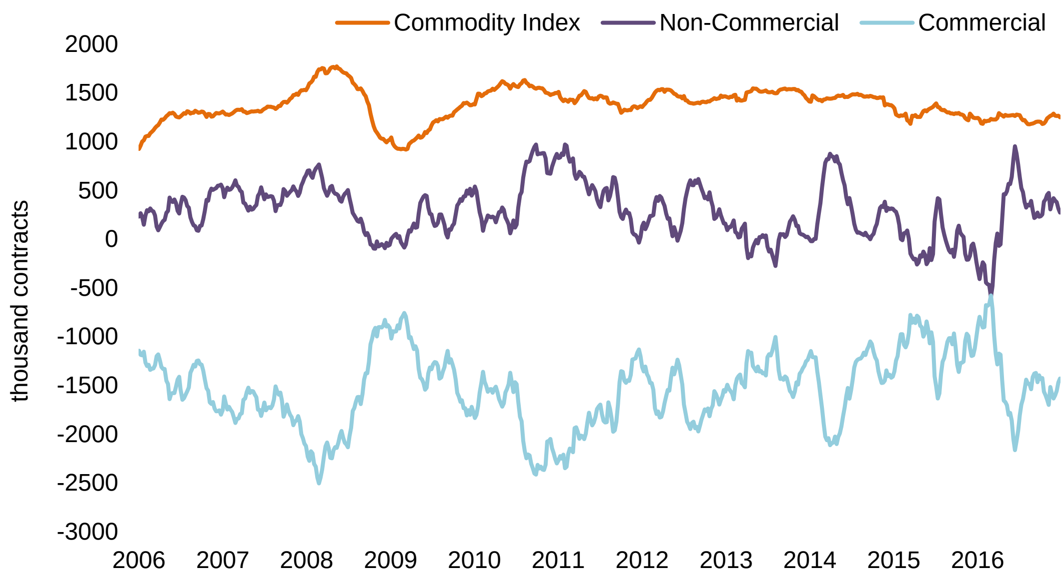


Figure 2: Net positions in thousands of contracts held in an aggregate of 13 agricultural commodities by commodity index investors and other market participants. The other participants are classified as either commercial or non-commercial according to the COT report definitions.

The view that financialization caused an across-the-board increase in commodity prices found backing in the so-called "Masters Hypothesis" (Masters, 2008), according to which CIIs distorted prices by exerting excessive pressure on the long side of commodity futures.

In order to assess the validity of the Masters Hypothesis, a great deal of work has been devoted to establishing links between CIIs positions and commodity prices, usually in the form of Granger-causality tests (Büyükşahin and Harris, 2011; Irwin and Sanders 2012; Stoll and Whaley 2010; Ribeiro et. al 2009). Results generally indicate that the inflows of CIIs and other financial investors have no predictive power for futures returns or volatility.

Assuming the demand elasticity for oil is non-zero, a speculator accumulating oil inventory in order to profit from a tightening market would cause an increase in the current spot price.  Hence, speculation on oil should be identifiable by its effect of raising spot price and inventory level simultaneously. Using this insight, Kilian and Murphy (2014) employed sign restriction identification in a structural VAR model to distinguish speculative demand from ordinary demand shocks[7]. The authors found evidence of speculation in various noteworthy events, such as the Iranian Revolution and Iraq's invasion of Kuwait. The run up in prices during the 2000s, however, was found to be driven primarily by ordinary demand shocks. Although not explicitly accounted for in the model, speculation on commodity futures by CIIs would also lead to accumulation of inventory given the no-arbitrage relation between spot and futures prices.

A significant challenge to the notion of speculation driving commodity prices is the lack of a precise definition of speculation. Some works classify as speculation all the trading carried out by CIIs and other financial investors. Alternatively, speculation was also interpreted as buying a position in a commodity for

[7] A drawback of this methodology is its dependence on accurate inventory data. Kilian and Lee (2014) demonstrate that results are sensitive to the choice of alternate data sources.

anticipating its appreciation. The works based on Granger-causality tests follow the former definition, while Kilian and Murphy (2014) use the latter. As noted by Fattaouh et. al (2013), trades considered speculative by either of these two definitions are a necessary component to the proper functioning of commodity markets. Hence, it is not clear to what degree these trades can be seen as undesirable.

Moreover, many of the commodities that reached record highs in the late 2000s have since experienced considerable devaluations in spite of the enduring prominence of CIIs as traders of commodity futures. As commodity prices dwindled, the interest in the commodity speculation debate and the Masters Hypothesis started to fade. Accordingly, this dissertation steers away from these topics albeit their significance to the early literature on commodity financialization.

# 3
# Modelling the Term Structure of Commodity Futures

With few exceptions[8], the task of modelling the term structure of commodity futures has been largely ignored by econometricians. This stands in contrast to the vastness of the literature on the term structure of government bonds. The goal of this chapter is showing how recent advancements in term structure modelling of bonds can be adapted to the realm of commodity futures.

Affine models are among the most popular approaches to represent the joint dynamics of bonds (Piazzesi, 2010). Models in this class ensure pricing consistency by imposing no-arbitrage restriction on the cross-section of yields. This feature allows representing the entire term structure with a low-dimensional vector of state variables. Moreover, particularly useful to the dissertation goals, the risk premia for all durations and holding periods can be inferred from a single affine model. Lastly, the cross-sectional relations depend on a relatively small number of parameters, which ensures good estimation efficiency.

As the name suggests, affine models represent bond yields as exponential-affine functions of dynamic factors. There are also some other attributes necessary for a model being classified as affine: 1) the factors being governed by a multidimensional Itô process, 2) the short-rate process being affine on the factors, 3) the drift and diffusion of the factors' process being affine on the factors themselves. In a seminal work, Duffie and Kan (1996) demonstrated that, regularity conditions aside, the presence of these attributes is sufficient to guarantee the existence of a closed-form exponential-affine expression for the bond yields. This result can be interpreted as a generalization of the early single-factors models by Vasicek (1977) and Cox et al. (1985).



## 3.1
## Affine Model with Gaussian Factors

Similar to the case of bonds, the first step for building an affine model for commodity futures is determining the nature of the model factors. Term structure models for bond yields have traditionally relied on latent factors, i.e. factors which can only be observed through the yields. Following Ang and Piazzesi (2003), many works started to incorporate observable macro factors along with the latent in order to study how the yield curve reacts to macroeconomic shocks.

Employing observable factors has no influence on the model's evaluation of the risk premium dynamics unless these factors are unspanned by the price information contained in the term structure, such as in the bonds model of Joslin et al. (2014). Nonetheless, there is considerable disagreement on the extent to which bonds risk premia depend on information unspanned by the yield curve[9]. Searching for observable factors related to commodity futures with meaningful risk premia information is beyond the scope of this work. Therefore, latent factors are deemed

[8] Aside from Hamilton and Wu (2014), other examples of term structure models for commodity futures can be found in Casassus and Collin-Dufresne (2005) and Trolle and Schwartz (2009).

[9] Bauer and Hamilton (2018), Bauer and Rudebusch (2016), and Duffee (2013) offer different perspectives on the subject.

sufficient.

When the factors are specified with constant conditional volatilities, their generating process becomes a Gaussian diffusion. Under a discrete-time setting and assuming a Gaussian process for the dynamics of the factors vector $X_t$:

$$X_{t+1} = c + \rho X_t + \Sigma\varepsilon_{t+1} \quad \varepsilon_{t+1} \sim N(0,1) \tag{3.1}$$

This setup ignores the heteroskedasticity commonly observed in futures prices, but allows for complete flexibility in terms of the signs and magnitudes for the correlations among factors (Dai and Singleton, 2000). As detailed in the next chapter, the introduction Markov regime-switching can partially compensate for not taking into account stochastic volatility.

The absence of arbitrage opportunities is equivalent to the existence of an equivalent risk-neutral measure for the process above. Therefore, there must exist a Radon-Nikodym derivative converting the risk-neutral measure $\mathbb{Q}$ into physical measure $\mathbb{P}$ with an associated density process $\tau_{t+1}$ such that $E_t^Q(X_{t+1}) = E_t(\tau_{t+1}X_{t+1})/\tau_t$. From the Girsanov's theorem:

$$\frac{\tau_{t+1}}{\tau_t} = \exp\left(-\frac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\varepsilon_{t+1}\right) \tag{3.2}$$

In the expression above, $\lambda_t$ refers to the price of risk:

$$\lambda_t = \lambda_0 + \lambda_1 X_t = \Sigma^{-1}(c - c^Q) + \Sigma^{-1}(\rho - \rho^Q)X_t \tag{3.3}$$

where $c^Q$ and $\rho^Q$ are the risk-neutral counterparts of the parameters in Eq. (3.1). It should be noticed that the quantity $\lambda_t'\lambda_t$, which corresponds to the instantaneous variance of the pricing kernel, is not affine on $X_t$ unless $\lambda_1 = 0$ (Duffee, 2002). For this reason, models in which the price of risk is defined according to Eq. (3.3) are referred to as essentially affine in opposition to the more restricted completely affine models.

Considering the one-period risk free rate $r_t$ and the density process given in Eq. (3.2), the pricing kernel is defined as

$$M_{t,t+1} = \exp\left(-r_t - \tfrac{1}{2}\lambda_t^{'}\lambda_t - \lambda_t^{'}\varepsilon_{t+1}\right) \tag{3.4}$$

Given that the model fulfills the conditions outlined by Duffie and Kan (1996), an exponential-affine expression for the commodity futures prices must exist:

$$F_t^n = \exp\left(\alpha_n + \beta_n^{'}X_t\right) \tag{3.5}$$

In particular, for $n = 0$,

$$F_t^0 = S_t = \exp\left(\alpha_0 + \beta_0^{'}X_t\right) \tag{3.6}$$

If the coefficients $\alpha_0$ and $\beta_0$ determining the spot price process are known, the remaining pricing coefficients can be obtained recursively:

$$\alpha_n = \alpha_{n-1} + \beta'_{n-1} c^Q + \frac{1}{2} \beta'_{n-1} \Sigma \Sigma' \beta_{n-1} \tag{3.7a}$$

$$\beta'_n = \beta'_{n-1} \rho^Q \tag{3.7b}$$

Details on the derivation of these equations can be found in Appendix I. Affine models for zero-coupon bonds also rely on recursive expressions to determine the pricing coefficients, but these expressions differ from Eqs. (3.7a) and (3.7b) by the addition of terms related to the risk-free interest earned over a period. The additional terms reflect the fixed contribution of the risk-free rate to the overall return on bonds. Unlike bonds, no transaction is made when commodity futures are acquired. Rewarding long futures investors by a risk-free rate would therefore violate no arbitrage.

A desirable feature of affine models is providing a parsimonious representation to the risk premia across all contract durations. Assuming a long position held for a single period:

$$R_p^n \sim rp^n = \ln E_t \left( \frac{F_{t+1}^{n-1}}{F_t^n} \right) = \beta'_{n-1} \Sigma \lambda_t \tag{3.8}$$

In the context of commodity index investing, risk premium as defined above would correspond to an investor employing monthly contract rolling. It is straightforward to generalize this notion to longer rolling periods. As Duffee (2010) points out, the risk premium of Eq. (3.8) with some adjustments can be written in the form of an annualized Sharpe Ratio:

$$S_R^n = \sqrt{12} \frac{\beta'_{n-1} \Sigma \lambda_t}{\sqrt{\beta'_{n-1} \Sigma \Sigma' \beta_{n-1}}} \tag{3.9}$$



## 3.2
## Normalization and Estimation

After some minor adaptations, the model presented in the previous section can be succinctly represented in a state-space form:

$$X_{t+1} = c + \rho X_t + \epsilon_{t+1} \qquad \epsilon_{t+1} \sim N\left(0, \Sigma \Sigma'\right) \tag{3.10a}$$

$$f_{t+1} = A(\theta^{CS}) + B(\theta^{CS}) X_{t+1} + \epsilon_{t+1}^e \qquad \epsilon_{t+1}^e \sim N\left(0, \Sigma^e \Sigma^{e'}\right) \tag{3.10b}$$

The first equation follows directly from Eq. (3.1), whereas the second derives from the application of Eq. (3.5) to each of the contract durations $D = \{d_1, d_2, \dots, d_M\}$ for which prices are known. For the sake of simplicity, the covariances of the measurement error $\epsilon_{t+1}^e$ are assumed to be zero, rendering $\Sigma^e$ diagonal.

Attention must be given to the form of the coefficients in Eq. (3.10b). By construction, these coefficients are equivalent to $A = [\alpha_{d1}\alpha_{d2} \dots \alpha_{dM}]'$ and $B = [\beta'_{d1}\beta'_{d2} \dots \beta'_{dM}]'$, meaning that the cross-section relations across the term structure of future prices are subjected to restrictions imposed by Eqs. (3.7). Therefore, Eq. (3.10b) differs from an unrestricted regression model.

As implied by the definition of Eq. (3.10b), the parameter set $\theta^{CS} = \{c^Q, \rho^Q, \alpha_0, \beta_0, \Sigma\}$ contains all parameters necessary for determining the cross-section of futures prices. There are two other relevant parameter sets: $\theta^{DYN} = \{c, \rho\}$ and $\theta^{\lambda} = \{\lambda_0, \lambda_1\}$, corresponding to the parameters describing the dynamics of factors and the parameters in the price of risk, respectively. Parameters related to the measurement error variance are less important for neither influencing the factors' dynamics nor the cross-section relations.

Assuming $\theta^{CS}$ is known, either $\theta^{DYN}$ or $\theta^{\lambda}$ suffices for fully determining the model. Hence, for $N$ factors, the model has a total of $(2 + 5N^2 + 7N)/2$ free parameters. The fact that the number of parameters is not dependent on the number $M$ of futures durations illustrates one of the main advantages of the cross-section relations in affine models.

One strategy for estimating the full set of parameters is applying the Kalman Filter directly on Eqs. (3.10), as done in a handful of works (Duffee, 2010; Duffee, 2011; Joslin et al., 2013). Alternatively, maximum likelihood estimation (MLE) can be made feasible with some assumptions on the structure of the measurement error.

Considering the number of futures contract durations used for estimation is typically larger than the number of factors, not all contract durations can be priced without error. More specifically, at least $M - N$ linear combinations of contract durations will be priced with error. Defining as $Y_{2,t}$ and $Y_{1,t}$ the linear combinations priced with and without error, respectively,

$$Y_{1,t} = W_1 f_t \tag{3.11a}$$

$$Y_{2,t} = W_2 f_t \tag{3.11b}$$

Follows from this assumption that there must exist a linear mapping between $Y_{1,t}$ and $X_t$, although the form of this mapping is undetermined. In fact, all models in which $X_t$ is defined as some linear transformation of $Y_{1,t}$ are observationally equivalent. Therefore, some arbitrary normalization is required for the identification of $X_t$. The simplest choice is taking $X_t = Y_{1,t}$, which leads to

$$Y_{1,t+1} = c + \rho Y_{1,t} + \Sigma\varepsilon_{t+1} \tag{3.12a}$$

$$Y_{2,t+1} = W_2 A\left(\theta^{CS}\right) + W_2 B\left(\theta^{CS}\right) Y_{1,t+1} + \Sigma^{e,norm}\epsilon_{t+1}^{e,norm} \tag{3.12b}$$

with restrictions

$$W_1 A\left(\theta^{CS}\right) = 0 \tag{3.12c}$$

$$W_1 B\left(\theta^{CS}\right) = I \tag{3.12d}$$

where the superscript $norm$ is used to distinguish the normalized residuals from the higher-dimensional measurement errors in the state-space model of Eqs. (3.10).

Joslin et al. (2011) introduced an MLE-based approach for the estimation of bonds term structure models normalized as in Eqs. (3.12). Fortunately, the key features of this approach can be easily adapted to commodity futures. First, the normalization allows for a separate ordinary least square (OLS) estimation of the parameters governing the factors' dynamics $\theta^{DYN} = \{c, \rho\}$. This point is made clear from the decomposition of the conditional likelihood function:

$$\begin{aligned} & f\left(Y_{2,t+1} | Y_{1,t};\ \theta^{CS}, \theta^{DYN}, \Sigma^{e,norm}\right) \\ & = f\left(Y_{2,t+1} | Y_{1,t+1};\ \theta^{CS}, \Sigma^{e,norm}\right) f\left(Y_{1,t+1} | Y_{1,t};\ \theta^{DYN}, \Sigma\right) \end{aligned} \tag{3.13}$$

If maximization of the second term is carried out separately, its global optimum should correspond to the OLS estimate of Eq. (3.12a). Given that the parameters belonging to $\theta^{DYN}$ appear only in the second term, their OLS estimates are also global maxima for the likelihood function. Hence, only the parameters in $\theta^{CS}$ are left for estimation by MLE. It should be noticed that the OLS estimate of $\Sigma$ is not guaranteed to a global maximum because this parameter also appears in the first term as part of $\theta^{CS}$. Nonetheless, the OLS estimate of $\Sigma$ serves as an appropriate initial guess for the likelihood function maximization.

The approach by Joslin et al. (2011) also involves a reparameterization of the model which reduces the dimension of the parameter space while ensuring that the restrictions in Eqs. (3.12c) and (3.12d) are satisfied. An analogous reparameterization is available for commodity futures models:

$$\rho^Q = f_1(\zeta) \tag{3.14a}$$

$$\beta_0 = f_2(\zeta) \tag{3.14b}$$

$$c^Q = f_3(\zeta, \alpha_0, \Sigma) \tag{3.14c}$$

where $\zeta$ is a vector containing the eigenvalues of $\rho^Q$. The specific forms of functions $f_1$, $f_2$ and $f_3$ are derived in Appendix II. Taking advantage of the relations expressed in Eqs. (3.14), the set of cross-sectional parameters can be redefined: $\overline{\theta^{CS}} = \{\zeta, \alpha_0, \Sigma\}$. Replacing $\theta^{CS}$ with $\overline{\theta^{CS}}$ typically leads to easier log-likelihood maximization.

While the estimation of $\theta^{DYN}$ by OLS is only possible for single-regime models, the benefits of reparametrization by Eqs. (3.14) are also available for the Markov-switching model introduced in the next chapter.

## 3.3
## Specifying a Model for Crude Oil Futures

West Texas Intermediate (WTI) crude oil is the main component of the GSCI and the largest energy commodity allocation in the BCOM. Unlike any other commodity, the price of oil has significant repercussions to the economy at large (Hamilton, 1983), justifying the attention it receives. The trading of WTI futures takes place at the New York Mercantile Exchange (NYMEX), currently owned by the CME Group. Reflecting the importance of its underlying asset, the market for WTI futures is among the deepest and most liquid commodity futures markets, being an ideal candidate for an application of the model presented. Furthermore, using the same commodity futures as HW allows the comparison of results to be more meaningful.

Estimating a term structure model for WTI futures requires price data for multiple contract durations. The selected dataset spans the period from 01/1990 to 12/2018 and consists of end-of-month NYMEX crude oil contract prices adjusted by the US Consumer Price Index.

Choosing which of the contract durations available in the dataset should be used for estimation is the first specification issue to be addressed. Liquid contracts are preferable because their pricing is more efficient. WTI futures expiring in few months are typically the most liquid. On the other hand, contracts with durations exceeding a year tend to be relatively illiquid. A set of fairly liquid contract durations is defined as the baseline specification: $D^b = \{1\text{m}, 2\text{m}, 3\text{m}, 6\text{m}, 12\text{m}\}$. For the purpose of benchmarking, an extended set containing $D^b$ is also considered: $D^e = \{1\text{m}, 2\text{m}, 3\text{m}, 4\text{m}, 5\text{m}, 6\text{m}, 9\text{m}, 12\text{m}\}$.

Another issue central to the model specification is determining the portfolios of contract durations to be priced without error. A natural approach is opting for portfolios that capture most to the price variations across the term structure, which is easily achieved through principal component analysis. This approach is commonplace for bonds, as the three largest principal components (PC) are deemed sufficient to model the entire term structure[10]. Moreover, the first, second and third PCs can be conveniently interpreted as level, slope and curvature factors, respectively, reflecting their specific contributions to the shape of the term structure (Litterman and Scheinkman, 1991).

Table I evaluates the three largest PCs for sets $D^b$ and $D^e$. Panel A shows the weights a PC ascribes to each contract durations. Panel B evaluates the fraction of the price variance explained by each PC in a given set.

[10]Duffee (2010) shows that models with more than three factors display unreasonably high Sharpe ratios as a consequence of overfitting.

**Panel A: Weights for Contract Durations of 1m, 2m, 3m, 4m, 5m, 6m, 9m, 12m**

| $PC_1$ | | $PC_2$ | | $PC_3$ | |
|---|---|---|---|---|---|
| $D^b$ | $D^e$ | $D^b$ | $D^e$ | $D^b$ | $D^e$ |
| 0.4421 | 0.3483 | -0.5178 | -0.5375 | 0.6129 | 0.6217 |
| 0.4448 | 0.3506 | -0.3263 | -0.3665 | -0.1218 | 0.0315 |
| 0.4467 | 0.3522 | -0.1549 | -0.2128 | -0.4629 | -0.2550 |
| | 0.3535 | | -0.0782 | | -0.3572 |
| | 0.3545 | | 0.0416 | | -0.3455 |
| 0.4502 | 0.3553 | 0.2459 | 0.1471 | -0.4563 | -0.2644 |
| | 0.3568 | | 0.3971 | | 0.1055 |
| 0.4521 | 0.3572 | 0.7355 | 0.5868 | 0.4324 | 0.4685 |

**Panel B: Fraction of Term Structure Variance Explained**

| $PC_1$ | | $PC_2$ | | $PC_3$ | |
|---|---|---|---|---|---|
| $D^b$ | $D^e$ | $D^b$ | $D^e$ | $D^b$ | $D^e$ |
| 0.99439 | 0.99565 | 0.00548 | 0.00422 | 0.00012 | 0.00011 |

Table I: Principal components evaluated for two sets of futures contracts

Similar to the case of bonds, the distribution of weights suggests that the three PCs can be interpreted as factors representing the level, slope and curvature of the term structure. Nonetheless, in both sets considered, the combination of the two largest PCs is able to explain at least 99.9% of the variance, rendering the third PC unnecessary.

Owining to the existance of price correlations across the entire term structure of WTI futures, the PCs obtained from $D^e$ and $D^b$ should be similar in spite of additional information contained in the extended set $D^e$. This can be formally demonstrated by regressing each PC of $D^e$ on its $D^b$ equivalent:

$$\widehat{PC_{n,t}^{D^e}} = a_n + b_n \widehat{PC_{n,t}^{D^b}} + u_t \qquad n = 1,2 \qquad (3.15)$$

where the hat denotes that the PC is normalized by its standard deviation. Eq. (3.15) is used for testing the null hypothesis of $b_n = 1$. Given that the error term is non-spherical, standard erros are obtained using heteroskedasticity and autocorrelation consistent (HAC) covariance estimators with bandwidths determined according to Andrews and Monahan (1992).

| PC | $b_n - 1$ | Standard Error | P-value |
|---|---|---|---|
| 1 | -9.865e-05 | 0.001393 | 0.9436 |
| 2 | -1.636e-04 | 0.002328 | 0.9440 |

Table II: Regression for testing the correlation between PCs obtained from different sets

As seen in Table II, the null hypothesis is accepted in both regression, which confirms that the PCs from the different sets are indeed extremely correlated. Given that adding more contract durations to the baseline specification set $D^b$ seem to have little effect on the dynamics of the PCs produced, it can be argued that the information contained in this set is sufficient to model the entire term structure.

# 4
# Modelling Regime Changes

In spite of most econometric models being built upon the assumption of linearity, non-linear behavior is frequently observed in economic time series. A relatively straightforward way of incorporating non-linearities to an otherwise linear model is by allowing its parameters to vary according to some underlying process. In the most common setting, parameters are assumed to switch between a predetermined number of regimes governed by a discrete Markov chain.

Markov-switching was initially conceived as a way of modeling the dynamics of aggregate output during different phases of the business cycle, having successfully produced a distinct regime for periods identified as recessions[11] (Hamilton, 1989). Other macroeconomic applications followed from this seminal work, most notably the study of changes in monetary policy rules (Sims and Zha, 2006). In the field of finance, Markov-switching has been employed in problems ranging from modeling different exchange rate regimes to detecting changes in the equity returns predictability during market turmoil (Ang and Timmerman, 2012).

The financialization of the commodity futures market can be broadly interpreted as a technological change with potential repercussions to the risk premium. Concurrent to this change, commodity futures are expected to undergo periods of bull and bear markets also capable of affecting the risk premium dynamics. Distinguishing the effects of financialization from transitory changes is crucial for the goals of this dissertation. Markov-switching handles this issue seamlessly because it captures both structural changes arising from new technologies or regulations and the transient market regimes occurring recurrently.

Aside from the theoretical motivations, the use of Markov-switching can also be justified from an empirical perspective. More specifically, the presence of multiple regimes helps the model to reproduce fundamental features of the commodity futures data. To illustrate this point, Table III presents some descriptive statistics on WTI futures excess log-returns and a conditional heteroskedasticity test. Returns are calculated according to a one-month rolling methodology based on 3-month contracts. Heteroskedasticity is assessed through the Lagrange multiplier test of Engle (1982) applied to the demeaned log-return series.

The first column in Table III refers to the entire series available, whereas the second and third columns analyze subintervals resulting from the division of the original series in two contiguous periods. Similar to HW, this division aims to outline differences arising from financialization.

| | 01/1990 – 12/2018 | 01/1990 – 12/2004 | 01/2005 – 12/2018 |
|---|---|---|---|
| Skewness | -0.2950 | 0.2655 | -0.8410 |
| Excess kurtosis | 2.0023 | 1.7630 | 1.9571 |
| Engle's test statistic | 11.2925 | 5.2878 | 4.2026 |
| p-value | 0.0008 | 0.0215 | 0.0404 |

Table III: Stylized facts in WTI futures returns

[11] Determined by the Business Cycle Dating Committee of the National Bureau of Economic Research (NBER).

Table III shows that non-zero skewness and fat tails are pervasive features of WTI futures excess returns. Moreover, the hypothesis of homoscedasticity was rejected in all intervals considered. Timmerman (2000) demonstrated that these stylized facts can be reproduced by a Gaussian model provided its conditional mean and variance-covariance matrix are regime-dependent. Therefore, regime-switching can be seen as a relatively simple adaptation that enables a model based on Eq. (3.1) to better accommodate the problem data.

## 4.1
## Testing for Multiple Regimes

Before delving deeper into regime-switching modelling, the existence of multiple regimes must be rigorously tested. Most research dedicated to testing parameter variability in econometric models have stressed the need for specific testing methodologies[12]. Conventional approaches such as the likelihood test fail to display their expected asymptotic distributions in the presence of nuisance parameters, i.e. parameters only identified under the alternative hypothesis. Consequently, the null cannot be tested against entire classes of non-linear models in which some unobserved process controls regimes changes. In the case of Markov-switching, for instance, nuisance parameters appear in the form of transition probabilities.

Carrasco et al. (2014) proposed an asymptotically optimal test for parameter variability based on the autocorrelations of the process controlling the parameters' change and the first two derivatives of the likelihood function. One key advantage of their approach is that the model needs to be estimated only under the null, which is particularly useful when estimating the alternative involves MLE with potentially multiple local maxima. Their approach is also flexible enough to accommodate alternative hypotheses with non-linearities other than Markov-switching as long as the covariance structure of the changing parameters is known. Unfortunately, bootstrapping is needed for the evaluation of critical values because the distribution of the test statistic is a function of nuisance parameters.

From the previous chapter, an affine model for WTI futures is appropriately specified with the two largest PCs being used as factors. Hence, the process governing the factors' dynamics can be written as

$$PC_{n,t+1} = c_n + \rho_n PC_{n,t} + \sigma_n \varepsilon_{t+1} \qquad n = 1, 2 \qquad (4.1)$$

The equation above is tested for variations of conditional mean $c_n$ or variance $\sigma_n^2$. Appendix III describes the methodology for calculating the statistic used for jointly testing these parameters.

Obtaining a bootstrapped distribution requires simulating several synthetic series from Eq. (4.1) estimated under the null hypothesis. A precise evaluation of the critical values requires at least some hundreds of simulations. In order to reduce the computational burden involved in this task, Eq. (4.1) is presented as sequence of regressions that can be tested separately as opposed to a vector model. Ignoring the residuals covariances is not unreasonable considering that the factors were specified as PCs, which are orthogonal by construction.

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[12] Hansen (1992), Andrews (1993), Cho and White (2007) just to mention a few.

Table IV presents the test results and critical values obtained from a thousand bootstrapped samples. The hypothesis of constant parameters was overwhelmingly rejected in both equations tested, confirming the need for a Markov-switching formulation.

| n | Test statistic | Bootstrapped critical value | P-value |
|---|---|---|---|
| 1 | 7.4268 | 3.8403 | 0.0000 |
| 2 | 7.1091 | 3.4407 | 0.0010 |

Table IV: Parameter variability tests

## 4.2 Factors with Regime-Dependent Dynamics

Motivated by the previous discussion, this section presents a model for the factor's dynamics in which the parameters are regime-dependent. This model belongs to the Markov-switching class because the regimes are assumed to be states of a Markov chain process. Given a $K$-state Markov chain with states denoted by $s_t$, the process in Eq. (3.12a) is rewritten as

$$Y_{1,t+1} = c^{s_{t+1}} + \rho Y_{1,t} + \Sigma^{s_{t+1}} \varepsilon_{t+1} \tag{4.2}$$

It follows from Eq. (4.2) that the conditional probability density for the one-step ahead forecast of the factors depends exclusively on the subsequent regime:

$$\begin{aligned} & f\left(Y_{1,t+1} | Y_{1,t};\ s_t = j;\ s_{t+1} = k\right) \\ & = f\left(Y_{1,t+1} | Y_{1,t};\ s_{t+1} = k\right) \sim N\left(c^k + \rho Y_{1,t}, \Sigma^k \Sigma^{k\prime}\right) \end{aligned} \tag{4.3}$$

Before obtaining an expression for likelihood function of the model described by Eq. (4.2), it is first necessary to introduce the concept of filtered probabilities, i.e. the regimes' probabilities conditional on the current information:

$$\xi_{t|t} = \begin{bmatrix} \text{prob}(s_t = 1 | \mathcal{F}_t) \\ \vdots \\ \text{prob}(s_t = K | \mathcal{F}_t) \end{bmatrix} \tag{4.4}$$

From the definition of Markov chain process, the filtered probabilities can be used for h-step ahead forecasts of the regimes' probabilities:

$$\xi_{t+h|t} = (\pi')^h \xi_{t|t} \tag{4.5}$$

where $\pi$ is the transition probability matrix of the Markov chain, i.e.

$$\pi = \begin{bmatrix} \pi_{11} & \cdots & \pi_{1K} \\ \vdots & \ddots & \vdots \\ \pi_{K1} & \cdots & \pi_{KK} \end{bmatrix} = \tag{4.6}$$

$$\begin{bmatrix} \text{prob}(s_t = 1|s_{t+1} = 1) & \dots & \text{prob}(s_t = 1|s_{t+1} = K) \\ \vdots & \ddots & \vdots \\ \text{prob}(s_t = K|s_{t+1} = 1) & \dots & \text{prob}(s_t = K|s_{t+1} = K) \end{bmatrix}$$

Hence, the probability density of a new observation $Y_{1,t+1}$ conditional on the current information can be constructed as

$$f\left(Y_{1,t+1}|\mathcal{F}_t\right) = \mathbf{1}'\left(\xi_{t+1|t} \odot \eta_{t+1}\right) \tag{4.7}$$

where $\eta_t$ is a vector concatenating the probability density of Eq. (4.3) conditional on each of the existing regimes:

$$\eta_t = \begin{bmatrix} f\left(Y_{1,t}|Y_{1,t-1};\ s_t = 1\right) \\ \vdots \\ f\left(Y_{1,t}|Y_{1,t-1};\ s_t = K\right) \end{bmatrix} \tag{4.8}$$

Filtered probabilities can be updated by a simple application of the Bayes rule based on Eq. (4.7):

$$\xi_{t|t} = \frac{\left(\xi_{t|t-1} \odot \eta_t\right)}{\mathbf{1}'\left(\xi_{t|t-1} \odot \eta_t\right)} \tag{4.9}$$

Noticing that the vector of initial probabilities $\xi_{1|0}$ is an additional model parameter to be estimated[13].

Finally, the log-likelihood function in the presence of Markov-switching can be calculated by applying Eq. (4.7) to each new observation and updating the filtered probabilities according to Eq. (4.9):

$$\mathcal{L}(\theta^{DYN,MS}, \Sigma^1, \dots, \Sigma^K) = \sum_{t=1}^{T} \log\left[\mathbf{1}'\left(\xi_{t|t-1} \odot \eta_t\right)\right] \tag{4.10}$$

where $\theta^{DYN,MS} = \{c^1, \dots, c^K, \rho, \pi\}$.

Because the parameters representing probabilities are limited in range, the maximization of the Markov-switching likelihood function is a constrained optimization problem. One way of avoiding the additional complexity of dealing with parameter constraints is by representing the model probabilities as sigmoid functions of constraint-free parameters.

A general feature of models based on Markov-switching is that the econometrician is uncapable of observing the regime realizations directly but only their associated probabilities. In contrast with the filtered probabilities of Eq. (4.9), smoothed probabilities $\xi_{t|T}$ are conditioned on the entire set of observations, being thus the best indicators of the prevailing regime at a given moment. The procedure for obtaining these probabilities involves a backward recursion starting from the last vector of filtered probabilities $\xi_{T|T}$:

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[13] If the Markov Chain is ergodic, an alternative approach would be taking $\xi_{t|0}$ as the unconditional probabilities of the regimes.

$$\xi_{t|T} = \xi_{t|t} \odot \{\pi'[\xi_{t+1|T}(/)\xi_{t+1|t}]\} \quad (4.11)$$

where $(/)$ denotes element-wise division. A step-by-step demonstration of Eq. (4.11) can be found in Kim and Nelson (1999).

The simplest strategy for regime classification based on smoothed probabilities is assigning to date $t$ the regime $k$ with $\xi_{t|T}^{k} > 0.5$, provided that such regime exists. This classification scheme works best when the regimes are clearly defined. More generally, results from a Markov-switching model are more useful if it is uncommon for multiple regimes to display considerable probabilities simultaneously. Taking this into account, Ang and Bekaert (2002) proposed a regime classification measure (RCM) as a diagnostic check for Markov-switching results:

$$RCM(K) = 100K^2 \frac{1}{T} \sum_{t=1}^{T} \left( \prod_{k=1}^{K} \xi_{t|T}^{k} \right) \quad (4.12)$$

In the case of two regimes, it is straightforward to show that RCM is small if the regimes are clearly defined and approaches 100 otherwise. For multiple regimes, however, it is possible to have near-zero RCM and yet poorly defined regimes provided that there is at least one very unlikely regime. A simple fix is applying the product of Eq. (4.12) solely to the two most likely regimes at each $t$.



## 4.3 Markov-Switching in an Affine Term Structure Model

In addition to the definition of a process governing the dynamics of the factors, an affine model also needs expressions for the cross-section relations across the term structure. These relations are derived from the imposition of no arbitrage, which in turn implies the existence of a risk-neutral measure and a pricing kernel. Consistent with the features desired for the model, the following risk-neutral dynamics is proposed:

$$Y_{1,t+1} = c^Q + \rho^Q Y_{1,t} + \Sigma^{s_{t+1}} \varepsilon_{t+1} \quad (4.13a)$$

$$\pi^Q = \begin{bmatrix} \pi_1^Q & \cdots & \pi_K^Q \\ \vdots & \ddots & \vdots \\ \pi_1^Q & \cdots & \pi_K^Q \end{bmatrix} \quad (4.13b)$$

Contrary to the physical-measure dynamics of Eq. (4.2), only the matrix $\Sigma^{s_{t+1}}$ is regime-dependent in Eq. (4.13a). This matrix is also the only parameter common to both measures, satisfying the minimal condition for an essentially affine model. The transition probability matrix defined by Eq. (4.13b) fully describes the risk-neutral dynamics of the Markov chain underlying the parameter changes in the model. Given that it has equal lines, the Markov chain under the risk-neutral measure behaves as an independent switching process i.e. a process in which regime probabilities are independent from the current regime.

Analogous to the single-regime case, the parameters in Eq. (4.13a) are linked to their ℙ-measure counterparts by a regime-dependent version of the price of risk:

$$\lambda_t^{s_{t+1}} = \lambda_0^{s_{t+1}} + \lambda_1^{s_{t+1}} Y_{1,t} = {\Sigma^{s_{t+1}}}^{-1}(c^{s_{t+1}} - c^Q) + {\Sigma^{s_{t+1}}}^{-1}(\rho - \rho^Q) Y_{1,t} \qquad (4.14)$$

For the model transition probabilities, the link between the ℙ- and ℚ-measures is established by the introduction of a new term:

$$\Gamma_{s_t, s_{t+1}} = \ln\left(\frac{\pi_{s_t, s_{t+1}}}{\pi_{s_{t+1}}^Q}\right) \qquad (4.15)$$

The model is completed with the postulation of a regime-dependent pricing kernel:

$$M_{t,t+1} = \exp\left(-r_t - \Gamma_{s_t, s_{t+1}} - \frac{1}{2} {\lambda_t^{s_{t+1}}}' \lambda_t^{s_{t+1}} - {\lambda_t^{s_{t+1}}}' \varepsilon_{t+1}\right) \qquad (4.16)$$

Considering the dynamics described by Eqs. (4.13), an exponential-affine representation of the commodity futures prices in the form of Eq. (3.5) is available and its pricing coefficients are given by

$$\alpha_n = \alpha_{n-1} + \beta_{n-1}' c^Q + \ln\left[\sum_{k=1}^{K} \pi_k^Q \exp\left(\frac{1}{2}\beta_{n-1}' \Sigma^k {\Sigma^k}' \beta_{n-1}\right)\right] \qquad (4.17a)$$

$$\beta_n' = \beta_{n-1}' \rho^Q \qquad (4.17b)$$

Details on the derivation of these expressions can be found in Appendix IV. In spite of factors’ dynamics in the physical measure being regime-dependent, the cross-section relations shown in Eqs. (4.17) are regime-independent. In fact, Eqs. (4.17) only differ from the single-regime Eqs. (3.7) by the presence of a modified Jensen term.

Compared to previous works dealing with Markov-switching in the context of an affine term structure framework, this model presents some interesting innovations. Ang and Bekaert (2002), for instance, did not considered a distinct risk-neutral dynamics for the process underlying the parameter changes. Therefore, their evaluation of risk premia does not account for the uncertainty in the regime transitions. Dai et al. (2007) introduced a pricing kernel similar to Eq. (4.17), but with some important differences. First, their price of risk is conditioned on the current regime instead of the next. Second, $\Gamma$ is defined such that $\pi^Q$ is unrestricted. The outcome is a model with regime-dependent cross-section relations, more parameters and a greater susceptibility to overfitting. Lastly, these works dealt with the term structure of zero-coupon bonds instead of commodity futures.

Estimating a Markov-switching affine model is slightly more complicated than the unrestricted dynamics of factors of the previous section because of the additional parameters necessary for describing the cross-section relations.

Fortunately, the reparameterization described in Appendix II still applies, reducing considerably the set of additional parameters: $\theta^{CS,MS} = \{\alpha_0, c^Q, \zeta, \pi^Q, \Sigma^1, \dots, \Sigma^K\}$. Hence, the log-likelihood function can be represented as

$$\mathcal{L}(\theta^{CS,MS}, \theta^{DYN,MS}) = \sum_{t=1}^{T} \log f\left(Y_{2,t} | Y_{1,t};\ \theta^{CS,MS}\right) + \log\left[\sum_{k=1}^{K} \xi_{t|t-1}^{k} f\left(Y_{1,t} | Y_{1,t-1};\ s_t = k; \theta^{DYN,MS}, \Sigma^1, \dots, \Sigma^K\right)\right] \quad (4.18)$$

The first term in Eq. (4.18) is the log-likelihood of Eq. (3.12b) with the cross-section relations given by Eqs. (4.17). The second comes directly from Eq. (4.10).

It should be noticed that the regime-dependent condicional volatility $\Sigma^s$ appears in both terms of the log-likelihood function, meaning that this parameter is overidentified. Ideally, the role $\Sigma^s$ plays in the pricing equations should not interfere with its contribution to the dynamics of factors. One way of verifying if this condition is met is by comparing the inferred smoothed probabilities from the affine model with those from an unrestricted model.

Conditional on the current regime $s_t = j$, the risk premium for holding a long position over a single-period is given by

$$rp_t^{n,j} = \ln E_t\left[\frac{F_{t+1}^{n-1}}{F_t^n} | s_t = j\right] = \sum_k \pi_{j,k}^P \frac{e^{\beta_{n-1}' \Sigma^k \lambda_t^k + \frac{1}{2}\beta_{n-1}' \Sigma^k \Sigma^{k'} \beta_{n-1}}}{\sum_l \pi_l^Q e^{\frac{1}{2}\beta_{n-1}' \Sigma^l \Sigma^{l'} \beta_{n-1}}} \quad (4.19)$$

An unconditional risk premium can be evaluated from Eq. (4.19) using the filtered probabilities:

$$rp_t^n = \sum_{j=1}^{K} \xi_{t|t}^j rp_t^{n,j} \quad (4.20)$$

Given the goals of this dissertation, there must exist a quantity that encapulates the overall risk premium behavior in a given regime. A regime-specific risk premium average is defined for this purpose:

$$\overline{rp^{n,J}} = \frac{1}{\sum_{t=1}^{T} \boldsymbol{I}\left(\xi_{t|T}^j > 0.5\right)} \sum_{t=1}^{T} \boldsymbol{I}\left(\xi_{t|T}^j > 0.5\right) rp_t^{n,j} \quad (4.21)$$

where $\boldsymbol{I}$ is the indicator function.

# 5
# Results

Having introduced a Markov-switching affine term structure model, the next step is taking advantage of this model to investigate how financialization affected the risk premium in commodity futures. More importantly, the proposed model is used to produce results suitable for comparison with the previous literature, particularly HW.

Dealing with multiple regimes involves some additional specification decisions, namely the number of regimes and the form of the transition probability matrix. In the absence of a generally recognized method for handling these decisions systematically, multiple specifications need to be attempted and have their results analyzed.

Table V outlines the three specifications considered. For an easier reference, each specification is assigned to a case number.

| | |
|---|---|
| **Case I** | 2 regimes with unrestricted transition probability matrix |
| **Case II** | 2 regimes with zero probability of reverting to initial regime |
| **Case III** | 3 regimes with unrestricted transition probability matrix |

Table V: Description of different regime specifications

In order to ensure that the restrictions on the factors' dynamics imposed by the affine formulation are not influencing the regime changes, each specification case is estimated in both the affine and unrestricted forms. The smoothed probabilities obtained are displayed in juxtaposition for comparision. From this comparison, it can be verified that the restrictions associated with the affine formulation had no noticiable effect on the regimes' probabilities in all three specification cases considered.

As a preliminary analysis, Case I consists of a specification with two regimes and no restrictions on the transition probability matrix. Smoothed probabilities shown in Figure 3 indicate that Case I is characterized by fairly defined regimes which alternate frequently. According to Table VI, the second regime can be distinguished from the first by a highly significant volatility increase. Hence, Case I seems suitable to represent market turmoil alternating with moments of normalcy.

However, the regime dynamics of Case I offers no insight into the effects of financialization. In contrast to the frequent regime switching seen in Figure 3, the popularization of commodity index investment is a one-time event, which translates as a permanent regime switch. Fortunately, Markov-switching models can easily reproduce structural changes by setting to zero the probability of an "old regime" being revisited. More generally, non-recurrent regimes can be created by placing zeros in appropriate positions of the transition probability matrix. Case II is specified with two regimes, one of them being non-recurrent.

Consistent with the expansion period of commodity index investment, Figure 4 shows that the structural break in Case II takes place around the mid-2000s. According to Table VI, the period following the break is characterized by slightly higher volatility, albeit much lower than the turmoil regime of Case I. From the term structure of risk premium averages shown in Figure 6, the late regime was also marked by a sharp decline in risk premium, particularly in the smaller durations. This risk premium drop is compatible with the predictions from the hedging pressure theory. More specifically, assuming no changes in the hedging demands of producers, better integration of the commodity futures market to the wide financial market would lead to a smaller risk premium.

Interestingly, the structural break in Case II is almost coincident with the date chosen by HW to mark the beginning of the prevalence of CIIs in the WTI futures market. Nonetheless, their conclusions deviate from the results of Case II in some aspects. Most notably, the increase in volatility following the emergence of CIIs is far more pronounced in their study. This extra volatility can likely be attributed to the use of a sample of different size. Given the sample available to HW ended in 2011, their post-financialization period overlapped considerably with the great financial crisis, a moment when volatility was indeed much larger than usual.

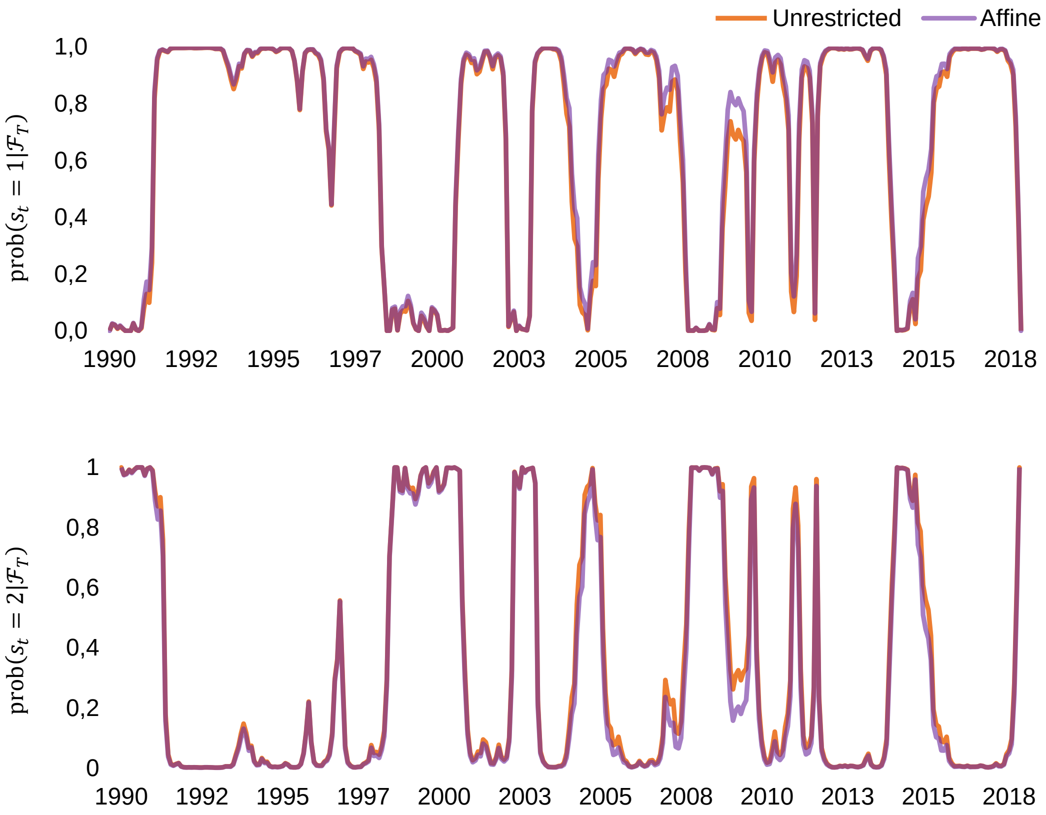


Figure 3: Smoothed regime probabilities for Case I obtained with the unrestricted and affine models. Regimes one and two are displayed respectively at the top and bottom.

The main drawback of Case II is being uncapable of reproducing recurrent events with potential implications to the risk premium, such as the turmoil regime of Case I. A direct comparison of the risk premium before and after financialization only makes sense if the fluctuations emerging from these recurrent events can be averaged out, which is not guaranteed. In order to overcome the limitations of Case II, Case III is specified with three regimes and no restrictions on the transition probability matrix.

According to the smoothed probabilities in Figure 5 and the parameter estimates in Table VI, Case III consists of two persistent regimes interrupted by brief occurrences of a high-volatility third regime. This pattern prompts interpreting the third regime as being representative of times of turmoil, being equivalent to the second regime of Case I.

Regimes in Case III are very well-defined, which is reflected in a low RCM value. As a consequence, the occurrences of the turmoil regime can be easily ascribed to noteworthy events of market stress, such as the Operation Desert Storm, the emerging market crises of the late 1990s, the great financial crisis of 2008 and the oil glut of the mid-2010s.

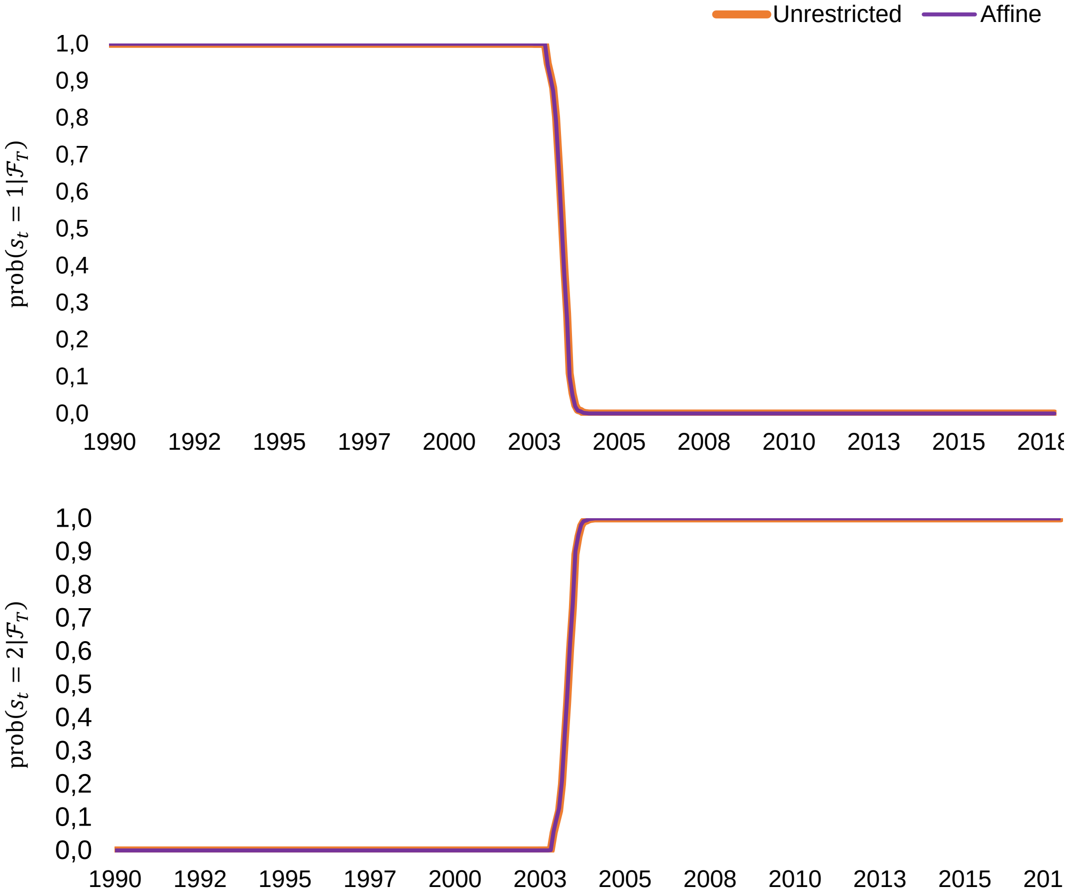


Figure 4: Smoothed regime probabilities for Case II obtained with the unrestricted and affine models. Regimes one and two are displayed respectively at the top and bottom.

From Table VI, the second persistent regime is slightly more volatile than the first. More importantly, Figure 6 reveals that the risk premium average is higher for the second regime across all durations of the term structure. It remains true, however, the conclusion drawn from Case II that the risk premium is overall smaller after the popularization of CIIs. This apparent contradiction is caused by the risk premium behavior of the third regime. The early instances of this high volatility regime present much higher risk premia than the later ones. Hence, the risk premium is generally larger after financialization, but its overall average is smaller due to brief appearances of the high volatility regime.

The predominance of a regime with high risk premium after the mid-2000s seems at odds with the previously established notion that the CIIs would decrease the costs of hedging for commodity producers. Nonetheless, this result is compatible with the evidence of increased correlations between the prices of commodities and financial assets after financialization. If correlations are no longer negative, commodity risk premium must increase to compensate for the greater exposure to systematic risk. In fact, assuming that no regime change takes place, the Sharpe Ratios for holding one-month long positions during the predominant post-financialization regime are somewhat comparable to traditional asset classes, as shown in Figure 7.

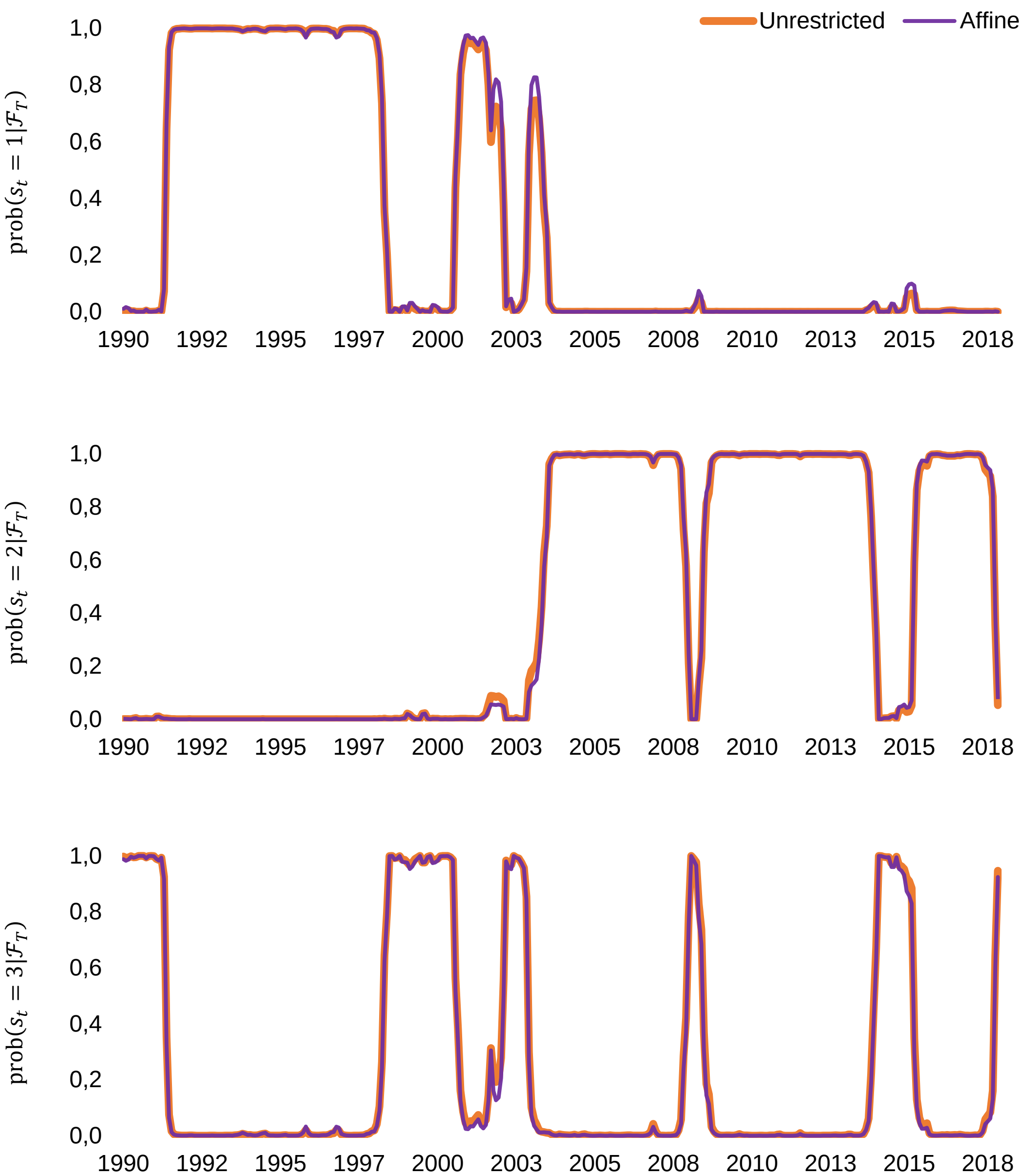


Figure 5: Smoothed regime probabilities for Case III obtained with the unrestricted and affine models. Regimes one, two and three are displayed respectively at the top, centre and bottom.

| | Case I | | Case II | | Case III | | |
|---|---|---|---|---|---|---|---|
| $c^1$ | 0.01533 | -0.01791 | 0.41336 | -0.03700 | 0.44255 | -0.05552 | |
| | (0.06443) | (0.01393) | (0.12854) | (0.02272) | (0.11146) | (0.02059) | |
| $c^2$ | -0.01567 | -0.01638 | 0.51775 | -0.03751 | 0.60071 | -0.06991 | |
| | (0.06970) | (0.01473) | (0.16245) | (0.02841) | (0.14162) | (0.02600) | |
| $c^3$ | ----- | ----- | ----- | ----- | 0.45021 | -0.06062 | |
| | | | | | (0.12098) | (0.02324) | |
| $\rho$ | 1.00016 | 0.07145 | 0.94694 | -0.03191 | 0.94156 | -0.02646 | |
| | (0.00718) | (0.11541) | (0.01694) | (0.11647) | (0.01444) | (0.10882) | |
| | 0.00106 | 0.89987 | 0.00327 | 0.89598 | 0.00614 | 0.88887 | |
| | (0.00154) | (0.02545) | (0.00296) | (0.02577) | (0.00265) | (0.02457) | |
| $\Sigma^1\Sigma^{1\prime}$ | 0.01493 | -0.00246 | 0.02671 | -0.00654 | 0.01226 | -0.00303 | |
| | (0.00177) | ---- | (0.00304) | ---- | (0.00200) | ---- | |
| | -0.00246 | 0.00070 | -0.00654 | 0.00231 | -0.00303 | 0.00099 | |
| | (0.00031) | (0.00008) | (0.00083) | (0.00027) | (0.00049) | (0.00015) | |
| $\Sigma^2\Sigma^{2\prime}$ | 0.06535 | -0.01074 | 0.03244 | -0.00339 | 0.02100 | -0.00194 | |
| | (0.01076) | ---- | (0.00344) | ---- | (0.00259) | ----- | |
| | -0.01074 | 0.00343 | -0.00339 | 0.00077 | -0.00194 | 0.00053 | |
| | (0.00221) | (0.00063) | (0.00045) | (0.00008) | (0.00033) | (0.00006) | |
| $\Sigma^3\Sigma^{3\prime}$ | ----- | ----- | ----- | ----- | 0.06123 | -0.01262 | |
| | | | | | (0.01086) | ---- | |
| | | | | | -0.01262 | 0.00426 | |
| | | | | | (0.00253) | (0.00075) | |
| $\pi$ | | | | | 0.96636 | 0.00792 | 0.02572 |
| | 0.94837 | 0.05163 | 0.99430 | 0.00570 | (0.02229) | (0.01334) | ----- |
| | (0.01997) | ---- | (0.00585) | ---- | 0.00026 | 0.98182 | 0.01793 |
| | 0.11464 | 0.88536 | 0 | 1 | (0.00168) | (0.01162) | ----- |
| | (0.04158) | ---- | ---- | ---- | 0.05397 | 0.02917 | 0.91686 |
| | | | | | (0.03274) | (0.02427) | ----- |
| $\xi_0$ | 0.00000 | 1.00000 | 1 | 0 | 0.05542 | 0.04769 | 0.89689 |
| | (0.00000) | --- | ---- | ---- | (0.24827) | (0.21705) | ----- |
| $\zeta$ | 0.99781 | 0.89781 | 0.99778 | 0.89791 | 0.99778 | 0.89787 | |
| | (0.00065) | (0.00404) | (0.00065) | (0.00405) | (0.00065) | (0.00403) | |
| $\alpha_0$ | 0.00684 | | 0.00761 | | 0.00652 | | |
| | (0.00363) | | (0.00360) | | (0.00362) | | |
| $\pi^Q$ | 0 | 1 | 0.99977 | 0.00023 | 0.00236 | 0.00256 | 0.99508 |
| | (0.00000) | ----- | (0.00971) | ----- | (0.02745) | (0.02672) | ----- |
| **RCM** | 23.15 | | 1.717 | | 10.63 | | |

Table VI: Parameter estimates and RCM

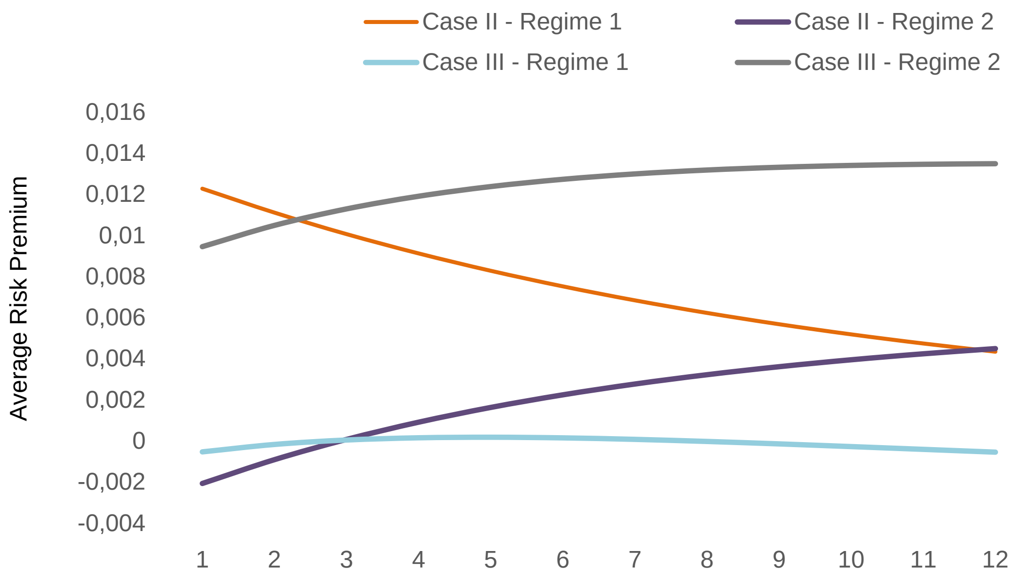


Figure 6: Risk premium averages for one-month holding periods across multiple durations.

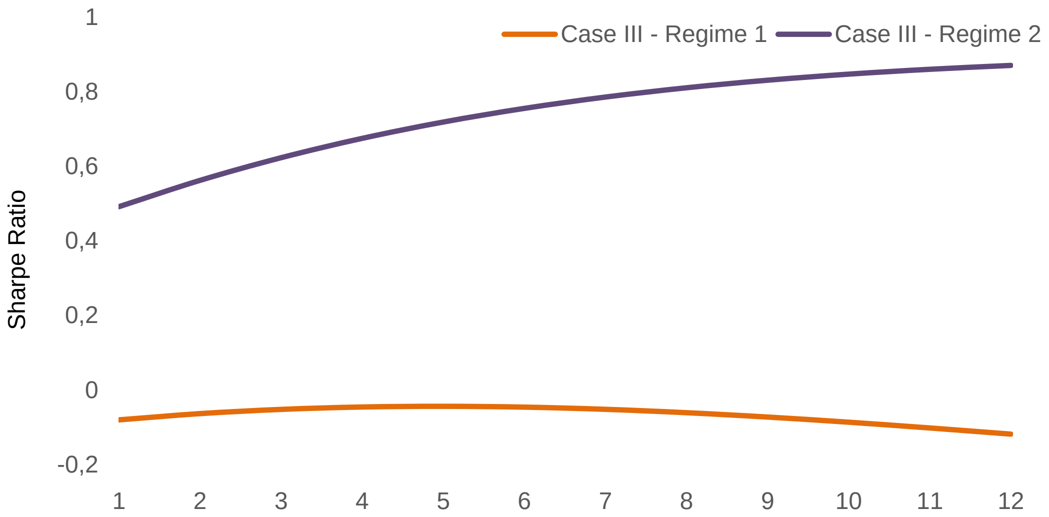


Figure 7: Sharpe Ratios conditional on staying in the same regime for one-month holding periods across multiple durations.

# 6
# Conclusion

The results obtained in this dissertation agree in many aspects with the previous literature on the effects of commodity index investing to the risk premium of commodities. In particular, the emergence of a distinct risk premium dynamics around the mid-2000s is consistent with the period generally accepted as when CIIs became dominant players in the commodity futures market. Furthermore, the risk premium was found to be on average smaller after this transformation took place, which aligns with both theoretical predictions and some previous empirical works on the subject.

Delving deeper into the problem, the results also reveal some important and previously ignored features of the period following the popularization of commodity index investing: most of its near-zero and negative risk premium was observed during brief instances of a high volatility regime, whereas the predominant regime in this period is characterized by high risk premium and moderate volatility. The existence of a persistent regime with high risk premium might explain the continuing popularity of commodity futures among financial investors during times of increased correlations in the prices of commodities and traditional financial assets. Nonetheless, further research is necessary to understand if the emergence of a systematic component in the commodity futures risk premium is linked to the process of financialization underwent by the commodity futures market.

From a methodological perspective, this dissertation introduced an affine term structure model for commodity futures with Markov-switching. The model builds upon several of the recent advancements in term structure modelling developed in the context of zero-coupon bonds. Its innovation comes in the form of a regime-dependent pricing kernel that ensures flexibility and yet avoids unrealistic regime-dependence in the cross-section relations of the term structure. The outcome is a model with fewer parameters, easier estimation and less susceptibility to overfitting. Moreover, it was verified that this model produces a dynamic of regime change nearly identical to that of an unrestricted Markov-switching benchmark. Therefore, the model offers the advantages of an affine formulation while not sacrificing goodness-of-fit.

# 7
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# APPENDIX I

Ignoring both margin and transactional costs, no payment is made for acquiring a future contract. Therefore, from risk-neutral pricing:

$$0 = E_t^Q[e^{-r_t}(F_{t+1}^{n-1} - F_t^n)] \tag{A.1}$$

Substituting the exponential-affine form of Eq. (3.5) into Eq. (A.1):

$$0 = E_t^Q[\exp(\alpha_{n-1} + \beta'_{n-1}X_{t+1}) - \exp(\alpha_n + \beta'_n X_t)] \tag{A.2}$$

Reorganizing the expression:

$$\exp(\alpha_n + \beta'_n X_t) = \exp(\alpha_{n-1})E_t^Q[\exp(\beta'_{n-1}X_{t+1})] \tag{A.3}$$

The expectation term is equivalent to the well-known Laplace transform of a multivariate normal variable. Given a variable $X$ such that $X \sim N(\mu, \Omega)$ and a vector with the same dimension as $X$:

$$L_X(u') = \exp\left(u'\mu + \frac{1}{2}u'\Omega u\right) \tag{A.4}$$

Adapting the transform in Eq. (A.4) to Eq. (A.3)

$$\begin{aligned} &\exp(\alpha_n + \beta'_n X_t) \\ &= \exp\left(\alpha_{n-1} + \beta'_{n-1}c^Q + \frac{1}{2}\beta'_{n-1}\Sigma\Sigma'\beta_{n-1} + \beta'_{n-1}\rho X_t\right) \end{aligned} \tag{A.5}$$

Finally, the Eqs. (3.7) are obtained by equating the coefficients on each side of Eq. (A.5). Hamilton and Wu (2014) employed an ad hoc approach instead of risk-neutral pricing, but reached the exact same pricing equations.

## APPENDIX II

Based on the argument of observational equivalence between the model of Eqs. (3.12) and any model in which the factors are a linear transformation of $Y_{1,t}$, the following alternative model is proposed:

$$X_{t+1} = \widetilde{c^Q} + \widetilde{\rho^Q} X_t + \widetilde{\Sigma} \varepsilon_{t+1}^Q \tag{A.6a}$$

$$\ln S_t = \widetilde{\alpha_0} + \widetilde{\beta_0'} X_t \tag{A.6b}$$

with

$$\widetilde{c^Q} = 0 \tag{A.6c}$$

$$\widetilde{\rho^Q} = J(\xi) \tag{A.6c}$$

$$\widetilde{\beta_0'} = \mathbf{1}' \tag{A.6e}$$

where $J(\xi)$ is a Jordan form with eigenvalues $\xi$ assumed to be real and distinct. As with the original model, the alternative model can be used to represent the futures log-prices:

$$f_t = \gamma(\xi, \widetilde{\alpha_0}, \widetilde{\Sigma}) + \Gamma'(\xi) X_t \tag{A.7}$$

The form of matrices $\gamma(\xi, \widetilde{\alpha_0}, \tilde{\Sigma})$ and $\Gamma'(\xi)$ can be derived from Eqs. (3.7). Given the assumptions regarding the eigenvalues, $J(\xi)$ must be diagonal and

$$\Gamma'(\xi) = \begin{bmatrix} \xi_1^{d1} & \cdots & \xi_N^{d1} \\ \vdots & \ddots & \vdots \\ \xi_1^{dM} & \cdots & \xi_N^{dM} \end{bmatrix} \tag{A.8}$$

Multiplying both sides of Eq. (A.7) by $W_1$ and reordering its terms leads to an expression linking $Y_{1,t}$ to the factors $X_t$ in the alternative model:

$$X_t = \left(W_1 \Gamma'(\xi)\right)^{-1} [Y_{1,t} - W_1 \gamma(\xi, \widetilde{\alpha_0}, \widetilde{\Sigma})] \tag{A.9}$$

Substituting Eq. (A.9) into Eqs. (A.6a) and (A.6b):

$$\begin{aligned} Y_{1,t+1} = &\left\{I - \left[W_1 \Gamma'(\xi)\right] J(\xi) \left[W_1 \Gamma'(\xi)\right]^{-1}\right\} W_1 \gamma(\xi, \widetilde{\alpha_0}, \widetilde{\Sigma}) + \\ &[W_1 \Gamma'(\xi)] J(\xi) [W_1 \Gamma'(\xi)]^{-1} Y_{1,t} + [W_1 \Gamma'(\xi)] \tilde{\Sigma} \varepsilon_{t+1}^Q \end{aligned} \tag{A.10a}$$

$$\ln S_t = \widetilde{\alpha_0} - \mathbf{1}' \left[W_1 \Gamma'(\xi)\right]^{-1} W_1 \gamma(\xi, \widetilde{\alpha_0}, \widetilde{\Sigma}) + \mathbf{1}' \left[W_1 \Gamma'(\xi)\right]^{-1} Y_{1,t} \tag{A.10b}$$

Therefore, by coefficient matching:

$$\rho^Q = \left[W_1 \Gamma'(\xi)\right] J(\xi) \left[W_1 \Gamma'(\xi)\right]^{-1} \tag{A.11a}$$

$$\beta_0' = \mathbf{1}' \left[W_1 \Gamma'(\xi)\right]^{-1} \tag{A.11b}$$

Leading to expressions for the reparameterization functions $f_1$ and $f_2$ in Eqs. (3.14a) and (3.14b). Considering that $\widetilde{\rho^Q}$ is the Jordan form of $\rho^Q$, the parameters in vector $\xi$ are indeed the eigenvalues of $\rho^Q$. Under the new parametrization, knowing these eigenvalues is sufficient to determine all the $\beta_n$ coefficients. In this aspect, there is no distinction between the approach for commodity futures being currently presented and the original approach introduced by Joslin et al. (2011) for bonds. However, some subtle distinctions appear when obtaining the reparameterization of Eq. (3.14c).

Defining the matrices $T(\xi)$ and $U(\xi, \alpha_0, \Sigma)$ such that

$$A(\xi, \alpha_0, \Sigma) = \begin{bmatrix} \alpha_{d1} \\ \vdots \\ \alpha_{dM} \end{bmatrix} = U(\xi, \alpha_0, \Sigma) + T(\xi) c^Q \tag{A.12}$$

After some calculations based on Eqs. (3.7):

$$U(\xi, \alpha_0, \Sigma) = \begin{bmatrix} \alpha_0 + \frac{1}{2} \sum_{j=0}^{d1-1} \beta_j' \Sigma \Sigma' \beta_j \\ \vdots \\ \alpha_0 + \frac{1}{2} \sum_{j=0}^{dM-1} \beta_j' \Sigma \Sigma' \beta_j \end{bmatrix} \tag{A.13a}$$

$$T(\xi) = \begin{bmatrix} \sum_{j=0}^{d1-1} \beta_j' \\ \vdots \\ \sum_{j=0}^{dM-1} \beta_j' \end{bmatrix} \tag{A.13b}$$

An expression for the reparameterization function $f_3$ in Eq. (3.14c) is found by substituting Eq. (A.12) into Eq. (3.12c) and rearranging:

$$c^Q = -\left(W_1 R(\zeta)\right)^{-1} W_1 U(\zeta, \alpha_0, \Sigma) \tag{A.14}$$

## APPENDIX III

Under the alternative hypothesis, the variations of the parameter vector are governed by a linear function of some underlying process $s_t$:

$$\theta_t = \theta_0 + m\omega s_t \tag{A.15}$$

where $\theta_0$ is the parameter vector under the null hypothesis, $m$ is a scalar denoting the magnitude of the parameter change and $\omega$ is a unit vector representing the direction of change within the parameter space. The only structure imposed to the underlying process is its autocorrelation:

$$\mathrm{corr}(s_{t+r}, s_t) = \rho^r \tag{A.16}$$

Markov-switching models are a special case of the formulation described by Eqs. (A.15) and (A.16). For instance, Eq. (A.16) accommodates a two-state ergodic Markov chain by setting $\rho = \pi_{11} + \pi_{22} - 1$, where $\pi_{11}$ and $\pi_{22}$ are the same-state transition probabilities. Furthermore, whenever this Markov chain transitions to a different state, Eq. (A.15) dictates that $\theta_t$ must switch to a different set of values, as expected in a Markov-switching model.

Letting $\hat{\theta}$ be the MLE of $\theta_0$, a process $\gamma_t$ is defined such that

$$\gamma_t(\rho, \omega; \hat{\theta}) = \omega' \left[ \left( \frac{\partial^2 l_t}{\partial\theta\partial\theta'} \right) + \left( \frac{\partial l_t}{\partial\theta} \right) \left( \frac{\partial l_t}{\partial\theta} \right)' + 2 \sum_{r=0}^{t} \rho^{t-r} \left( \frac{\partial l_t}{\partial\theta} \right) \left( \frac{\partial l_r}{\partial\theta} \right)' \right] \omega \tag{A.17}$$

where $l_t$ is the conditional log-likelihood of observation $t$ given the past observations. The first and second derivatives of $l_t$ are taken with respect to the entire parameter vector, being thus a gradient vector and a Hessian matrix, respectively.

Also necessary for the calculating the test statistic, the process $e_t$ is defined as the residual of the regression

$$\gamma_t(\rho, \omega; \hat{\theta}) = \delta_0 + \delta_1 \frac{\partial l_t}{\partial\theta}(\hat{\theta}) + e_t \tag{A.18}$$

The test statistic is written in the form of a supremum:

$$TS = \sup \left\{ \max \left[ 0, \left( \frac{\sum_{t=1}^{T} \gamma_t(\rho, \omega; \hat{\theta})}{2\sqrt{e_t' e_t}} \right)^2 \right] \right\} \tag{A.19}$$

Noticing that the supremum is taken with respect to hyperparameters $\rho$ and $\omega$ governing the dynamics of $\theta_t$. The hyperparameter $m$, on the other hand, is irrelevant to the test statistic. The supremum is numerically approximated by the maximization of the expression between braces.

Taking into account the specific form of the alternative hypothesis in the tests conducted for this dissertation, some restrictions must be imposed to $\rho$ and $\omega$. First,

$\rho$ should be within a range compatible with a persistent Markov chain:

$$\rho(\vartheta) = \underline{\rho} + \overline{\rho} \frac{|\vartheta|}{\sqrt{1 + \vartheta^2}} \tag{A.20}$$

A test specification with $\underline{\rho} = 0.2$ and $\overline{\rho} = 0.9$ seems suitable for this purpose. Moreover, the form of $\omega$ should reflect the fact that only the conditional mean $c_n$ and variance $\sigma_n^2$ in Eq. (4.1) are being tested for variability. Without loss of generality, $c_n$ and $\sigma_n^2$ are assumed to be the two first parameters in vector $\theta_0$, hence

$$\omega_i = \begin{cases} \cos \kappa & i = 1 \\ \sin \kappa & i = 2 \\ 0 & otherwise \end{cases} \tag{A.21}$$

Given the reparameterizations of Eqs. (A.20) and (A.21), the test statistic is calculated by maximizing the right-hand side of Eq. (A.19) over the parameters $\vartheta = (-\infty, \infty)$ and $\kappa = (0, 2\pi]$.

# APPENDIX IV

For single-regime affine term structure models, Appendix I shows that the imposition of no arbitrage leads to recursive expressions for the pricing coefficients. Provided that a regime-dependent pricing kernel exists, the approach used in the single-regime case can be generalized to multiple regimes.

As with the single-regime approach, it is assumed that no transaction is made for acquiring a commodity future. Hence, if the term structure of futures is said to be arbitrage-free, the one-period future return conditional on the current regime $s_t = j$ must satisfy

$$0 = \sum_{k=1}^{K} \pi_{jk} E_t\left[M_{t,t+1}^{jk}\left(F_{t+1}^{k,n-1} - F_t^{j,n}\right) | \, s_t = j; s_{t+1} = k\right] \qquad \text{(A.22)}$$

Substituting the pricing kernel of Eq. (4.16) into Eq. (A.22):

$$0 = \sum_{k=1}^{K} \pi_{jk} \, E_t \left[\exp\left(-r_t - \Gamma^{jk} - \frac{1}{2}\lambda_t^{k\prime}\lambda_t^k - \lambda_t^{k\prime}\varepsilon_{t+1}\right)\left(F_{t+1}^{k,n-1} - F_t^{j,n}\right) | \, s_t = j; s_{t+1} = k\right] \qquad \text{(A.23)}$$

Rearranging the terms and using Eq. (4.15) to replace the $\mathbb{P}$-measure transition probabilities with their risk-neutral counterparts:

$$F_t^{j,n} = \frac{\sum_{k=1}^{K} \pi_k^Q \, E_t\left[\exp\left(-\frac{1}{2}\lambda_t^{k\prime}\lambda_t^k - \lambda_t^{k\prime}\varepsilon_{t+1}\right) F_{t+1}^{k,n-1} | \, s_{t+1} = k\right]}{\sum_{k=1}^{K} \pi_k^Q \, E_t\left[\exp\left(-\frac{1}{2}\lambda_t^{k\prime}\lambda_t^k - \lambda_t^{k\prime}\varepsilon_{t+1}\right) | \, s_{t+1} = k\right]} \qquad \text{(A.24)}$$

It can be observed that the future price $F_t^{j,n}$ is being equated to a completely regime-independent expression, implying that its dependence on the current regime $j$ should be dropped. By induction, the same reasoning applies to the dependence of $F_{t+1}^{k,n-1}$ on $k$. Moreover, the denominator of the right-hand side is a summation that adds up to one, leading to further simplification:

$$F_t^n = \sum_{k=1}^{K} \pi_k^Q \, E_t\left[\exp\left(-\frac{1}{2}\lambda_t^{k\prime}\lambda_t^k - \lambda_t^{k\prime}\varepsilon_{t+1}\right) F_{t+1}^{n-1} | \, s_{t+1} = k\right] \qquad \text{(A.25)}$$

The next steps consist of plugging the exponential-affine form of Eq. (2.5) into the equation and solving the expectation term. Details are skipped for being analogous to the single-regime case:

$$\exp(\alpha_n + \beta_n' X_t) = \sum_{k=1}^{K} \pi_k^Q \exp\Big(\alpha_{n-1} + \beta_{n-1}' c^k + \beta_{n-1}' \rho X_t + \frac{1}{2}\beta_{n-1}' \Sigma^k \Sigma^{k\prime} \beta_{n-1} + \beta_{n-1}' \Sigma^k \lambda_t^k\Big) \tag{A.26}$$

Using the definition of regime-dependent price of risk from Eq. (4.14) and simplifying:

$$\exp(\alpha_n + \beta_n' X_t) = \exp(\alpha_{n-1} + \beta_{n-1}' c^Q + \beta_{n-1}' \rho^Q X_t) \sum_{k=1}^{K} \pi_k^Q \exp\Big(\frac{1}{2}\beta_{n-1}' \Sigma^k \Sigma^{k\prime} \beta_{n-1}\Big) \tag{A.27}$$

Deriving Eqs. (4.17) from Eq. (A.27) involves coefficient matching in the same way as the single-regime case.