Pontifícia Universidade Católica do Rio de Janeiro

Pedro Henrique da Silva Castro

Essays on macroeconomics and monetary policy

Tese de Doutorado

Thesis presented to the Programa de Pós–graduação em Economia of PUC-Rio in partial fulfillment of the requirements for the degree of Doutor em Economia.

Advisor : Prof. Márcio Gomes Pinto Garcia
Co-advisor: Prof. Tiago Couto Berriel

Rio de Janeiro
Abril 2018

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Bibliographic data

da Silva Castro, Pedro Henrique

Essays on macroeconomics and monetary policy / Pedro Henrique da Silva Castro; advisor: Márcio Gomes Pinto Garcia; co-advisor: Tiago Couto Berriel. – Rio de janeiro: PUC-Rio, Departamento de Economia, 2018.

v., 169 f: il. color. ; 30 cm

Tese (doutorado) - Pontifícia Universidade Católica do Rio de Janeiro, Departamento de Economia.

Inclui bibliografia

1. Economia – Teses. 2. Macroeconomia;. 3. Política monetária;. 4. Crédito direcionado;. 5. Canal de custo;. 6. Investimento;. 7. Fluxos de capitais;. 8. Ciclo de negócios;. 9. Portfolio-balance;. 10. Intervenções esterilizadas.. I. Gomes Pinto Garcia, Márcio. II. Couto Berriel, Tiago. III. Pontifícia Universidade Católica do Rio de Janeiro. Departamento de Economia. IV. Título.

CDD: 330

Acknowledgments

I would like to start by expressing my deep gratitude for my beloved wife Dany. *Amore*, you have sacrificed a lot in order to stay by my side. You have given me strength when I most needed it even though you had so much to deal with yourself. For that and for everything else, thank you.

I also wish to thank my advisors, Márcio and Tiago. Márcio, you have accepted me as your student in a critical moment in the development of this thesis and has ever since been a very present mentor. Your support gave me stability to keep on track. Tiago, when I was taking my first steps as a reasearcher we have explored so many paths, and had so many insightful discussions. You have then found time to guide me despite all your obligations as Deputy Governor at the Central Bank.

I can hardly imagine a better examination comitte than the one I had, and for that I am also very grateful. Bonomo and Bernardo, your suggestions were invaluable and your thoughts gave me a lot of perspective regarding the value of my own work. Dudu, thank you for caring for the macro students (such as myself) who needed a free second though on their work, specially when this good was in such low supply. Finally, to you Carlos my special gratitude. You had no formal responsability for my work and certanly had many other things to be occupied with, as Deputy Governor. Still, you helped me a lot.

Last but not least, thanks to my all family, whom I could always count on. Specially you, *Mãe* and *Pai*: you have always given your best for me, and now is my time to retribute. I hope to make you proud not only with this achievement, but in all my endevours.

Abstract

da Silva Castro, Pedro Henrique; Gomes Pinto Garcia, Márcio (Advisor); Couto Berriel, Tiago (Co-Advisor). **Essays on macroeconomics and monetary policy**. Rio de Janeiro, 2018. 169p. Tese de doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

This thesis is comprised of three essays. The first two investigate the relationship between monetary policy power and the prevalence of earmarked credit (featuring interest rates that are insensitive to the monetary cycle) in the economy. The first shows that the available microeconometric evidence is not necessarily informative about the macroeconomic phenomenon of interest, and illustrates this result with a simples New-Keynesian model with working capital credit. Giving sequence, the second essay extends the analysis with a medium-sized DSGE model where earmarked credit is used to finance the acquisition of physical capital by firms. The model is estimated to Brazil using Bayesian techniques. Under the prior distribution it is shown that the presence of earmarked credit does not necessarily reduces monetary policy power over inflation. Under the posterior it is shown that a reduction of power is likely, but small. Finally, the third essay studies to what extent the effects of capital flows on a small open economy's business cycle depend on the type of the inflow (e.g., whether a bond or a stock inflow, a liability or an asset flow), and for such it build an open economy New-Keynesian model with financial frictions. Direct mechanisms through which inflows may have differentiated effects depending or their type are identified. Using a calibrated version of the model it concludes that the differences are probably of little significance.

Keywords

Macroeconomics; Monetary policy; Earmarked credit; Cost channel; Investment; Capital flows; Business cycle; Portfolio-balance; Sterilized interventions.

Resumo

da Silva Castro, Pedro Henrique; Gomes Pinto Garcia, Márcio; Couto Berriel, Tiago. **Ensaios em macroeconomia e política monetária**. Rio de Janeiro, 2018. 169p. Tese de Doutorado – Departamento de Economia, Pontifícia Universidade Católica do Rio de Janeiro.

Esta tese é composta de três ensaios. Os dois primeiros investigam a relação entre a potência da política monetária e a prevalência do crédito direcionado (concedido à taxas de juros insensíveis ao ciclo monetário) na economia. O primeiro mostra que a evidência microeconométrica disponível não é necessariamente informativa sobre o fenômeno macroeconômico de interessee ilustra esse resultado com um modelo Novo-Keynesiano simples com financiamento de capital de giro. Dando sequência, o segundo ensaio estende a análise usando um modelo DSGE de médio porte no qual crédito direcionado é utilizado pelas firmas para financiar a aquisição de capital. O modelo é estimado para o Brasil usando técnicas Bayesianas. Sob a distribuição *priori* mostra-se que a presença de crédito direcionado não reduz necessariamente a potência da política monetária sobre a inflação. Sob a distribuição *posteriori* mostra-se que a redução de potência é provável, mas pequena. Finalmente, o terceiro ensaio estuda em que medida o efeito de fluxos de capitais sobre o ciclo de negócios depende do tipo do influxo (e.g., se para títulos ou para ações, se um fluxo de ativo ou de passivo), construindo para tanto um modelo Novo-Keynesiano de economia aberta com fricções financeiras. Identifica-se mecanismos diretos através dos quais o influxo pode ter efeito diferenciado dependendo do seu tipo. Conclui-se, usando uma versão calibrada do modelo, que as diferenças são provavelmente pouco significativas.

Palavras-chave

Macroeconomia; Política monetária; Crédito direcionado; Canal de custo; Investimento; Fluxos de capitais; Ciclo de negócios; Portfolio-balance; Intervenções esterilizadas.

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1
Earmarked credit and monetary policy power: micro and macro considerations

Is monetary policy power reduced in the presence of earmarked credit with subsidized interest rates, insensitive to the monetary cycle? I argue this question has not yet been reasonably answered even though a virtual consensus seems to have been reached. I show that the available microeconometric evidence is not necessarily informative about the macroeconomic effect of interest, due to the presence of general equilibrium effects. Also, power may be increased over one variable and reduced over another. To provide an example of these possibilities, I build a simple New Keynesian model where firms take credit (from both the market and the government) to finance working capital needs. Due to a cost-channel, the presence of earmarked credit reduces the power of monetary policy shocks over output, but increases it over inflation.

Keywords: Monetary economics, Earmarked credit, Cost channel

JEL Classification: E51, E52, H81

1.1
Introduction

Government is responsible for a large share of the credit supply in Brazil, by owning banks and by earmarking credit, channeling it to desired sectors and modalities. In December 2017 credit provided by government controlled banks amounted to 54.1% of total outstanding bank loans; earmarked loans corresponded to 48.7%.[1]. A significant share of these earmarked loans have interest rates that are lower than the prevailing market rate, and insensitive to the monetary policy rate (Selic). Both these features can be seen in Figure 1.1, which compares the trajectories of the Selic and TJLP[2] rates from 2000

[1] Note that there is significant overlap between earmarked loans and loans provided by government controlled banks. For example, BNDES is a government controlled bank and most of its loans are earmarked. Nonetheless, the concepts are different. Banco do Brasil is counted as a government controlled bank, but many of its loans are in the 'free' (i.e., not earmarked) segment. Bradesco is private controlled, but some of its loans are earmarked.

[2] The benchmark rate for BNDES credit operations, from December 1994 to December 2017. We focus on the TJLP because it certainly has drawn most of the attention. BNDES

to 2017.

Figure 1.1: Selic × TJLP (% p.a.)

It has been argued[3] that this pervasiveness of earmarked credit reduces the efficacy of monetary policy. This would occur because earmarked credit does not tighten in response to monetary policy tightening, or at least not as much as market credit does, and agents who can access it would not have to adjust their spending and investment as much as they would if they faced market interest rates. Such obstruction of monetary policy's transmission channel would make harder the job of the Central Bank in stabilizing the economy and might imply more volatile interest rates, as the Central Bank would have to increase its policy rate by more to achieve a given contraction in demand, if needed. In fact, such concern was one of the motivations for a recent policy change, as made clear by MP 777's exposition of motives[4]. This *Medida Provisória*, later converted into Law 13,438/2017, created a new benchmark rate for BNDES operations, the TLP, in substitution to the TJLP. Unlike its predecessor, the TLP is linked to the yield on 5-year inflation-indexed government bonds and, hence, affected by changes in policy rate. The effective

credit operations, for both households and non-financial companies, amounts to 36.5% of the total stock of earmarked credit in Brazil (as of October 2017). BNDES credit to companies amount to 69.2% of earmarked credit to companies. Other modalities of earmarked credit are real-estate (41.1%), rural (15.8%) and others (0.6%), which includes micro-credit.

[3] For instance, (1), (2), (3), (4), (5), among many others.

[4] Which can be found here (in Portuguese): `http://www.planalto.gov.br/ccivil_03/_ato2015-2018/2017/Exm/Exm-MP-777-17.pdf`

TLP is phased in over 5 years, linearly increasing from the TJLP to the new benchmark.

A broad agreement was reached, thus, despite the fact that few are the academic works dedicated to study the relationship between earmarked credit and the monetary policy power. On the empirical front the exception is (6), who use firm-level employment and credit micro-data to assess how monetary policy transmission is affected by government-driven (both earmarked and by government controlled banks) loans, exploring variation in earmarked credit access across firms. They find that access to government-driven credit does help insulate firms from the effects of interest rate changes: for instance, after a 1.p.p. hike in the policy rate, employment growth falls 1.2 p.p. in firms without access to earmarked credit, but only 0.7 p.p. in firms totally financed by the government.

But how informative is this result about the question of interest, namely, the extent to which earmarked credit reduces monetary policy traction on the aggregate economy? Can we extrapolate the results from the cross-section domain (micro effects) to the aggregate domain (macro effects), thus corroborating the hypothesis that interest-insensitive earmarked credit renders monetary policy less effective?

Of course, the external validity of a result is not necessarily warranted and one must be cautious with extrapolations. I show there is a good reason why caution should be applied here as well. A firm's output response to monetary policy does not depend only on its own access to earmarked credit but also on all other firms'. Because of that, the macroeconomic effect depends not only on how firms' response to monetary shocks is affected by how much earmarked credit they receive (microeconomic effect), but also by how it is affected by other firms receiving it (external effect). In fact, I show that we can decompose the macro effect into the sum of average micro and average external effects.

This general result is then explored in the context of a very simple New-Keynesian model which includes a *working-capital channel*, through which monetary policy shocks affect firms differently, depending on their reliance on earmarked credit. The model is able to reproduce the microeconometric evidence that employment is less responsive to monetary shocks in firms with more access to earmarked credit. In the model aggregate output is also less responsive to monetary shocks the more important government is in supplying credit. But the magnitude of micro and macro effects differ, the later usually being considerably larger than the former.

Another interesting result, in the model, is that inflation becomes more responsive the higher is the importance of earmarked credit — contrarily to the popular view. This happens both in the micro and macro level, but again with different magnitudes. The reason is the presence of a cost-channel induced by firms' working capital needs: an interest rate hike increase firms marginal costs, offsetting in part the deflationary pressure that comes from aggregate demand. But this cost-channel is weaker the more insulated firms are from variation in the market interest rate.

Related literature. As emphasized, the academic literature on the relationship between earmarked credit and monetary policy power is sparse. In (7), BNDES lending is countercyclical and reduce the response of the economy to monetary shock. But in his model the credit policy follows (8)'s model of unconventional monetary policy meaning that interest rate on government credit is no different from the one in the private market, which is at odds with the data and with our motivation. (9) builds a DSGE model where earmarked credit finances firms' working capital needs. In his model government credit is entirely financed with distortionary taxation on households' labor income. A balanced-budget is assumed and this, together with a fixed tax-rate, implies that earmarked credit interest rates must endogenously respond to monetary policy, which is at odds with the observations of insensitiveness[5]. (10) is the closest to this paper. They extend the model of (11) by assuming that a share of the monopolistically competitive banks is government-owned and provide cheaper credit at a constant interest rate. Firms take credit in order to finance their working capital needs, opening space to a cost-channel. They find that both output and inflation responses to a monetary shock become more muted when the presence of earmarked credit is higher. But, importantly, because they find a significant *price-puzzle*[6], what happens is that inflation *rises less* following a monetary policy tightening. In a sense, this is similar to my result that inflation *falls more*.

This paper also relates to the cost-channel literature — see (12), (13), (14). This literature posits that interest rates changes not only work through demand channels (such as households' consumption-savings decisions) but also through supply channels, as higher interest rates may increase firms' operational costs. This, in turn, could be a possible explanation for the *price-puzzle*. The cost channel arises in our model because credit to firms is introduced through working capital needs — as in other DSGE models with this feature —, but the cost-channel is more general than firms relying on working capital credit. It arises whenever there is a delay between paying

[5]Unfortunately, the paper does not show how the interest rate on earmarked credit respond to a monetary shock.

[6] *Price-puzzle* is the name given to the phenomenon of higher inflation following a monetary contraction.

production costs and receiving for sales.

Finally, this papers is also close to literatures showing that macro and micro elasticities can be very different. Classical papers are (15) — showing that the aggregation of fixed inputs-technology firms (hence, with zero elasticity of substitution) can give rise to an aggregate Cobb-Douglas production function (hence, with unitary elasticity), due to extensive margins — and (16) — providing an example of asymmetric hiring and firing on the firm level that do not occur in the aggregate. For a sample of recent papers who take seriously these difference between micro and macro elasticities, see (17), (18), (19) and (20).

Guideline. Section 2 provides a general analysis (i.e., not model-specific) of the relationship between the micro and macro elasticities of IRFs with respect to earmarked credit. Section 3 provides specific analysis, based on a New Keynesian model with a cost-channel. Section 4 concludes.

1.2 A general analysis

1.2.1 Distinguishing between macro and micro effects

When discussing whether, and to what extent, the presence of earmarked credit makes monetary policy less effective our interest mostly lies in the response of *aggregate* output and inflation to monetary shocks, and how these responses change with the importance of earmarked credit in the economy. In this work I use the expression *macroeconomic effect* to describe this sort of consequences, distinguishing it from *microeconomic effects* that take place at firm level. In order to be precise I provide formal definitions of these objects:

Definition: The **macroeconomic effect** that earmarked credit has over variable Z's response to a monetary shock is given by:

$$\frac{\partial}{\partial \zeta}\left(\frac{\partial Z}{\partial R}\right)$$

where R is the policy rate and ζ is measure of the overall importance that earmarked credit has in the economy. Both changes in policy (earmarked credit and monetary) must be exogenous in order not to be confounded with other factors.

Definition: The **microeconomic effect** that government credit has over firm i variable Z's response is given by:

$$\frac{\partial}{\partial \zeta_i}\left(\frac{\partial Z_i}{\partial R}\right)$$

Note that the effect is measured by exogenously changing firm i's access to earmarked credit (ζ_i) while holding fixed all other firms' access to government credit. If firm i's size is negligible economy-wise, as we assume, then the overall importance of government credit (ζ) is also fixed.

Before I present the main result of this paper it will prove useful to introduce one more definition. This is motivated by the fact that firm i's behavior is not only affected by its own access to government credit, but also by all other firms access. For instance, its business is likely to be harmed if its competitors are able to find cheaper credit.

Definition: The **external effect** that government credit has over firm i variable Z's response is given by:

$$\frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial Z_i}{\partial R}\right)$$

where ζ_{-i} is a measure of the overall importance that government credit has to all other (than i) firms in the economy.

With these definitions in place we are ready to proceed to one of the main results of this paper. In order to focus on the essence of the argument, in the main text I only provide a proof of the proposition for a case with a finite number of firms, leaving the extension for infinitely countable and uncountable number of firms for the appendices A.1 and A.2. For concreteness I focus on output, but the analysis is similar for other variables.

Let N be the number of firms in the economy and denote by Y_i firm i's output. The equilibrium value for this variable potentially depends on many factors and, among them, the stance of monetary policy (R) and the government credit access of each firm in the economy ($\zeta_1, \zeta_2, \ldots, \zeta_N$). Because of that we write $Y_i = Y_i\big(\ R\ ;\ \zeta_1, \zeta_2, \ldots, \zeta_N\ ;\ \cdot\ \big)$. Let us define aggregate output as an average of firm's output, i.e., $Y = \frac{1}{N}\sum_{i=1}^{N} Y_i$. We use the average and not the sum for convenience. First, note that this can be considered just a choice of scale. Second, this is more consistent with the definition of aggregate output in a model with a unit measure continuum of firms, as is typical in DSGE models. In the same spirit, let us also define the aggregate importance of earmarked credit in the economy as the cross-section average of firms' access: $\zeta = \frac{1}{N}\sum_{i=1}^{N}\zeta_i$. Accordingly, $\zeta_{-i} = \frac{1}{N}\sum_{j\neq i1}\zeta_j$.

Proposition: The macroeconomic effect of interest is given by the sum of microeconomic and external effects averaged over the set of firms. I.e.,

$$\underbrace{\frac{\partial}{\partial\zeta}\left(\frac{\partial Y}{\partial R}\right)}_{\text{Macro effect}} = \underbrace{\mathbb{E}_i\left[\frac{\partial}{\partial\zeta_i}\left(\frac{\partial Y_i}{\partial R}\right)\right]}_{\text{Average of micro effects}} + \underbrace{\mathbb{E}_i\left[\frac{\partial}{\partial\zeta_{-i}}\left(\frac{\partial Y_i}{\partial R}\right)\right]}_{\text{Average of external effects}}$$

Proof: Total differentiation of aggregate output in respect to government credit variables $\{\zeta_j\}$ yields:

$$dY = \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j=1}^{N}\frac{\partial Y_i}{\partial\zeta_j}d\zeta_j\right)$$

Now, let us consider changes in government access such that $d\zeta_i = d\zeta$, for all i. Hence,

$$\begin{aligned}\frac{\partial Y}{\partial\zeta} &= \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{\partial Y_i}{\partial\zeta_j} \\ &= \frac{1}{N}\sum_{i=1}^{N}\frac{\partial Y_i}{\partial\zeta_i} \quad + \quad \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j\neq i}\frac{\partial Y_i}{\partial\zeta_j}\right)\end{aligned}$$

Note that the term inside parenthesis in the last expression captures the how firm i's output is affected by changes in all other firms ($j \neq j$) access to government credit. We can write $\frac{\partial Y_i}{\partial\zeta_{-i}} = \sum_{j\neq i}\frac{\partial Y_i}{\partial\zeta_j}$. Derive the resulting expression in respect to R to complete the proof.

As the derivation makes clear the result above is pretty much an identity. It just relies on the fact that a firm's output potentially depends not only on its own access to earmarked credit, but also on all other firms access. How to define the aggregate variable may have practical implications [7], but it does not change the essence of the argument. We have framed the proposition for our objects of interest (which are second-order mixed partial derivatives with respect to monetary and earmarked credit policies), but it is clear that a similar result is valid for many objects[8]. Thus, the result is very general and does not

[7] Defining the aggregate as a sum, instead of an average, would make the macro effect to be the sum of total micro and external effects. Also, if the aggregate is a weighted average then the weights would be carried over to the decomposition.

[8] For instance, we could be interested in whether the presence of earmarked credit increases steady-state aggregate output. And it can also be useful to study things unrelated to government credit policy as well, for instance, elasticicties of substitutions as in (15).

hinge on strong hypothesis. In particular, it is model-independent.

The generality of the result is a strength and, at the same time, a weakness. It carries no information about the sign and magnitude of each of the defined effects (macro, micro and external). In other words, the result tells us nothing about whether and to what extent the presence of government credit reduces monetary policy power. Nonetheless, it is still useful because it helps us to better understand the available microeconometric evidence, making clear how misleading extrapolating it can be.

1.2.2 External effect and general equilibrium

What is the nature of the "external effect"? Pragmatically, it depends only on the hypothesis that a firm's output depend not only on its own access to government credit but also on other firms access. Why would it be the case?

One does need to rely on the existence of real (or technological) externalities in order to justify this assumption. In fact, what we have in mind is the existence of pecuniary externalities associated with general equilibrium forces. Consider the case with atomistic firms. The microeconomic effect captures a partial equilibrium effect in the sense that prices (including factor prices) and hence, the allocation of all other agents — are unchanged when a single atomistic firm is given more cheap credit. One would expect this firm to be able to hire more workers, capital, etc, and to produce more. When all firms in the economy are granted cheaper credit, however, prices are expected to change. For instance, if all firms want to hire more workers in response to the increased availability of credit then wages should rise, and this in turn should mitigate the initial partial equilibrium effect (on the marginal cost). This general equilibrium force can be isolated by giving all other firms more credit, and then examining the unfavored atomistic.

1.2.3 A naive extrapolation

Consider this reduced form equation estimated by (6):

$$\Delta Y_{it} = \eta G_{i,t-1} + \pi \Delta R_t + \beta \left(G_{i,t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_{it} + a_i + \varepsilon_{it}$$

where Y_{it} is an output[9] in firm i in year t, $G_{i,t-1}$ is firm's government-credit access in the previous year, R_t is the policy interest rate and $\boldsymbol{X}_{it}$ is a vector of controls. The microeconomic effect is here captured by the parameter β.

[9] They use employment, but this is the same for our purpose.

Because this equation is assumed to be valid for all firms in the cross section, a naive analyst could be tempted to aggregate it in order to obtain an estimate of the macro effect. For instance, defining $Y = \int Y_i \mathrm{d}i$ as the aggregate output, and doing the same for government credit, controls and error terms, a simple integration of the equation (across firms) yields

$$\Delta Y_t = \eta G_{t-1} + \pi \Delta R_t + \beta \left(G_{t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_t + a + \varepsilon_t$$

and one would conclude that the macro effect would also be given by β. But from our decomposition we know that this is generally not the case. What is wrong with this procedure is that the estimated cross-section equation omits the external effect. For instance, suppose that we also include terms associated with the overall importance of government credit:

$$\begin{aligned} \Delta Y_{it} = \eta G_{i,t-1} + \pi \Delta R_t + \beta \left(G_{i,t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_{it} + a_i + \varepsilon_{it} \\ + \tilde{\eta} G_{t-1} + \tilde{\beta} \left(G_{t-1} \cdot \Delta R_t \right) \end{aligned}$$

Now the aggregation yields:

$$\Delta Y_t = \left(\eta + \tilde{\eta} \right) G_{t-1} + \pi \Delta R_t + \left(\beta + \tilde{\beta} \right) \left(G_{t-1} \cdot \Delta R_t \right) + \boldsymbol{\gamma}' \boldsymbol{X}_t + a + \varepsilon_t$$

and it becomes clear that the macro effect is now given by $\beta + \tilde{\beta}$, which is the sum of the micro effect and the external effect, as defined.

In principle this approach may be tried in order to disentangle micro and macro effects. But a problem that arises is that a good estimate of $\tilde{\beta}$ is much harder to obtain than a good estimate of β, since the identification of $\tilde{\beta}$ relies only on the time series dimension of the data, taking no advantage of the cross-section dimension. All aggregate time-varying effects that are correlated with $G_{t-1} \cdot \Delta R_t$ must be accounted for and, at the same time, one can not use time-effect dummies.

1.3 A model as example

I have argued that the available microeconometric evidence is not necessarily informative about the macroeconomic effect we are interested in. But in principle the external effect could be zero or very small, implying that micro and macro effects are quantitatively similar. As I have emphasized, one weakness of general decomposition is that it is silent about the sign and magnitudes of each effect.

In this section I examine the sign and magnitude of each of the effects in the context of a very simple model. The model is a textbook-like[10] New Keynesian model which includes working-capital needs by firms and, hence, a cost-channel, as in (13), (14) and (22). A share of these loans is provided by the government, with interest rates that are subsidized and constant (hence, insensitive to the monetary policy rate). I allow for firm-level heterogeneity in access to government-driven credit in order to capture the microeconomic effect as well as the macro one.

Why would I work with a model of earmarked credit that emphasizes working capital credit instead of investment credit, as would be expected given the recent discussion on the TJLP rate, the importance of BNDES in the total supply of earmarked credit (around 40%) and its focus on financing investments (around 95%)? First, by ignoring capital accumulation I can work with an analytically solvable model, giving formulas for the micro, macro and external effects. This is fine since one of this paper's main objectives is to give an example of the decomposition and of the fact that one cannot rely on microeconomic estimates to draw conclusion on the macroeconomic effect of interest. Second, related theoretical works on this subject — (7), (10) — also embed earmarked credit in a model of working capital needs. But they do not explore the differences in macro and micro effects; and they do not thoughtfully examine the mechanism driving their results. Hence, in some sense this paper completes a previous literature. Third, we do see earmarked credit financing working capital needs. Although working capital credit to firms corresponds only to 2.5% of BNDES outstanding loans, working capital is very common (74% of the value lent[11]) in the rural credit, which amounts to 15% of earmarked outstanding loans. The model should thus be useful when discussing such modality. Fourth, the cost-channel surpasses the existence and extent of working capital credit. All that is needed for it to be operative is for payments for input and factor use to occur before the production revenues. This time lag between payments and incomes introduces the opportunity cost of money in the marginal production cost, and is passed to prices. That is why (12) measure the importance of working capital and the value of inventories plus trade receivables (net of trade payables). Finally, there is nothing specific to investment in the claim that monetary policy becomes less effective when earmarked credit is present. The same obstruction-based argument could be applied every time a decision depends on the interest rate.

[10] Based on (21)'s chapter 3.

[11] Source: BCB — *Matriz de Dados do Crédito Rural*, for the year 2016. We consider as working capital the contracts financing current expenditures (*custeio*) and commercialization. The other major modality is investment.

1.3.1 Model description

Because the model is very standard and in order to conserve space, in what follows I present the model without fully deriving it. Anyway, the full set of equilibrium conditions that characterizes the model is presented in appendix A.5, and the log-linearized version of the model is presented in appendix A.6.

Households. The representative household chooses consumption (C_t), labor supply (H_t) and security holdings, both real (D_t) and nominal (D_t^n), so as to maximize his expected lifetime utility

$$\max_{\{C,H,D,D^n\}} \qquad \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{C_{t+s}^{1-\sigma}}{1-\sigma} - \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\}$$

subject to a set of flow budget constraints

$$C_t + D_t + \frac{D_t^n}{P_t} = W_t H_t + R_t D_{t-1} + R_t^n \frac{D_t^n}{P_t} + T_t$$

where T_t captures government net transfers and dividends from the ownership of firms.

Final Good Assemblers. The final good assembler operates in a perfectly competitive environment, producing the final consumption good from a continuum of varied retail goods, indexed by i. Its production technology is given by $Y_t = \left(\int_0^1 Y_{it}^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}}$. Conditional demand for each variety can be found by cost-minimization, and is given by $Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t$. Free entry in this market drives profit down to zero in each period and imply that the aggregate price level is given by: $P_t = \left(\int_0^1 P_{it}^{1-\varepsilon} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}}$.

Firms. There is a unit-mass continuum of monopolistically competitive firms indexed by i. Each produces a differentiated good, but the technology used is the same, represented by the production function $Y_{it} = H_{it}^{1-\alpha}$.

As in (22) we introduce a 'working capital channel' by requiring that a fraction ψ of each firm's wage bill is to be externally financed. It should be acknowledged, however, that this is a simple modeling device used in the literature and that the existence of a cost-channel is much more general and can be derived from other micro-foundations. Let R_{it}^w be the gross real interest rate on working capital loans that firm i faces. Its real total cost is given by

$\text{Cost}_{it} = W_t H_{it}\Bigg(1+\psi\left(R^w_{it}-1\right)\Bigg)$. The real marginal cost of production is given by $\text{MC}_{it} = \left(\frac{1}{1-\alpha}\right)\Bigg(1+\psi\left(R^w_{it}-1\right)\Bigg)W_t\frac{H_{it}}{Y_{it}}$.

A fraction ζ_i of firm i's financing needs is supplied the government at the constant real rate R^s (s for subsidized). The other fraction must be financed at the market rate R_t. The average (and also the marginal) real interest rate firm i faces is then given by $R^w_{it} = R_t + \zeta_i\Big(R^s - R_t\Big)$.[12]

Firms are subject to Calvo nominal price rigidities, and with probability θ they are unable to reset prices. Retailer i price-setting problem is

$$\max_{P^*_{i,t}} \qquad \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\theta^s\Lambda_{t,t+s}\left(\frac{P^*_{i,t}}{P_{t+s}} - \text{MC}_{i,t+s|t}\right)\left(\frac{P^*_{i,t}}{P_{t+s}}\right)^{-\varepsilon} Y_{t+s}\right\}$$

where we have already substituted in the expression for demand it faces, found in the last section. Let $p^*_{it} = P^*_{it}/P_t$ be the real optimal reset price. Taking into account the relationship between the marginal cost of firms setting prices in time t, $\text{MC}_{i,t|t}$ and the average marginal cost of firm of the same type, $\text{MC}_{i,t}$,[13] the first order condition for this problem can be rewritten as

$$p^*_{it} = \left[\left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\theta^s\Lambda_{t,t+s}\Pi_{t,t+s}^{\frac{\varepsilon}{1-\alpha}}Y_{t+s}\text{MC}_{i,t+s}\right\}}{\mathbb{E}_t\left\{\sum_{s=0}^{\infty}\theta^s\Lambda_{t,t+s}\Pi_{t,t+s}^{\varepsilon-1}Y_{t+s}\right\}}\right]^{\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}}$$

and states that the optimal reset price is a constant mark-up of a weighted average of expected marginal costs (which is just the price of the wholesale goods). This optimality condition is the core of the New-Keynesian Phillips curve. Firms of the same type (i.e., with the same level of access to government credit) chose the same $p^*_{i,t}$ when allowed to reset prices in the same period. But firms of different types choose different prices, and this gives rise to a multitude of Phillips curves — one for each type of firm.

[12] A remark on this specification. One could alternatively have specified that the government provides the firm a fixed *amount* in credit, instead of a fixed *fraction* of a firm's total credit need. This alternative specification might be seen as more plausible but its implications are, maybe, less appealing. This is because what should matter to a firm is the marginal credit, not the average credit. In a *fixed amount* setting firms output and hiring decisions would only be affected by earmarked credit if the amount of credit the firm needs is lower then the amount the government is willing to provide. But most firms are not totally financed by the government and, for these, private credit would be the marginal credit and hence the one affecting decision-making.

[13] Technically, we consider that for each firm i there is another continuum of identical firms, some able to readjust their prices and some not. This is necessary in order for the usual Calvo pricing algebra to follows.

Credit policy. Credit policy is defined by the cost of the government credit (R^s) and by the distribution of ζ_i, which represents accessibility. The interest rate is constant and, hence, is not influenced by monetary policy. I also assume that each firms' access to government credit (ζ_i) is exogenous and fixed. The cost of this credit policy depends on the interest rate differential and on the amount of loans extended by the government. I assume it is entirely financed through lump-sum taxes.

Monetary policy. The central bank is assumed to fix nominal interest rates following a simple Taylor rule: $R_t^n = \left(R^n\right)\Pi_t^{\phi} U_t^m$, where the monetary policy shock is assumed to follow an AR(1) process with auto-regressive coefficient ρ. It is assumed that fiscal policy is passive: the government uses lump-sum taxes in order to satisfy its inter-temporal budget constraint for any sequence of price levels.

Market clearing

In this simple model there is no government spending, investment or foreign trade. Hence, final goods are all consumed: $Y_t = C_t$. Clearing in the labor market requires households' supply to equal wholesalers' demand: $H_t = \int_0^1 H_{jt} \mathrm{d}j$.

Equilibrium and solution

Equilibrium is defined as a sequence for endogenous variables that satisfies households optimality conditions, firms' optimality conditions, the government policy rule, and market clearing conditions, simultaneously, given the realized sequence of the exogenous stochastic process. In order to solve the model I log-linearize it around the deterministic steady-state. In linearized models shocks enter additively and, because our goal is to compute impulse responses, there is no need to detail other stochastic process besides the monetary shock of interest.

1.3.2
A representative firm

Our model has a continuum of heterogeneous firms and this may be a nuisance for the solution of the model. For instance, if we approximate the model to have a hundred firms this would lead to $10*100+9 = 1009$ equations, according to the model's summary in appendix A.5. Of course one can simplify the equations before going for the solution, but it would still be the case that we would have at least one Phillips curve for each type of firm.

Fortunately our simple model admits a representative firm, up to a first order approximation, as I show in appendix A.7. This is very useful as it allows us to ignore the distribution of ζ_i in the population of firms when computing the response of aggregate variables, like GDP and inflation. We only have to use the distribution of ζ_i to compute an appropriate average importance of government credit in the economy, ζ, and work as if all firms in this economy has this same ζ access to earmarked credit. Given the solution for aggregate variables, we can go back and compute the solution for any given zero-measure firm with arbitrary access ζ_i. Hence, we are able to study both micro and macro effects with minimum computational difficulty. In fact, analytically.

1.3.3
Solving for macro variables

Because the model admits a representative firm we can solve for aggregate variables while ignoring what is happening to individual firms. As we show in appendix (A.8) the model can be reduced to a 3-equations system — comprised of an IS curve, a Phillips curve and a policy rule — for three variables — output, inflation and the real interest rate:

$$\begin{aligned} y_t &= \mathbb{E}_t\Big\{y_{t+1}\Big\} - \sigma^{-1} r_t \\ \pi_t &= \beta \mathbb{E}_t\Big\{\pi_{t+1}\Big\} + \kappa y_t + \gamma r_t \\ r_t &= \Big(\phi \pi_t + u_t^m\Big) - \mathbb{E}_t\left\{\pi_{t+1}\right\} \end{aligned}$$

where u_t^m follows an AR(1) process with root ρ. In addition to the structural parameters we have the following reduced-form parameters:

$$\lambda = \left(\frac{(1-\theta)(1-\beta\theta)}{\theta}\right)\frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \qquad \kappa = \lambda\left(\sigma + \frac{\eta+\alpha}{1-\alpha}\right)$$

$$\gamma = \lambda\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w-1)}\right)$$

Note that the parameters λ and κ were defined exactly as in (21)[14]. Additionally this model also features a parameter γ, which captures the strength of the cost-channel. Note that γ/λ is is the elasticity of the representative firm's real marginal cost to changes in the real interest rate, and that by setting $\gamma = 0$ (from either $\psi = 0$ or $\zeta = 1$) we recover the canonical textbook model. Also note that ζ, the overall importance of earmarked credit, only affects the

[14] See chapter 3, which introduces the New-Keynesian model.

equilibrium through this reduced form parameter γ and that:

$$\frac{\partial\gamma}{\partial\zeta} = \frac{-\lambda\psi\beta^{-1}\left(1+\psi\left(R^s-1\right)\right)}{\left[1+\psi(R^w-1)\right]^2} \quad < \quad 0$$

such that the higher the importance of earmarked credit the lower is the strength of the cost-channel. This is to be expected, since the less firms rely on private credit to finance their working capital needs then less they suffer when the market interest rate rises.

We find the solution to this model by guess-and-verify. First, we assume that that for any variable z_t the solution is given by $z_t = b_z\, u_t^m$. This and the AR(1) nature of the driving force u_t^m imply that $\mathbb{E}_t\{z_{t+1}\} = \rho z_t$. Substituting the policy rule in the other two equations, and also the expectational terms:

$$\pi_t = \left[\frac{-\sigma(1-\rho)}{\phi-\rho}\right] y_t + \left[\frac{-1}{\phi-\rho}\right] u_t^m \qquad \text{IS, demand}$$
$$\pi_t = \left[\frac{\kappa}{1-\beta\rho-\gamma(\phi-\rho)}\right] y_t + \left[\frac{\gamma}{1-\beta\rho-\gamma(\phi-\rho)}\right] u_t^m \qquad \text{PC, supply}$$

This is a linear system of two equations for (y_t, π_t) where the exogenous term depends linearly on u_t^m. Hence the solution will be linear in u_t^m and the guess is verified. The solution for inflation and output has the following coefficients:

$$b_\pi = \frac{-\left[\kappa-\gamma(1-\rho)\right)\right]}{(1-\beta\rho)(1-\rho)\sigma+(\phi-\rho)\left(\kappa-\gamma(1-\rho)\right)}$$
$$b_y = \frac{-(1-\beta\rho)}{(1-\beta\rho)(1-\rho)\sigma+(\phi-\rho)\left(\kappa-\gamma(1-\rho)\right)}$$

Models with a cost-channel may feature a “wrong” inflation response to monetary policy shocks, in principle, since interest rate changes trigger two effects with different signs. First, there is the usual aggregate demand effect, which decreases inflation for any given output level. Second, there is also an aggregate supply, cost-channel, effect, where inflation rises along with marginal costs. If the later dominates the former then inflation may rise after a contractionist monetary shock. Fortunately this awkward response does not arise in this model[15].

[15] To see this, note that the inflation's response will be well-behaved if $\kappa-\gamma(1-\rho)>0$. Also, note that $\kappa-\gamma>0$ is sufficient, since $\rho\in[0,1]$. This condition boils down to

$$\left(1+\frac{\eta+\alpha}{1-\alpha}\right) > \frac{\psi(1-\zeta)\beta^{-1}}{(1-\psi)+\psi\zeta R^s+\psi(1-\zeta)\beta^{-1}}$$

and it is clear that the left-hand side is bigger than one while the right-hand side is smaller

Macro effects. With these solutions in hand we can then find the (macro) effects that earmarked credit has on output and inflation. Remember that

$$\text{Macro effect: output} \equiv \frac{\partial}{\partial \zeta}\left(\frac{\partial y_t}{\partial u_t^m}\right) = \frac{\partial b_y}{\partial \zeta} = \frac{\partial b_y}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta}$$
$$\text{Macro effect: inflation} \equiv \frac{\partial}{\partial \zeta}\left(\frac{\partial \pi_t}{\partial u_t^m}\right) = \frac{\partial b_\pi}{\partial \zeta} = \frac{\partial b_\pi}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta}$$

where the last equalities comes from the fact that the overall importance of earmarked credit in the economy (ζ) only affects the economy through γ. We already have the value of $\frac{\partial \gamma}{\partial \zeta}$, and it has negative sign. Now:

$$\frac{\partial b_y}{\partial \gamma} = \left[\frac{(\phi-\rho)(1-\rho)}{(1-\beta\rho)(1-\rho)\sigma + (\phi-\rho)\Big(\kappa - \gamma(1-\rho)\Big)}\right] b_y \quad < \quad 0$$
$$\frac{\partial b_\pi}{\partial \gamma} = \left[\frac{-\sigma(1-\rho)^2}{(1-\beta\rho)(1-\rho)\sigma + (\phi-\rho)\Big(\kappa - \gamma(1-\rho)\Big)}\right] b_y \quad > \quad 0$$

Hence, $\frac{\partial b_y}{\partial \zeta} > 0$ and $\frac{\partial b_\pi}{\partial \zeta} < 0$. Because both b_y and b_π are negative we conclude that earmarked credit reduces the power of monetary policy shocks over aggregate output — as the common sense predicts — but increases the power over inflation — contrary to the common sense.

Figure 1.2 illustrates what is happening[16]. Suppose the economy is initially at steady-state, represented by $(0,0)$. An interest rate hike shifts aggregate demand (IS) curve inwards from IS to IS', as consumption spending is cut down in favor of savings. If the cost-channel is not operative then aggregate supply PC does not shift, and the new equilibrium is represented by $(-1,-1)$: both inflation and output fall. If the cost-channel is operative the supply curve shifts up, however, as the rise in marginal cost caused by the higher interest rates is passed prices, generating inflation. The curve PC_1' represents the case where the cost-channel is operative but not sufficiently strong to dominate the aggregate demand effect — the case that always happens in our model. The equilibrium is now $(-1.5,-0.5)$: inflation and output still fall, output more than before while inflation less[17]. Now, remember

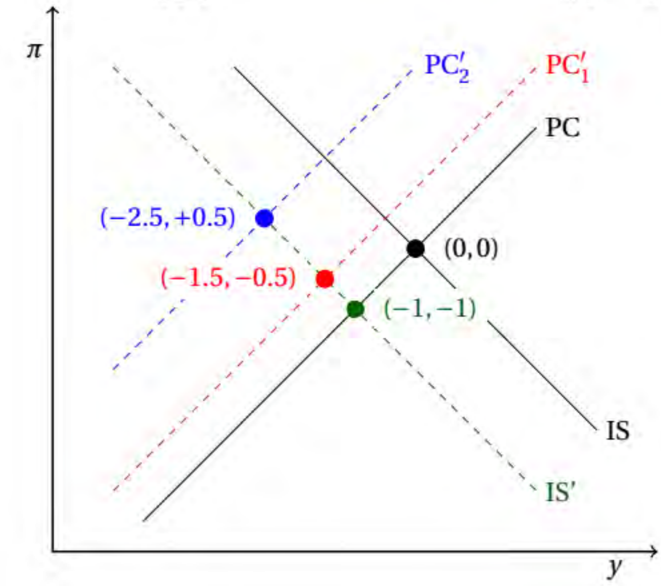
than one ($\psi \in [0,1]$). Hence, $b_\pi < 0$ and $b_y < 0$.

[16] The graph used in this example is not a precise description of the demand and supply curves we have found — for instance, it ignores the fact that changes in γ changes not only the shift size of the supply curve but the slope of this curve. I do this to simplify the exposition.

[17] PC_2' represents cases where the cost-channel dominates the aggregate demand channel, and it gives rise to a price-puzzle. Again, this does not arise in this model, but may arise in others.

that the presence of earmarked credit reduces the strength of the cost-channel. Hence, it mitigates the effect of the monetary policy on aggregate output but reinforces the effect over inflation.

Figure 1.2: Shifts in supply and demand due to monetary policy tightening



1.3.4 Solving for micro variables

The Phillips curve for a given individual firm can be written as:

$$\pi_{i,t} = \beta \mathbb{E}_t \Big\{ \pi_{i,t+1} \Big\} + \kappa y_t + \gamma_i r_t - \delta p_{it}$$

where γ_i is firm i's analogue of the aggregate γ, and δ how pricing decisions depend on firms' relative price, given aggregate conditions (the higher the relative price, less prices need to be increased):

$$\gamma_i = \lambda \left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)} \right) \qquad \delta = \lambda \left(\frac{1-\alpha+2\alpha\varepsilon}{1-\alpha} \right)$$

Using the equation that determines the evolution of this firm's relative price

$$p_{it} = p_{i,t-1} + \pi_{it} - \pi_t$$

to substitute for π_{it} in the Phillips curve, and noting from the aggregate Phillips curve that $\pi_t - \beta \mathbb{E}_t \{\pi_{t+1}\} = \kappa y_t - \gamma_i r_t$, we can then write the following equation for the relative price:

$$p_{it} = \left(\frac{1}{1+\beta+\delta}\right) p_{i,t-1} + \left(\frac{\beta}{1+\beta+\delta}\right) \mathbb{E}_t \left\{p_{i,t+1}\right\} - \left(\frac{\gamma_i - \gamma}{1+\beta+\delta}\right) r_t$$

Note that in this equation the real interest r_t is the exogenous driving force, whose dynamics were already computed using the set of equations for the aggregate economy. As I show in appendix A.9 the solution for this firm's relative price is given by:

$$p_{i,t} = Ap_{i,t-1} + B\Big(\gamma_i - \gamma\Big) r_t$$

where:

$$A = \frac{\Big(1+\beta+\delta\Big) - \sqrt{\Big(1+\beta+\delta\Big)^2 - 4\beta}}{2\beta} \qquad \in [0,1]$$

$$B = \frac{1}{(1-\beta A) + \beta(1-\rho) + \delta} \qquad > 0$$

Note that neither A nor B depend on earmarked credit. Because $B > 0$ it is clear that the relative price of firms with below-average access to earmarked credit ($\gamma_i > \gamma$) increases after an interest rate hike, while the relative price of firms with above-average access decreases.

With the solution for p_{it} it is then possible to back-out the solution for the firm's output using the conditional demand for firms' products:

$$\begin{aligned} y_{i,t} &= \underbrace{y_t}_{\text{Aggregate demand}} + \underbrace{\Big(-\varepsilon\Big) p_{i,t}}_{\substack{\text{Relative price} \\ \Longrightarrow \text{Market share}}} \\ &= -\varepsilon A p_{i,t-1} + \Big[1 + \varepsilon\sigma(1-\rho)B\Big(\gamma_i - \gamma\Big)\Big] b_y u_t^m \end{aligned}$$

I have highlighted that variations in firm's output must be related to two causes: (i) variations in aggregate demand; (ii) variations in firm's relative price, which determines firm's market share. Now note that the aggregate demand effect is equal across firms, which means that heterogeneity in the response of firms to a monetary policy shock must come only from market share variations. This is another reason why it makes no sense to extrapolate the microeconomic effect — which relies on market share changes — to the macroeconomic level — for which there is no sense talking about market shares.

Note that it is possible for the output of some firms to rise after a contractionist monetary shock. This would occur for firms with $\Big(\gamma_i - \gamma\Big) < -1\Big/\varepsilon\sigma(1-\rho)B$ — i.e., for firms with a particularly *high* access to earmarked credit. For this to be possible it is necessary that the market-share effect is sufficiently strong, more than compensating than the aggregate demand effect which has the "correct" sign. Weak restrictions of the parametric space cannot rule out this possibility[18], but I have checked that this does not happen within conventional bounds for parameter values — at least on impact, which is our focus[19].

The solution for firm's inflation can be backed-out using its definition:

$$\begin{aligned}\pi_{i,t} &= \pi_t + p_{i,t} - p_{i,t-1} \\ &= (A-1)\, p_{i,t-1} + \Big[b_\pi - \sigma(1-\rho)B\Big(\gamma_i - \gamma\Big)b_y\Big]\, u_t^m\end{aligned}$$

and, again, it is theoretically possible that this solution has the wrong sign — firm's inflation rising after a contractionist shock. This would happen for firms with $\Big(\gamma_i - \gamma\Big) > [\kappa - \gamma(1-\rho)] \Big/ \varepsilon\sigma(1-\beta\rho)(1-\rho)B$ — i.e., for firms with a particularly *low* access to earmarked credit. Although there is this possibility, I again have checked that this does not happen with conventional values for the parameters.

Micro effects — output:

$$\begin{aligned}\text{Micro effect : output} \quad &\equiv \frac{\partial}{\partial\zeta_i}\left(\frac{\partial y_{i,t}}{\partial u_t^m}\right) \\ = \left[\varepsilon\sigma(1-\rho)B\left(\frac{\partial\gamma_i}{\partial\zeta_i} - \frac{\partial\gamma}{\partial\zeta_i}\right)\right] b_y &+ \Big[1 + \varepsilon\sigma(1-\rho)B\Big(\gamma_i - \gamma\Big)\Big]\frac{\partial b_y}{\partial\gamma}\frac{\partial\gamma}{\partial\zeta_i} \\ = \varepsilon\sigma(1-\rho)Bb_y\frac{\partial\gamma_i}{\partial\zeta_i} \quad &> \quad 0\end{aligned}$$

where the last equality uses the fact that $\gamma = \int \Big(p_i\Big)^{1-\varepsilon}\gamma_i \mathrm{d}i$ (see appendix A.7) and, hence, that $\frac{\partial\gamma}{\partial\zeta_i} = \Big(p_i\Big)^{1-\varepsilon}\mathrm{d}i \approx 0$. Intuitively, giving more subsidized credit to a zero-measure firm has negligible effect on the overall importance of government credit in the economy. The positive sign means that the output of a firms *falls less* when it has more access to earmarked credit, following a

[18] For instance, we can generate such response pattern for $\gamma_i = 1$ firms using a basic calibration for all parameters except for the inverse elasticity of substitution, for which we set $\gamma = 1000$.

[19] It happens, though, for firms' response to have the "wrong sign" over longer horizons. In fact, figure 1.3 exemplifies this.

contractionary monetary shock[20]. Averaging across firms:

$$\begin{aligned}\text{Avg. micro effect : output} &\equiv \int_0^1 \Big(p_i\Big)^{1-\varepsilon}\left[\frac{\partial}{\partial\zeta_i}\left(\frac{\partial y_{i,t}}{\partial u_t^m}\right)\right] \mathrm{d}i \\ &= \varepsilon\sigma(1-\rho)Bb_y\frac{\partial\gamma}{\partial\zeta} \quad > \quad 0\end{aligned}$$

where the weights take into account the fact that firms have different steady-state output levels. Also, I have used the fact that $\gamma = \int \Big(p_i\Big)^{1-\varepsilon}\gamma_i \mathrm{d}i$.

Micro effects — inflation:

$$\begin{aligned}\text{Micro effect : inflation} &\equiv \frac{\partial}{\partial\zeta_i}\left(\frac{\partial\pi_{i,t}}{\partial u_t^m}\right) \\ &= \frac{\partial b_\pi}{\partial\gamma}\frac{\partial\gamma}{\partial\zeta_i} - \sigma(1-\rho)B\left[\Big(\gamma_i-\gamma\Big)\frac{\partial b_y}{\partial\gamma}\frac{\partial\gamma}{\partial\zeta_i} + b_y\left(\frac{\partial\gamma_i}{\partial\zeta_i} - \frac{\partial\gamma}{\partial\zeta_i}\right)\right] \\ &= -\sigma(1-\rho)Bb_y\frac{\partial\gamma_i}{\partial\zeta_i} \quad < \quad 0\end{aligned}$$

and, averaging:

$$\begin{aligned}\text{Avg. micro effect : inflation} &\equiv \int_0^1 \Big(p_i\Big)^{1-\varepsilon}\left[\frac{\partial}{\partial\zeta_i}\left(\frac{\partial\pi_{i,t}}{\partial u_t^m}\right)\right] \mathrm{d}i \\ &= -\sigma(1-\rho)Bb_y\frac{\partial\gamma}{\partial\zeta} \quad < \quad 0\end{aligned}$$

where again I have used the fact that $\frac{\partial\gamma}{\partial\zeta_i} \approx 0$.

Relation between output and inflation average micro effects. Note that:

$$\text{Avg. micro effect : output} = \Big(-\varepsilon\Big) \text{ Avg. micro effect : inflation}$$

meaning that (i) these effects have opposite signs; and that (ii) the higher the elasticity os substitution across goods varieties the higher is the micro effect over output, given the micro effect over inflation. This is to be expected, since the micro effect comes from market-share variations induced by variation in relative prices.

External effects — output: Remember that the external effect is defined as the change in the impulse response function of a firm when the

[20] Or *rise more*, if the individual firm response has the wrong sign. This is unusual, however.

access to earmarked credit *of all other firms* varies. Hence:

$$\begin{aligned}\text{External effect : output} \quad &\equiv \frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial y_{i,t}}{\partial u_t^m}\right)\\ = &\left[\varepsilon\sigma(1-\rho)B\left(\frac{\partial \gamma_i}{\partial \zeta_{-i}} - \frac{\partial \gamma}{\partial \zeta_{-i}}\right)\right] b_y + \left[1 + \varepsilon\sigma(1-\rho)B\Big(\gamma_i - \gamma\Big)\right]\frac{\partial b_y}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta_{-i}}\\ = &\left[-\varepsilon\sigma(1-\rho)Bb_y + \Big(1 + \varepsilon\sigma(1-\rho)B\Big(\gamma_i - \gamma\Big)\Big)\frac{\partial b_y}{\partial \gamma}\right]\frac{\partial \gamma}{\partial \zeta}\end{aligned}$$

where we used the fact that $\frac{\partial \gamma_i}{\partial \zeta_{-i}} = 0$ and that $\zeta_{-i} = \zeta - \Big(p_i\Big)^{1-\varepsilon} \mathrm{d}i \approx \zeta$, and, hence, $\frac{\partial \gamma}{\partial \zeta_{-i}} \approx \frac{\partial \gamma}{\partial \zeta}$. The average external effect is less complicated:

$$\begin{aligned}\text{Avg. external effect : output} \quad &\equiv \int_0^1 \Big(p_i\Big)^{1-\varepsilon}\left[\frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial y_{i,t}}{\partial u_t^m}\right)\right]\mathrm{d}i\\ = \left[\frac{\partial b_y}{\partial \gamma} - \varepsilon\sigma(1-\rho)Bb_y\right]\frac{\partial \gamma}{\partial \zeta} \quad &\gtreqless 0\end{aligned}$$

but its sign is still ambiguous, related to the fact the a firm's output is affected by both aggregate demand and market-share considerations. Aggregate demand follows the macro effect: when all other firms (but i) have more access to earmarked credit aggregate demand falls less when there is a contractionary shock, so demand for firm i's goods also falls less, *given* relative prices. But relative price of a firm also changes: when all other firms have more access to earmarked credit their prices fall by more following a contractionary shock, meaning that firm i's relative price rises, reducing the demand for its goods, *given* aggregate demand.

External effects — inflation:

$$\begin{aligned}\text{External effect : inflation} \quad &\equiv \frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial \pi_{i,t}}{\partial u_t^m}\right)\\ = &\frac{\partial b_\pi}{\partial \gamma}\frac{\partial \gamma}{\partial \zeta_{-i}} - \sigma(1-\rho)B\Big(\gamma_i - \gamma\Big)\frac{\partial b_y}{\partial \gamma} - \sigma(1-\rho)Bb_y\left(\frac{\partial \gamma_i}{\partial \zeta_{-i}} - \frac{\partial \gamma}{\partial \zeta_{-i}}\right)\\ = &\left[\frac{\partial b_\pi}{\partial \gamma} - \sigma(1-\rho)B\Big(\gamma_i - \gamma\Big)\frac{\partial b_y}{\partial \gamma} + \sigma(1-\rho)Bb_y\right]\frac{\partial \gamma}{\partial \zeta}\end{aligned}$$

and, averaging:

$$\begin{aligned}\text{Avg. external effect : inflation} \quad &\equiv \int_0^1 \Big(p_i\Big)^{1-\varepsilon}\left[\frac{\partial}{\partial \zeta_{-i}}\left(\frac{\partial \pi_{i,t}}{\partial u_t^m}\right)\right]\mathrm{d}i\\ &= \left[\frac{\partial b_\pi}{\partial \gamma} + \sigma(1-\rho)Bb_y\right]\frac{\partial \gamma}{\partial \zeta} \quad \gtreqless 0\end{aligned}$$

The sign of this average external effect is also ambiguous, and again there are two forces. On one hand a firm needs to raise its prices when there is inflation if it wants to keep its relative price fixed: when all other firms (but i) have more access to earmarked credit inflation falls by more following a contractionary monetary shock and, by this channel, firm i also wants to cut the price it charges. On the other hand the firm may want to change its relative price: when all other firms have more access to government credit the marginal cost of firm i increases by more than the marginal cost of its competitors, and this is an incentive for firm i to increase the price it charges.

The decomposition works. With the formulas for the average micro and external effects it is easy to check that the decomposition

$$\text{Macro effect} \quad = \quad \text{Avg. micro effect} \quad + \quad \text{Avg. external effect}$$

works for both output and inflation.

1.3.5 A quantitative assessment

The analysis so far has been all analytical, and this approach was very useful to find some answers that are not conditional on the parameterization and also to better understand the forces at play. For instance, it allowed us to show that in the model the presence of government credit reduces the power of monetary policy shocks over output, but increases it over inflation, and allowed us to understand how it is linked to the cost-channel. Also, we could check that the micro and macro effects are indeed different objects, with different formulas for their computation.

However, some answers could not be obtained by relying only on analytical derivation. For example, the signs of the external effects are ambiguous, and its not clear how big they are. In order to proceed we need to to put some values on the parameters. To this end I consider two approaches: (i) looking at a particular parameter vector of interest; and (ii) considering a prior distribution for the parameters and computing the resulting distribution of macro, micro and external effects.

Table 1.1 presents the considered priors I use in the analysis that follows. For the distribution shapes, we consider beta or gamma distributions depending on the parametric space. For simplicity, I choose as prior means values from (21)[21]. For standard deviations I set somewhat ad hoc values

[21] Chapter 3, page 52.

Table 1.1: Prior distributions

Parameter	Meaning	Dist.	Mean	Std. Dev
β	Discount factor	Beta	0.99	0.01
α	Capital share	Beta	0.33	0.05
σ	Inverse elasticity of intertemporal subs.	Gamma	1	0.5
η	Inverse Frisch Elasticity	Gamma	1	0.5
θ	Calvo nominal rigidity	Beta	0.66	0.2
$\varepsilon - 1$	Elasticity of substitution among goods	Gamma	5	5
$\phi_m - 1$	Taylor rule coefficient	Gamma	0.5	0.25
ρ_m	Monetary shock persistence	Beta	0.5	0.25
R_s	Governmental credit interest rate	Fixed	1	-
ψ	Working capital need	Fixed	1	-
ζ	Overall importance of gov. credit	Fixed	$\{ 0 \ , \ 1 \}$	-

reflecting our own uncertainty about parameter values. Of course, the textbook can not guide the choice of values for ψ and R^s, as they are specific to this model. For these I just fix a value instead of specifying a distribution, because it is trivial how they affect our effects of interest. I set $\psi = 1$, implying that firms must finance the entirety of its wage bill. I do so not for realism but to maximize the potential effect of government credit on the economy. I set $R_s = 1$, so that the annualized real interest rate on government loans is 4 p.p. lower than the policy rate in steady-state. For ζ I just consider the values of 0 and 1 in order to compute macro and external effects[22].

For the approach using a specific parameter vector I employ prior mean shown in table 1.1, with one twist: $\alpha = 0$. I do so for a pedagogical purpose, in order to obtain more pronounced micro, macro and external effects. Also, $\alpha = 0$ is itself a benchmark case (constant returns to scale).

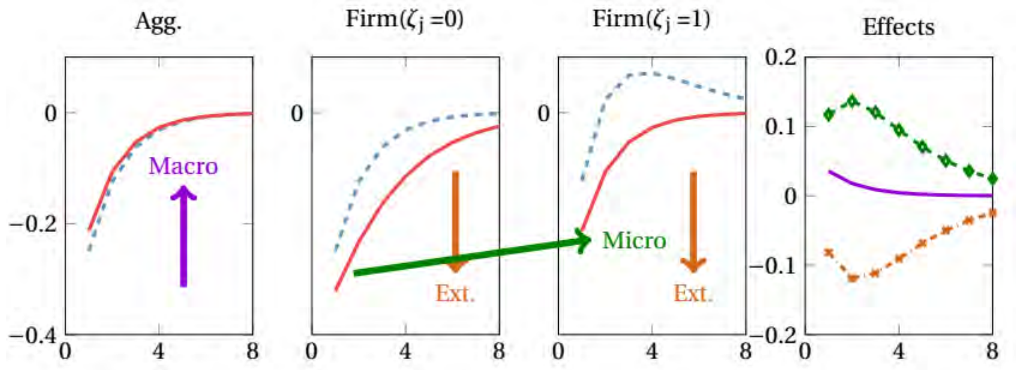
Assessment using a particular parameter vector. Figures 1.3 is a graphical representation of the macro, micro and external effects, for both output (upper panel) and annualized inflation (lower panel). In each panel, the leftmost box plots the response of the aggregate variable following a 1 p.p. contractionary monetary policy shock, both when $\zeta = 0$ (blue, dashed line) and $\zeta = 1$ (red, continuous line), and comparison between these two lines capture the "macro effect". The two central boxes plot firm-level impulse responses: one box plots the response of a firm without any access to government credit ($\zeta_j = 0$), the other the response of a firm completely financed by the government ($\zeta_j = 1$). Comparison between these central boxes capture the differences in responses across firms, i.e., the "micro effect". Again the lines correspond to different scenarios of the overall importance of government

[22] Appendix A.4 shows how to perform the decomposition when the considered changes in ζ and ζ_j are discrete (i.e. not infinitesimal). Anyway, it turns out that the function solution coefficients are almost linear in ζ or ζ_j on the domain $[0, 1]$, such that it does not matter whether one computes the marginal difference or a discrete difference.

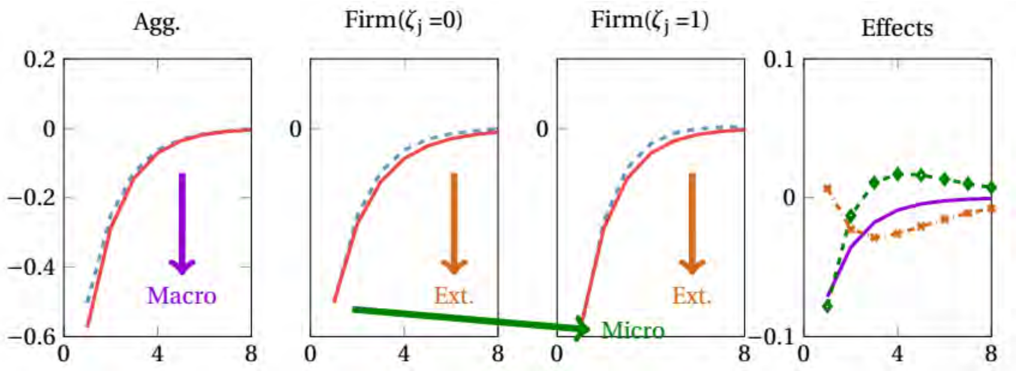
credit, and the difference between them captures the "external effect". Finally, the rightmost box simultaneously plots the macro, micro and external effects.

Figure 1.3: IRFs to a 1 p.p. contractionary M.P. shock; and macro, micro and external effects

Upper panel: Output



Lower panel: Inflation (annualized)

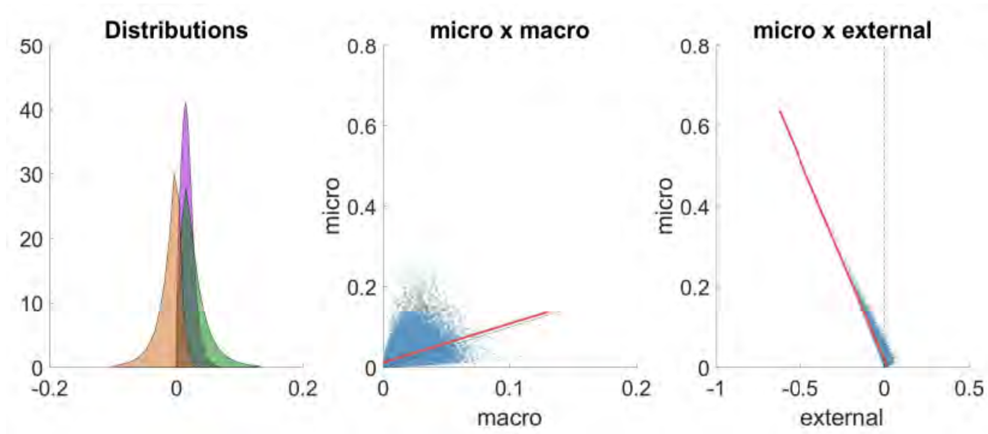


As expected, both aggregate inflation and output falls on impact. In accordance with our previous discussion, when earmarked credit is present output falls less — from -0.249% in the economy with $\zeta = 0$ to -0.213% in the economy with $\zeta = 1$, for a macro effect of +0.036 p.p. — and inflation falls more — from -0.503% in the economy with $\zeta = 0$ to -0.574% in the economy with $\zeta = 1$, in annualized terms, for a macro effect of -0.071 p.p. These macro effects are relatively small, in comparison to the respective IRF, barely noticeable. For output, the average micro effect (of +0.117) is three times larger than the macro effect. Hence, by observing the cross-section a large and significant effect of earmarked credit on firms' response does not necessarily means that the same large and significant effect is present at the aggregate level.

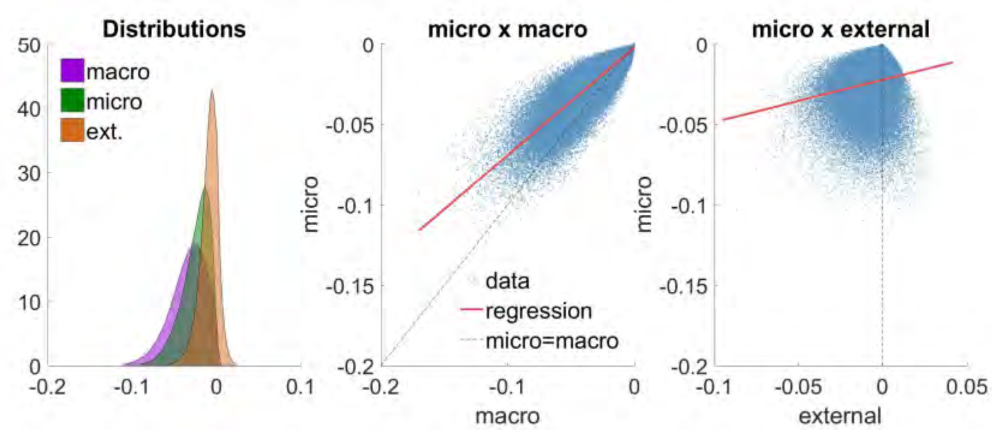
Sensitivity to other parameterizations. I consider a random sample of 100,000 draws from the prior distribution and for each I compute the associated micro, macro and external effects, for both output and inflation. Figure 1.4 plots the results.

Figure 1.4: Macro, micro and external effects associated with the prior parameter distribution

Upper panel: Output



Lower panel: Inflation (annualized)



For output (upper panel) we see that the distribution of macro and micro effects has support over positive numbers, as expected, which means that monetary policy power is always reduced when earmarked credit is present, both at the firm and at the aggregate level. The distribution of external effects is mostly concentrated on negative numbers, implying that in general the external effect mitigates the micro effect, implying a macro effect which is lower than the micro effect. But there are cases where the external effect is positive and, hence, the macro effect is higher. The size of the macro effect is positively correlated with the size of the micro effect, in this prior, but for some parameter variation — for instance, for ϵ — the correlation is negative (not shown).

For inflation (lower panel) we see that the distribution of macro and micro effects has support over negative numbers, as expected — meaning that monetary policy power is increasing in the importance of earmarked credit. The distribution of external effects is mostly concentrated on negative numbers — which means that micro and external effects generally reinforce each other and result in a larger macro effect.

1.4 Conclusion

It seems that a broad agreement has been reached among Brazilian economists, that monetary policy becomes significantly less effective in the presence of earmarked credit featuring subsidized and monetary cycle-insensitive interest rates. In this paper I argue that such question should be reexamined.

First, the available microeconometric evidence that firms with more access to government credit respond less to monetary policy shocks is not necessarily informative about the macroeconometric effect economists are mostly interested in. I show this theoretically and also in a toy exemplifying model. In particular I show the possibility of a large effect on the cross-section of firms to coexist with a small effect on the aggregate level.

Second, monetary policy affects many variables, and the presence of earmarked credit may affect differently each variable's responsiveness. In the toy model I show that aggregate output's response does indeed become milder, as expected, but that inflation's responsiveness becomes stronger. This is the case because, by financing firms' working capital, the government reduces the strength of the cost channel.

2
Earmarked credit, investment and monetary policy power

Is monetary policy power reduced in the presence earmarked credit with subsidized interest rates, insensitive to the monetary cycle? I investigate this question through the lens of a medium sized DSGE model in which capital is firm-specific and earmarked credit finances firms' investment. First, I show that the answer is not necessarily affirmative: under some possible parameterizations we can observe increased power, particularly over inflation. Then I estimate the model with Bayesian techniques and find that a reduction of monetary policy power is indeed the likely outcome of earmarked credit present. However, because output's responsiveness to a monetary shock is decreased more than inflation's, this is associated with a lower sacrifice ratio. Finally I also use the model to show that the subsidized credit is not very effective in boosting steady-state investment since most of it finances investment projects that would be viable even at market interest rates.

Keywords: Monetary economics, Earmarked credit, ...

JEL Classification: E51, E52, H81

2.1
Introduction

> *"(...) the cost of loans granted by both BNDES and CEF has consistently been inferior to the Selic rate or the money-market lending cost. This reduces the monetary policy power in comparison with an alternative scenario in which the Selic rate also affects credit from CEF and BNDES."* (Pérsio Arida, 2005)

It has been argued, for at least a decade as we can see from the quote, that earmarked credit provision in Brazil has a negative effect on monetary policy power — loosely understood as the size of the economy's response for a given policy rate change. The argument is simple and intuitive: because the interest rates charged on earmarked loans (e.g., the TJLP rate) are mostly insensitive to the policy rate (Selic), economic decisions which depends on them, such as investment, are not as affected as they would if those interest rates reflected broader monetary conditions. This might be a major hindrance to Central Bank's stabilization policies insofar the government is responsible for a large share of the credit supply in Brazil[1].

This claim has received support from most economists and had broad media repercussion in the last years. It also was one of the main motivations for a recent policy reform that changed the cost of the loans granted by the Brazilian federal development bank BNDES, making then reflect the yield on long-term government bonds (Law 13483/2017)[2]. Despite this apparent consensus, few are the number of academic studies dedicated to study this specific question: (7), (9), (10), (6). The overall takeaway is that, indeed, the presence of government credit reduces monetary policy power. I argue the question is not yet settled, though.

In the previous chapter I showed that (6)'s microeconometric evidences[3] are not necessarily informative as to monetary policy loss of power, since firm-level cross-section effects are conceptually different from the aggregate effects of interest. In particular, one should take into account the fact that firms' response depend not only on how much earmarked credit they receive (what I call *micro effect*) but also on how much earmarked credit other firms in the economy receive on average (what I call *external effect*). These effects sum to the *macro effect* of interest — how an aggregate variable's response depend on the overall measure of earmarked credit in the economy. I also show the possibility of inflation becoming not less, but more, responsive to monetary shocks in the presence of earmarked credit. This counterintuitive result is derived from a model where earmarked credit finances firms' working capital needs. In this setting, the presence of earmarked credit insulates firms' marginal costs from variation in market interest rates, mitigating the cost-channel of the monetary policy. A fair criticism to this result is its reliance on the cost-channel, considering that working capital financing is far from being the most representative modality of earmarked credit in Brazil. It should be said, however, that such criticism should be shared with two other works of the literature [(9) and (10)].

This motivates me to build a model where earmarked credit — with subsidized interest rates which are insensitive to the monetary cycle — is

[1] In December 2017 credit provided by government controlled banks amounted to 54.1% of total outstanding bank loans; earmarked loans corresponded to 48.7%.

[2] This law, first introduced as a *Medida Provisória* (777), created a new benchmark interest rate for BNDES operations, the TLP, in substitution to the TJLP. Unlike its predecessor, the TLP is linked to the yield on 5-year inflation-linked government bonds and, hence, affected by changes in policy rate. The change is phased in over 5 years.

[3] They find that the employment growth in firms with higher access to earmarked credit is less responsive to changes in the monetary policy rate than the employment growth in firms with lower access.

channeled to finance firms' investments. In order to make investment by firms meaningful I assume that capital is specific to and directly accumulated by each one. In order to study the differences in micro and macro effects, as defined in the previous chapter, I allow for heterogeneity in firms' access to earmarked credit. Still, I show that up to a first order approximation a representative firm framework captures the aggregate dynamics very well, and exploit this feature to simplify the solution without losing touch of the microeconomic phenomenon. Unlike the model in my parallel work this one is medium-sized and features, beside capital accumulation, variable capacity utilization, convex investment adjustment costs, habit formation, price and wage rigidity with indexation. This improves the fit to the data, *vis-a-vis* the previous simple model. Moreover, the model is estimated with Brazilian data using Bayesian techniques, allowing for a more quantitative discussion.

The relationship between the earmarked credit presence and monetary policy power is investigated considering both the prior and the posterior parameter distributions. The prior is useful for assessing *theoretical possibilities*, being a loose distribution with support over a wide range of parameter values. Using it I show that earmarked credit does not necessarily reduce monetary policy power over inflation. The outcome depends on general equilibrium forces, the resultant of both demand and supply channels. On the other hand, the posterior distribution embeds interpretation of macroeconomic data through lens of the model and is thus more useful for assessing the *likely outcome*. I find that a reduction of monetary policy power is indeed likely, as conventionally claimed. However the estimated effects over aggregate output and inflation's responsiveness are very small. Moreover I also find that the fall in inflation's responsiveness is smaller than the fall in output's, leading to a lower sacrifice ratio. In other words, a lower output loss is necessary in order to bring inflation down. In some sense this could be seen as an improvement to monetary policy transmission. Hence, even though monetary policy power is reduced, this does not seem to be significant hindrance to Central Bank's economic stabilization mandate.

Although the model was built with the purpose of studying the relationship between earmarked credit and monetary policy power, it can also be used to investigate other related issues also important for economic policy evaluation. Evaluating the model at the posterior mean I show that a large and permanent increase in subsidized credit is not very effective in boosting steady-state aggregate investment, because most of this credit end up financing projects that would be viable at market interest rates, anyway. This result and interpretation is largely consistent with the empirical findings of (23) and

(24).

Related literature: Among the few papers studying the relationship between earmarked credit and monetary policy power, (7) is the only one where the credit policy is related to investment. But the paper explores a different feature of the credit policy, focusing on the rapid expansion of BNDES's balance sheet after the crisis. This is interpreted as a policy akin to Quantitative Easing and is modeled following (8). Importantly, in his model BNDES purchase credit assets at the ongoing market price and, hence, the yield on its loans is not subsidized and is sensitive to the monetary policy stance. In other DSGE models [(9), (10), and also in the previous chapter, earmarked credit finance firms' working capital needs.

Guideline. Section 2.2 presents the model; section 2.3 discusses the estimation: data, method, priors and posteriors. Results are then presented in section 2.4 and, finally, section 2.5 concludes.

2.2 Model

The model is similar to (25)'s in many features. It is a medium-sized, closed economy model with external consumption habits, convex investment adjustment costs, variable capacity utilization, nominal rigidity of prices and wages together with some price/wage indexation. It also includes a similar shock structure: all their shocks (overall productivity shock, investment-specific productivity shock, price and wake mark-up shocks, risk-premium shock, monetary shock and a government expenditures shock) and one more, to subsidized credit policy's interest rate.

But there is also some differences to their model. The most important, of course, is the existence of earmarked credit. I assume that for each firm in the economy the government finances a given share of its investments. The interest rate on such credit is subsidized and, more importantly, completely independent from the monetary policy rate. Also important is the fact that in this model capital is firm-specific, as opposed to the more traditional assumption of a rental market for capital. Another difference is that I model nominal rigidity following (26)'s convex price adjustment approach, for computational convenience[4]. For the sake of simplicity I dispense with some other features of (25)'s model: the non-separability of consumption and labor in households' utility function and the endogenous time-variability of demand elasticity. Finally, I also consider a simpler Taylor rule (analogous the one in (28)) and a simpler dynamics for government spending.

[4] Calvo pricing interacts with capital specificity in a complicated way, as in (27).

2.2.1 Description

In what follows I provide a complete description of the model. For convenience, a summary with the model's equilibrium conditions is presented in appendices B.1 (the original non-linear set of equations) and B.2 (the log-linearized set of equations).

Households. The representative household chooses consumption (C_t), labor supply (H_t) and nominal bond holdings (D_t), so as to maximize his expected lifetime utility

$$\max_{\{C,H^h,D\}} \qquad \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{\left(C_{t+s} - \gamma \bar{C}_{t+s-1}\right)^{1-\sigma}}{1-\sigma} - \chi \frac{H_{t+s}^{1+\eta}}{1+\eta} \right] \right\}$$

subject to a set of flow budget constraints

$$C_t + \frac{D_t}{P_t} = W_t^h H_t^h + V_{t-1} R_{t-1}^n \frac{D_{t-1}}{P_t} + T_t$$

Note that consumption habits ($\bar{C}_t$) are present in its external form, a.k.a, *keeping up with the Joneses*. The household deem it exogenous even though, in equilibrium, $\bar{C}_t = C_t$. In the budget constraint, P_t denotes the price level, W_t^h is the real wage received by the household, T_t includes transfers/taxes from the government, profits distributed by firms and unions, and adjustment costs paid by firms and unions[5], and R_t^n is the market nominal interest rate. V_t is an exogenous premium in the return of market bonds *vis-a-vis* the return nominal R_t^n the Central Bank is able to set in the money market. The premium is assumed to follow an AR(1) process in logs:

$$\log\left(V_t\right) = \rho_v \log\left(V_{t-1}\right) + \sigma_v \xi_t^v, \qquad \xi_t^v \sim \text{i.i.d.}, \mathcal{N}\left(0,1\right)$$

Let $\Pi_t = \frac{P_t}{P_{t-1}}$ denote inflation, and let

$$R_t = V_{t-1} R_{t-1}^n \frac{1}{\Pi_t} \tag{2-1}$$

denote the ex-post real return on the market bonds. Then, the first order

[5] More precisely:

$$T_t = \int_0^1 \Omega_{j,t} + \Omega_{w,t} + \Psi_{p,t} + \Psi_{w,t} - \text{Tax}_t$$

Each of these terms will be properly presented in the next subs ections.

conditions can be written as:

$$\Big(C_t - \gamma \bar{C}_{t-1}\Big)^{-\sigma} = \beta \mathbb{E}_t \left\{ R_{t+1} \Big(C_{t+1} - \gamma \bar{C}_t\Big)^{-\sigma} \right\} \tag{2-2}$$

$$\chi \Big(C_t - \gamma \bar{C}_{t-1}\Big)^{\sigma} \Big(H_t^h\Big)^{\eta} = W_t^h \tag{2-3}$$

$$\Lambda_t = \beta \left(\frac{C_t - \gamma \bar{C}_{t-1}}{C_{t-1} - \gamma \bar{C}_{t-2}} \right)^{-\sigma} \tag{2-4}$$

which are the Euler equation, the labor supply equation and the definition of the real stochastic discount factor.

Final good assemblers. The final good, which is used for consumption and investment, is a composite of a continuum of varied retail goods, indexed by j. Its production technology is given by:

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{\varepsilon_t^p}{\varepsilon_t^p - 1}} \mathrm{d}j \right)^{\frac{\varepsilon_t^p - 1}{\varepsilon_t^p}} \tag{2-5}$$

where ε_t^p, related the elasticity os substitution among varieties, is assumed stochastic as a device to introduce mark-up shocks in the Phillips curve. It logarithm is assumed to follow an AR(1) process:

$$\log\left(\varepsilon_t^p\right) = (1 - \rho_p)(\epsilon_p - 1)\,\rho_p \log\left(\varepsilon_{t-1}^p\right) + \sigma_p \xi_t^p, \qquad \xi_t^p \sim \text{i.i.d.}, \mathcal{N}\Big(0, 1\Big)$$

The market for this good is perfectly competitive. The first order condition for the representative assembler's profit maximization problem yields the following conditional demand for each variety:

$$Y_{j,t} = (p_{j,t})^{-\left(1 + \varepsilon_t^p\right)} Y_t \tag{2-6}$$

where $p_{j,t} = P_{j,t} \big/ P_t$ is the relative price of the firms' product, in terms of the final good. Free entry in this market drives profit down to zero in each period. Using this condition we can find the appropriate restriction for the relative prices:

$$1 = \int_0^1 \Big(p_{j,t}\Big)^{-\varepsilon_t^p} \mathrm{d}j$$

Firms. There is an unit-mass continuum of monopolistically competitive firms indexed by j. Each face a negatively sloped demand curve for their products, given by equation (2-6). The technology is the same for all firms and is represented by the following constant return to scale production function:

$$Y_{j,t} = A_t\Big(U_{j,t}K_{j,t-1}\Big)^{\alpha}\Big(H_{j,t}\Big)^{1-\alpha},, \qquad \xi_t^a \sim \text{i.i.d.}, \mathcal{N}\Big(0,1\Big) \tag{2-7}$$

where A_t, an aggregate productivity shock, follows an AR(1) process in logs:

$$\log\Big(A_t\Big) = \rho_a \log\Big(A_{t-1}\Big) + \sigma_a \xi_t^a$$

Importantly, capital is firm-specific ($K_{j,t-1}$). Firms must accumulate it themselves instead of relying on a rental market, and this means that a firm capital stock is predetermined when production occurs. But firms are able to adjust how intensively ($U_{j,t}$) they use their capital stock, and to invest($I_{j,t}$) in order to increase their capital stock for the following periods. Using the capital more intensively raises production but makes the capital stock depreciate faster, and investing is subject to convex adjustment costs. Hence, the law of motion for a firm's capital stock is given by:

$$K_{j,t} = \Big[1 - \Delta(U_{j,t})\Big]K_{j,t-1} + Z_t I_{j,t}\left(1 - f\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right) \tag{2-8}$$

where the investment adjustment cost function $f(\cdot)$ is characterized by $f(1) = f'(1) = 0$ e $f''(1) = \kappa > 0$ and the depreciation function is characterized by $\Delta(1) = \delta_0 > 0$, $\Delta'(1) = \delta_1 > 0$ and $\Delta''(1) = \delta_1\delta_2 > 0$[6]. Also, Z_t is an exogenous aggregate shock in investment productivity whose log follow an AR(1) process:

$$\log\Big(Z_t\Big) = \rho_z \log\Big(Z_{t-1}\Big) + \sigma_z \xi_t^z,, \qquad \xi_t^z \sim \text{i.i.d.}, \mathcal{N}\Big(0,1\Big)$$

Nominal investment expenditures by the firm is given by $P_t I_t$, where again P_t is the price level of the final good. These expenditures are financed by both market and earmarked credit. The nominal interest rate on the former, R_t^s, is lower than the nominal interest rate prevailing on the market $V_t R_t^n$, but access to earmarked credit is rationed. In particular, it is assumed that firm j is only able to borrow from the subsidized credit policy only a fraction ζ_j of its investment needs. Access to earmarked credit, ζ_j, may vary across firms. The

[6] In practice, I use the following functions:

$$f(x) = \frac{1}{2}\left(e^{\sqrt{\kappa}(x-1)} + e^{-\sqrt{\kappa}(x-1)} - 2\right)$$
$$\Delta(x) = \delta_0 + \frac{\delta_1}{1+\delta_2}\left(x^{1+\delta_2} - 1\right)$$

but any functions characterized as in the main text yields identical results in a first order approximation.

nominal interest rate the firms face when investing is given by:

$$R^w_{j,t} = \zeta_j R^s_t + (1-\zeta_j) V_t R^n_t \tag{2-9}$$

Note that I do not force the firms to finance their investments with credit. Firms in the model are indifferent between financing their investments with equity or private debt, just like in (29)'s theorem. But earmarked credit is cheaper than these sources of financing, and hence credit is used. I assume that the government can effectively earmark the credit to a investment project, ruling out the possibility that these cheaper form of financing is used to pay wages, dividends, etc. Thus, firms' optimal behavior is akin to the case where investments are forced to be financed by credit, at the firm-specific rate. Importantly, have in mind that in the model the government is not *actively selecting* investment projects to finance. In particular, funds are not being channeled *only* to projects that would not take place had the government not stepped in.

Resuming the model description, firms are subject to costs when adjusting their nominal prices, *à la* Rotemberg[7], given by:

$$\Psi^p_{j,t} = \Psi_p \left(\frac{P_{j,t}}{P_{j,t-1} X^p_t} \right) Y_t$$

where the function $\Psi_p(\cdot)$ is characterized by $\Psi_p(1) = \Psi'_p(1) = 0$ and $\Psi''_p(1) = \psi_p > 0$[8]. Price adjustment costs increase linearly with aggregate output, as usual in this literature, and I assume they are paid to households and do not represent a direct waste of resources[9]. Also, note that firms who adjust their prices by a factor X^p_t do not pay any cost. This feature is included in the model in order to introduce some indexation of prices to past inflation, by defining:

$$X^p_t = \left(\Pi_{t-1} \right)^{\iota_p} \tag{2-10}$$

[7] I use Rotemberg pricing instead of Calvo pricing in this paper because it leads to a more tractable problem when capital is firm-specific. See (30) and (27) for the complications of mixing both features.

[8] In practice, I the usual quadratic adjustment function:

$$\Psi_p(x) = \frac{\psi}{2} (x-1)^2$$

but any function characterized as in the main text yields identical results in a first order approximation.

[9] Its more commonly assumed that it represents a waste of resources, and this affects the market clearing condition in the goods market. It is well known that up to a first order approximation this resource waste is negligible, so what is assumed here does not change my analysis.

Given this setup, a firm's real cash flow is given by:

$$\Omega_{j,t} = p_{j,t}Y_{j,t} - W_t H_{j,t} - I_{j,t-1}\frac{R^w_{j,t-1}}{\Pi_t} - \Psi_p\left(\frac{p_{j,t}}{p_{j,t-1}}\frac{\Pi_t}{X^p_t}\right)Y_t$$

Substituting the conditional demand into the cash-flow and the production function, firms' problem can be stated as of choosing $(H_j, K_j, U_j, I_j, p_j)$ to maximize

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\Lambda_{0,t}\left[\left(p_{j,t}\right)^{-\varepsilon^p_t}Y_t - W_t H_{j,t} - I_{j,t-1}\frac{R^w_{j,t-1}}{\Pi_t} - \Psi_p\left(\frac{p_{j,t}}{p_{j,t-1}}\frac{\Pi_t}{X^p_t}\right)Y_t\right]\right\}$$

subject to

$$\left(p_{j,t}\right)^{-\left(1+\varepsilon^p_t\right)}Y_t = A_t\left(U_{j,t}K_{j,t-1}\right)^{\alpha}\left(H_{j,t}\right)^{1-\alpha}$$

and

$$K_{j,t} = \left[1-\Delta\left(U_{j,t}\right)\right]K_{j,t-1} + Z_t I_{j,t}\left(1 - f\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right)$$

where $(p_{j,-1}\,,\, k_{j,-1}\,,\, i_{j,-1})$ is the vector of initial conditions. Let $\Lambda_{0,t}\mathcal{M}_{j,t}$ and $\Lambda_{0,t}Q_{j,t}$ denote the Lagrange multipliers of the production constraint and the capital law of motion constraint, respectively. $\mathcal{M}_{j,t}$ is interpreted as the real marginal production cost and $Q_{j,t}$ as the shadow value of a capital unit. Define the firm-specific marginal productivity of capital

$$R^k_{j,t} = \mathcal{M}_t\alpha\frac{Y_{j,t}}{K_{j,t-1}} \tag{2-11}$$

and the firm level "inflation" (nominal price change of the firm):

$$\Pi_{j,t} = \frac{p_{j,t}}{p_{j,t-1}}\Pi_t \tag{2-12}$$

Then, the first order conditions for H , K , U , I and p can respectively be written as:

$$W_t = \mathcal{M}_{j,t}(1-\alpha)\frac{Y_{j,t}}{H_{j,t}} \tag{2-13}$$

$$Q_{j,t} = \mathbb{E}_t\left\{\Lambda_{t+1}R^k_{j,t+1}\right\} + \mathbb{E}_t\Big\{\Big[1-\Delta(U_{j,t+1})\Big]\Lambda_{t+1}Q_{j,t+1}\Big\} \tag{2-14}$$

$$R^k_{j,t} = Q_{j,t}U_{j,t}\Delta'\big(U_{j,t}\big) \tag{2-15}$$

$$\begin{aligned}\mathbb{E}_t\left\{\frac{\Lambda_{t+1}}{\Pi_{t+1}}\right\}R^w_{j,t} = Q_{j,t}Z_t\left[1-f\left(\frac{I_{j,t}}{I_{j,t-1}}\right)-\left(\frac{I_{j,t}}{I_{j,t-1}}\right)f'\left(\frac{I_{j,t}}{I_{j,t-1}}\right)\right] \\ +\,\mathbb{E}_t\left\{\Lambda_{t+1}Q_{t+1}Z_{t+1}\left(\frac{I_{j,t+1}}{I_{j,t}}\right)^2 f'\left(\frac{I_{j,t+1}}{I_{j,t}}\right)\right\}\end{aligned} \tag{2-16}$$

$$\begin{aligned}p_{j,t} = \left(\frac{\varepsilon^p_t+1}{\varepsilon^p_t}\right)\mathcal{M}_{j,t} \\ -\left[\frac{\big(p_{j,t}\big)^{\varepsilon^p_t+1}}{\varepsilon^p_t}\right]\left[\Psi'_p\left(\frac{\Pi_{j,t}}{X^p_t}\right)\frac{\Pi_{j,t}}{X^p_t}-\mathbb{E}_t\left\{\Lambda_{t+1}\Psi'_p\left(\frac{\Pi_{j,t+1}}{X^p_{t+1}}\right)\frac{\Pi_{j,t+1}}{X^p_{t+1}}\frac{Y_{t+1}}{Y_t}\right\}\right]\end{aligned} \tag{2-17}$$

The first three optimality conditions are usual: (2-13) represents a traditional labor demand, (2-14) represents a traditional asset pricing condition for capital, and (2-15) is also common for models with capacity utilization.

Equation (2-16) features a novelty that is due to existence of subsidized earmarked credit for investment: the term $\mathbb{E}_t\left\{\Lambda_{t+1}\big/\Pi_{t+1}\right\}R^w_{j,t}$ on the left-hand side. In the case where this interest rate is equal to market rate, $\left(\varepsilon^b_t\right)R^n_t = \mathbb{E}_t\left\{\Lambda_{t+1}\big/\Pi_{t+1}\right\}^{-1}$, it boil-down to the typical investment optimality condition found in models with investment-adjustment costs, where the left-hand side is equal to one.

Finally, equation (2-17) is also common in models featuring Rotemberg-type nominal rigidity, but one remark should be added. These models usually have homogeneous firms and invoke a symmetric equilibrium where all of them choose the same price. In this case one would have $p_{j,t} = 1$ and $\Pi_{j,t} = \Pi_t$, and the first order condition for prices would be further simplified. Here firms are heterogeneous in terms of access to earmarked credit and hence the assumption of symmetry can only be invoked for firms with the same level of access (which is a constant in the model) and the same initial conditions.

Unions. Firms hire labor from unions. There is a continuum of unions indexed by z, each suppling a different type of labor. For firms what matters is the composite of these labor types, given by:

$$H_t = \left(\int_0^1 H_{z,t}^{\frac{\varepsilon_t^w}{1+\varepsilon_t^w}} \mathrm{d}z \right)^{\frac{1+\varepsilon_t^w}{\varepsilon_t^w}}$$

and they hire each labor type optimally in order to minimize costs. ε_t^w, related to elasticity of substitution among labor varieties, is assumed stochastic in order to introduce wage mark-up shocks. Its log is assumed to follow an AR(1) process:

$$\log\left(\varepsilon_t^w\right) = (1-\rho_w)\left(\epsilon_w - 1\right)\rho_w \log\left(\varepsilon_{t-1}^w\right) + \sigma_w \xi_t^w, \qquad \xi_t^w \sim \text{i.i.d.}, \mathcal{N}\left(0,1\right)$$

Conditional demand for each labor type is given by:

$$H_{z,t} = \left(\frac{W_{z,t}}{W_t}\right)^{-\left(1+\varepsilon_t^w\right)} H_t$$

where $W_{z,t}$ is the wage rate for union-z labor and W_t is the wage index faced by firms, which is given by:

$$1 = \int_0^1 \left(\frac{W_{z,t}}{W_t}\right)^{-\varepsilon_t^w} \mathrm{d}z$$

In turn, unions hire homogeneous labor supplied by households at the wage rate W_t^h and costlessly differentiates it into a specific labor type, supplied to firms at the wage rate $W_{z,t}$. Unions are monopolistically competitive when supplying its labor variety and explore its market power accordingly by fixing its wage rate in order to maximize an operational result

$$\Omega_{z,t} = \left[W_{z,t} - W_t^h\right] H_{z,t} - \Psi_{z,t}^w$$

which is transfered back to the households. Wages are nominally rigid *a la* Rotemberg[10]. The cost for a union to change the nominal wage it charges is given by

$$\Psi_{z,t}^w = \Psi_w \left(\frac{W_{z,t}\Pi_t}{W_{z,t-1}X_t^w}\right) W_t H_t$$

where the function $\Psi_w(\cdot)$ is just like the function $\Psi_w(\cdot)$ except for the fact that $\Psi_w''(1) = \psi_w$. Note that in my specification the cost increases linearly with the

[10] I could have used Calvo-type nominal rigidity here without further complications, but I have chosen to stick to the same framework of nominal rigidity for wages and prices.

aggregate wage bill, $W_t H_t$[11]. Again, the adjustment cost just entails transfers to the households, not a waste of resources. I allow for some indexation of wages to past inflation by specifying:

$$X_t^w = \Pi_{t-1}^{\iota_w} \tag{2-18}$$

The problem of a union is, hence, to maximize:

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t} W_t H_t \left[\left(\frac{W_{z,t}}{W_t} \right)^{-\varepsilon_t^w} - \frac{W_t^h}{W_t} \left(\frac{W_{z,t}}{W_t} \right)^{-\left(1+\varepsilon_t^w\right)} - \Psi_w \left(\frac{W_{z,t}\Pi_t}{W_{z,t-1} X_t^w} \right) \right] \right\}$$

and the first order condition is:

$$\frac{W_{z,t}}{W_t} = \left(\frac{1+\varepsilon_t^w}{\varepsilon_t^w} \right) \frac{W_t^h}{W_t}$$
$$- \left[\frac{\left(W_{z,t} / W_t \right)^{1+\varepsilon_t^w}}{\varepsilon_t^w} \right] \left[\Psi_w' \left(\frac{\Pi_{z,t}^w}{X_t^w} \right) \frac{\Pi_{z,t}^w}{X_t^w} - \mathbb{E}_t \left\{ \Lambda_{t+1} \Psi_w' \left(\frac{\Pi_{z,t+1}^w}{X_{t+1}^w} \right) \frac{\Pi_{z,t+1}^w}{X_{t+1}^w} \frac{H_{t+1} W_{t+1}}{H_t W_t} \right\} \right]$$

Where, for convenience, I have introduced a symbol for union level "wage inflation"

$$\Pi_{z,t}^w = \frac{W_{z,t}}{W_{z,t-1}} \Pi_t \tag{2-19}$$

Because unions are homogeneous I invoke a symmetric equilibrium where $W_{z,t} = W_t$ and $\Pi_{z,t}^w = \Pi_t^w$ for all firms. Hence, the equilibrium condition can be rewritten as:

$$W_t = \left(\frac{1+\varepsilon_t^w}{\varepsilon_t^w} \right) W_t^h$$
$$- \left(\frac{W_t}{\varepsilon_t^w} \right) \left[\Psi_w' \left(\frac{\Pi_t^w}{X_t^w} \right) \frac{\Pi_t^w}{X_t^w} - \mathbb{E}_t \left\{ \Lambda_{t+1} \Psi_w' \left(\frac{\Pi_{t+1}^w}{X_{t+1}^w} \right) \frac{\Pi_{t+1}^w}{X_{t+1}^w} \frac{H_{t+1} W_{t+1}}{H_t W_t} \right\} \right] \tag{2-20}$$

Government: monetary, fiscal and credit policies. The Central Bank is assumed to fix nominal interest rates following a smoothed Taylor rule that reacts to to inflation and to output deviations from its steady-state:

$$R_t^n = \left(R^n \right)^{1-\phi_r} \left[\left(\mathbb{E}_t \left\{ \Pi_{t,t+4} \right\} \right)^{\frac{\phi_\pi}{4}} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right]^{\phi_r} \left(\sigma_m \xi_t^m \right) \tag{2-21}$$

[11] This specification is unusual, and is used in order to obtain a simpler mapping between the Rotemberg adjustment cost parameter ψ_w and the nominal rigidity parameter θ_w of a similar model with Calvo nominal wage rigidity. For a recent paper on the mapping between Calvo and Rotemberg wage Phillips curve, see (31).

where $\mathbb{E}_t \{\Pi_{t,t+4}\}$ denotes expected cumulated inflation over the next four quarters, $\frac{Y_t}{Y}$ denotes the output gap computed as its deviation from steady-state, and $\xi_t^m \sim$ i.i.d.,$\mathcal{N}\Big(0, 1\Big)$ denotes the monetary shock. This monetary policy rule is analogous to the one in (28), except for the fact that here there the inflation target and growth trends are normalized to zero.

Credit policy is defined by the interest rate charged on government credit, R_t^s, and by the distribution of access, ζ_j. Both are assumed exogenous, the ζ_j's being fixed over time[12], and the subsidized rate following the process:

$$\log\Big(R_t^s\Big) = (1-\rho_s)\log\Big(R^s\Big) + \rho_s \log\Big(R_{t-1}^s\Big) + \sigma_s \xi_t^s, \qquad \xi_t^s \sim \text{i.i.d.}, \mathcal{N}\Big(0, 1\Big)$$

The real total amount lent by the government is:

$$L_t^s = \int_0^1 \zeta_j I_{j,t} \mathrm{d}j$$

Government consumption, G_t, is also modeled as an exogenous AR(1) process:

$$\log\Big(G_t\Big) = (1-\rho_s)\log\Big(G\Big) + \rho_s \log\Big(G_{t-1}\Big) + \sigma_g \xi_t^g, \qquad \xi_t^g \sim \text{i.i.d.}, \mathcal{N}\Big(0, 1\Big)$$

It is assumed that fiscal policy is passive, in the sense that government uses lump-sum taxes (TAX_t) in order to satisfy its inter-temporal budget constraint for any sequence of price levels. Because Ricardian equivalence holds in this model it is immaterial when exactly those taxes are charged. Nominal government debt evolves accordingly

$$D_t^b = \Big[P_t G_t + V_{t-1} R_{t-1}^n D_{t-1}^b + P_t L_t^s\Big] - \Big[P_t \text{TAX}_t + R_{t-1}^s P_{t-1} L_{t-1}^s\Big]$$

Market-clearing. Clearing in the market for final goods imply:

$$Y_t = C_t + I_t + G_t \tag{2-22}$$

where the final good is given by equation (2-5). In the labor market unions hire all labor supplied by the households, and supply all the labor demanded

[12] For simplicity. This is not very important since, at first order approximation and for aggregate outcomes, shocks to R_t^s and ζ_t (if allowed to be time-varying) are isomorphic, affecting the model only through R_t^w. For this same reason the model is useful for discussing the recent policy change in Brazil, moving from the TJLP to the TLP, even though the analysis focus on ζ and not on R^s.

by firms. Hence:

$$H_t = \int_0^1 H_{j,t} \mathrm{d}j \tag{2-23}$$

There is no market clearing condition for capital, since it is firm-specific[13]. Aggregate investment is defined as the sum of all firms' investment:

$$I_t = \int_0^1 I_{j,t} \mathrm{d}j \tag{2-24}$$

In the bonds market, households' holdings must equal government debt and firms' market financing needs:

$$D_t = D_t^b + P_t \int_0^1 (1 - \zeta_j)\, I_{j,t} \mathrm{d}j$$

This completes the description of the model.

2.2.2
A remembrance: macro, micro and external effects

I am interested in how the presence of government credit affects the response of economic variables after a monetary shock. As I discussed in the previous chapter, it is useful to distinguish between three types of effects:

1. **Macro effect**. The aggregate response of the economy to a monetary shock is expected to depend on the overall importance that government credit. The "macro effect over variable $\mathcal{Y}$" is the effect that changing this government credit importance (ζ) has on the response of an aggregate variable ($\mathcal{Y}$) to a monetary shock. At the margin it can be measured by:

$$\frac{\partial}{\partial \zeta}\left(\frac{\partial \mathcal{Y}}{\partial R}\right)$$

2. **Micro effect**. Firms with higher access to government credit are expected to react differently from low-access firms. Equivalently, a firm's response is expected to change depending on its own access to government credit. The "micro effect over variable $\mathcal{Y}$" is the effect that changing

[13] This also means that there is no need to define a measure of the aggregate capital stock. If one wants do so, however, it would also be necessary to define a price-index of capital goods. The only logical restriction for such definitions is that the real value of the capital stock must be equal to the sum of the real value of each firm's capital:

$$Q_t K_t = \int_0^1 Q_{j,t} K_{j,t} \mathrm{d}j$$

but we would still need another equation to pin-down both K_t and Q_t.

a given firm access to government credit (ζ_j) has one the response of this same firm (measured by the variable $\mathcal{Y}_j$) to a monetary shock. At the margin it is given by:

$$\frac{\partial}{\partial \zeta_j}\left(\frac{\partial \mathcal{Y}_j}{\partial R}\right)$$

3. **External effect**. The response of a firm also depends directly on the overall importance government credit has in this economy, even controlling for the firm's own access to government credit. This includes the existence of general equilibrium effects. The "external effect over variable $\mathcal{Y}$" is the effect that changing government credit access to all-firms-but-j (ζ_j) has on firm-j response (again, measured by $\mathcal{Y}_j$) to a monetary shock. At the margin, it is measured by:

$$\frac{\partial}{\partial \zeta_{-j}}\left(\frac{\partial \mathcal{Y}_j}{\partial R}\right)$$

My parallel work also shows that the following identity holds, somewhat independently of the model:

$$\underbrace{\frac{\partial}{\partial \zeta}\left(\frac{\partial \mathcal{Y}}{\partial R}\right)}_{\text{macro effect}} = \underbrace{\mathbb{E}_j\left[\frac{\partial}{\partial \zeta_j}\left(\frac{\partial \mathcal{Y}_j}{\partial R}\right)\right]}_{\text{avg. micro effect}} + \underbrace{\mathbb{E}_j\left[\frac{\partial}{\partial \zeta_{-j}}\left(\frac{\partial \mathcal{Y}_j}{\partial R}\right)\right]}_{\text{avg. external effect}}$$

where $\mathbb{E}_j[\cdot]$ is the average across all firms[14]. The main objects of interest for this study are the macro effects for variables such as investment, output and inflation. But in order to better understand the macro effects it is useful to how are they decomposed into micro and external effects, particularly because micro-level evidence is usually presented even when discussing aggregate phenomena.

2.2.3 Heterogeneity, aggregation, and zero measure firms

Firms in the model are ex-ante heterogeneous in how much earmarked credit they can borrow when investing. This ex-ante heterogeneity induces ex-post differences in investment, accumulated capital, production levels, relative prices, etc, both at the steady-state and through-out business cycles.

[14] For this decomposition, the overall level of government credit is defined as $\zeta = \mathbb{E}_j[\zeta_j]$ and the aggregate variable of interest is defined as $Z = \mathbb{E}_j[Z_j]$. Also, $\zeta_{-j} = \mathbb{E}_{i \neq j}[\zeta_i]$ is the overall acess of all other firms but j.

If one considers a rich heterogeneity structure where the distribution of ζ_j has support over many different levels, the resulting model would feature many equations and variables[15]. The computational cost of solving for the steady-state and for the model dynamics is increasing in the number of variables/equations. Because the estimation requires us to solve the model thousands of times (at least once for each vector of parameters considered, but in practice more given the posterior sampler I use), one may wonder whether it is possible to aggregate all these firms into a representative firm and focus only on aggregate quantities, instead of keeping track of the whole distribution of micro variables.

Strictly speaking, perfect aggregation is impossible even in a first-order approximation. This is because the aggregation weights for variables such as output, labor demand and investment are different from one another. For instance, aggregating the log-linearized production functions (one for each j):

$$y_{j,t} = a_t + \alpha\Big(u_{j,t} + k_{j,t-1}\Big) + (1-\alpha)h_{jt}$$

we obtain

$$y_t = a_t + \alpha\Big(u_t + k_{t-1}\Big) + (1-\alpha)h_t + \xi_t$$

where the approximation error is given by:

$$\begin{aligned}\xi_t = \alpha \left[\int \Big[\varpi_y(j) - \varpi_u(j)\Big] u_{j,t} \mathrm{d}j\right] + \alpha \left[\int \Big[\varpi_y(j) - \varpi_k(j)\Big] k_{j,t} \mathrm{d}j\right] \\ + (1-\alpha)\left[\int \Big[\varpi_y(j) - \varpi_h(j)\Big] h_{j,t} \mathrm{d}j\right]\end{aligned}$$

and the $\varpi_x(j)$'s denote aggregation weight of firm j in aggregate variable x[16].

Fortunately, turns out that for my model these approximation errors are very small in practice, as illustrated by Figure 2.1. Panel (a) plots a simulated trajectory for the true output, investment and inflation aggregates, comparing it to the simulated trajectory for the representative firm's corresponding variables. I consider the worst case scenario (for the approximation): an economy with only $N = 2$ types of firms, one with $\zeta_1 = 0$ and the other with $\zeta_2 = 1$, each type with a measure of half — thus maximizing the dispersion of $\{\zeta_j\}$. Panel (b) shows the approximation error for both an economy with $N = 2$ types of firms and for an economy with $N = 100$ types of firms

[15] For instance, if one considers one hundred types of firms with positive measure (each type is associated with a different ζ_j value), the resulting model would feature $11 \times 100 + 13 = 1113$ equations and variables.

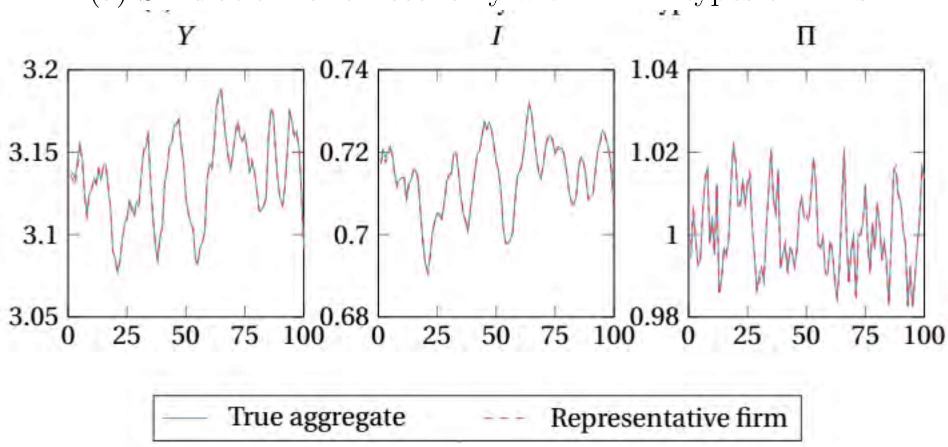
[16] Weights can be found in appendix B.3. In general, firm j's weight for aggregate variable x is $\varpi_x(j) = \frac{X_j}{X}$, measured at the steady-state.

(with ζ uniformly distributed over [0,1]), confirming that the error is bigger with for $N = 2$. The conclusion, thus, is that a representative firm economy provides a good approximation for the heterogeneous firms economy, and avoid unnecessary computational complexity.

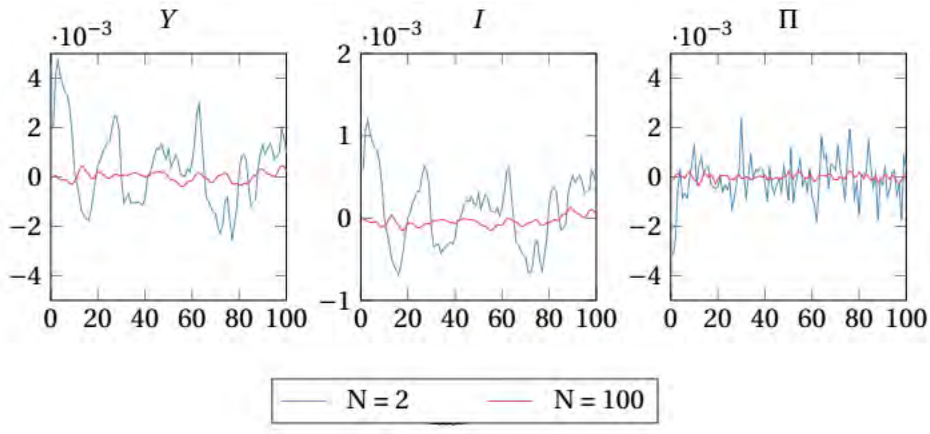
Importantly, by following this approach I do not have to sacrifice the analysis of micro and external effects of earmarked credit. This is because, even in a framework with a representative firm, I can consider the existence of *zero-measure* firms which differ in their level of access to earmarked credit. These firms' dynamics depend on aggregate dynamics, obviously, but the converse is not true. Hence, these firms need not be included in the model during estimation using aggregate data, but some can be later included for analysis. For instance, by including a zero-measure firm with $\zeta_j = 0$ and another with $\zeta_J = 1$ we can effectively compare how much differently they react after shocks (micro effect), and also how differently each react when the overall importance of government credit is changed (external effect).

Figure 2.1: Assessing the approximation error

(a) Simulation for an economy with $N = 2$ types of firms



(b) Approximation error, by number of types (N) of firms



2.3
Estimation

2.3.1
Data

Nine Brazilian time-series are used as observable variables: (i) output growth; (ii) consumption growth; (iii) government consumption growth; (iv) investment growth; (v) employment growth; (vi) real wage inflation; (vii) consumer price inflation — IPCA; (viii) monetary policy rate — Selic; and (ix) the subsidized credit rate — TJLP. Data frequency is quarterly and the full-sample covers the period from 1999:4 to 2017:3, for a total of 72 data points. The first 13 observations (1999:4-2002:4) are only used in the initialization of the Kalman Filter, though, so the effective sample comprises 59 points (2003:01-2017:3).

Detrending the variables is necessary in order for the variables used in estimation to conform to the stationarity of the model. I use a one-sided HP filter [(32)] for this purpose, with $\lambda = 1600$). Using the one-sided filter is recommended over the more traditional (two-sided) HP-filter because it is purely backward looking, not taking into account future realizations of the filtered variable. I also find this approach preferable to a simple demeaning, at least in this particular application, because filtering allows me to better deal with the downward trend in the Selic series. The same filter is also applied for all other series for a matter of consistency, in order for all variables to have the same business cycle frequency. These recommendations are due to (33). Additional information on the data is provided in appendix B.4.

The measurement equation, which links model variables to observed data, is given by:

$$\begin{bmatrix} dl\,\mathrm{GDP}_t \\ dl\,\mathrm{Consumption}_t \\ dl\,\mathrm{Investment}_t \\ dl\,\mathrm{Gov.Cons.}_t \\ dl\,\mathrm{Real\ Wage}_t \\ dl\,\mathrm{Employment}_t \\ dl\,\mathrm{IPCA}_t \\ \mathrm{Selic}_t \\ \mathrm{TJLP}_t \end{bmatrix} = \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ g_t - g_{t-1} \\ w_t - w_{t-1} \\ h_t - h_{t-1} \\ \pi_t \\ r_t^n \\ r_t^s \end{bmatrix} + \begin{bmatrix} \eta_t^y \\ 0 \\ 0 \\ 0 \\ \eta_t^w \\ \eta_t^h \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Note that I have included measurement error in the observation equations for output, real wage and employment. Three are three reasons for including a measurement error for output: (i) to account for changing weights in the true data; (ii) to take into account the existence of different deflators for each demand component in the true data; and, perhaps more importantly, (iii) because net-exports are a missing demand component in the model. Of course modeling the Brazilian economy as a closed implies a misspecification, but we proceed this way for simplicity. In the case of real wage and employment, measurement errors are included because these series are not as well measured. First, they cover only the most important capitals, not the whole national territory. Second, because of the series were constructed by chaining together three different households surveys, when they are discontinued and substituted by other. I assume for estimation that all these measurement errors are independent white noises.

2.3.2 Method

The parameters of the model are estimated with Bayesian methods, i.e., by combining prior and data's likelihood information to construct a posterior distribution for parameter values. The Bayesian approach is attractive not only because it allows the researcher to incorporate information from other sources (previous studies, microeconometric data, etc), but also because the priors mitigate the problem of the likelihood function being very irregular in usual DSGE applications, resulting in poor and often unreasonable maximum likelihood estimates.

Bayesian estimation relies on Markov chain Monte Carlo (MCMC) algorithms to sample from the posterior distribution. In this paper I use the *Tailored Randomized Block Metropolis-Hastings* (TaRB-MH) method proposed by (34). At each iteration of this algorithm the estimated parameters are randomly assigned to a random number of blocks. Block-specific proposal distributions are constructed, tailored with the mode and hessian of a new optimization for the block's parameters. The TaRB-MH is computationally costly since a mode-finding step is performed for each block at each iteration, but it pays off because, in comparison to the more traditional *Random Walk Metropolis-Hastings* (RW-MH): (i) it delivers much less auto-correlated MH draws, reducing the number of draws required to form a representative sample from the posterior; and (ii) it allows a better exploration of the parametric space in the case of irregular target distributions.

I use the Matlab toolbox Dynare[17] for the estimation, which allows for the TaRB-MH sampler. I set 0.5 the probability of forming a new block, such that

[17] `http://www.dynare.org/`

the average block size is 2. I run 3 independent chains of size 20,000, and from each chain I discard the first 2,000 draws as a burn-in. The actual combined sample is, hence, 54,000 draws. Running different independent chains allowed me to check that convergence to the target distribution was achieved.

2.3.3
A transformation of the nominal rigidity parameters

In the model price and wage nominal rigidities are modeled *a la* Rotemberg — i.e., with convex price adjustment costs — instead of the more traditional Calvo approach of i.i.d. lottery of price re-optimization. I follow this approach because the Rotemberg model is more tractable when capital is firm-specific, yielding an analytical representation for the marginal cost coefficient in the Phillips curve. This has one drawback for estimation, however. The Calvo parameter — which I shall denote by θ — has a straightforward economic interpretation: as $1/\theta$ captures the average time interval between price changes, a moment which which is easy to observe in the data[18]. This is a clear advantage over the Rotemberg adjustment parameter ψ, which is more obscure and without a straightforward data counterpart.

Fortunately there is an equivalence between Calvo and Rotemberg models up to a first order approximation. This is the case because they both imply a set of log-linearized equilibrium conditions where the respective nominal rigidity parameter (either θ or ψ) only appears inside the coefficient relating inflation to marginal cost, in the Phillips curve. This allows one to draw a mapping from one parameter to the other: if $\mathcal{C}\Big(\theta\Big)$ denotes the Phillips curve slope for a model with a Calvo-type rigidity and $\mathcal{R}\Big(\psi\Big)$ the same slope for a model with Rotemberg-type rigidity, than both models are equivalent whenever $\mathcal{C}\Big(\theta\Big) = \mathcal{R}\Big(\psi\Big)$. I make use of this equivalence in order to transform the clumsy parameter ψ into the more convenient θ, for which a prior distribution is easier to be established.[19]

For nominal wage rigidity the mapping is straightforward, analytical. It is precisely given by:

$$\psi_w = \frac{(\epsilon_w - 1)\theta_w}{(1-\theta_w)(1-\beta\theta_w)}$$

On the other hand, the precise mapping for nominal price rigidity does

[18] More precisely, between price re-optimizations. Typical models with price indexation lose these link to the data, as price is always changing (either optimally or by a mechanical rule).

[19] In fact it is very common for models with Rotemberg pricing to have ψ calibrated using a value of θ as benchmark.

not allow for a analytical representation in our model, because slope of the Phillips curve can not be analytically computed for the Calvo model, in the first place. I than rely on a approximate mapping, analogous to the one for nominal wage rigidity, but substituting the respective parameters.

2.3.4 Prior distributions of the parameters

The model features 35 parameters (20 technological/behavioral, 15 related to the shocks) while the set observation equations include 3 noisiness parameters. From this total of 38 parameters, only 2 are calibrated: the elasticity is substitution across labor types (ϵ_w) and the steady-state levels of the subsidized interest rate (R^s). Using (35)'s approach I check that both are identifiable in this model with the dataset assembled.[20]. I set $\epsilon_w = 11$, for a mark-up of 20% in the labor market, and $R^s = 1$, implying 0% real rate for the subsidized lending rate[21].

All the remaining parameters are estimated. For each parameter prior shapes are chosen respecting its theoretical bonds[22]. It is useful to divide those into four categories: (i) economic parameters with a close link to observable steady-state values, (ii) other economic parameters; (iii) shock parameters; and (iv) noise parameters. In the first group I include (β, α , δ, G/Y and ζ), and for these I set somewhat tight priors centered around values consistent with steady-state relations. I assume $100\left(\beta^{-1} - 1\right)$ is centered around 1.25 with a standard deviation of 0.05, consistent with the average annualized real interest rate around 5% I observe in the data, after 2005. The capital share α is assumed to be centered around 0.4 — a value which is close to the sum of gross operating surplus and mixed income (41.2% in 2014) in annual national account data — with a standard deviation of 0.03. For the depreciation rate, I assume 100δ is spread around 2.5 with a standard deviation of 0.5. For G/Y I set a prior centered around 0.2 with a standard deviation of 0.05, close to government's consumption of 19.15% GDP in 2014. Finally, the prior for government's share in investment financing ζ is also assumed to have a mean of 0.2 and a standard deviation of 0.05. The assumed prior matches the participation of BNDES in the credit market: BNDES amounts to roughly 40% of outstanding earmarked credit, which in turn correspond to roughly 50% total outstanding credit.

In the second group are (σ, η, γ, δ_2, κ, ϵ_p, θ_p, θ_w, ι_p, ι_w, ϕ_r, ϕ_π, ϕ_y), and for these I see a looser priors. σ's mean is set to 1.5, implying an inter-

[20] This is checked for many points of the parameter space.

[21] Which is not far from the average ex-post real TJLP rate of 0.5%$p.a.$ after 2005. This would imply $R^s = 1.00135$, but I fix $R^s = 1$ for simplicity.

[22] For instance, beta distributions for parameters located in the $[0, 1]$ interval.

temporal elasticity of 0.66, with a standard deviation of 0.1 . η is assumed to be distributed around 0.33 with a standard deviation of 0.2; its prior mean implies a Frisch elasticity of 3, typical of macro models as reported by (36). The convexity of the depreciation function, δ_2, is assumed to be have mean 5, with a standard deviation of 2.5. The convexity of the investment adjustment cost function, κ, fluctuates around 2.5 with a standard deviation of 1.5, implying an investment elasticity of 0.4 with respect to capital price, in line with (37)'s report. The prior for both transformed nominal rigidity parameters (θ_p and θ_w) are assumed to have a mean of 0.65 and a standard deviation of 0.1, while the prior for both indexation parameters (ι_p and ι_w) have mean 0.5 and a standard deviation of 0.15. For Taylor rule parameters ϕ_π and ϕ_y I set a prior mean of1.5 and 0.5, respectively, in line with Taylor's original prescription. The smoothing coefficient is assumed to be around 0.75. The standard deviations for these are set to 0.3, 0.1 and 0.1, respectively.

I set the same prior for all the shocks. For the autoregressive roots, a mean of 0.5 with a standard deviation of 0.15. For the volatilities 0.1 and 1, respectively. Finally, for measurement errors' volatilities I set a prior with mean and standard deviation of 0.01.

Prior distributions' shape, mean and standard deviation, and also the values set for calibrated parameters, are shown in Table 2.1.

2.3.5
Posterior estimates of the parameters

Table 2.1 also shows the mode, the mean, the standard deviation and the bounds of the 95% credible interval of each parameter's posterior distribution. Figures comparing the shapes of the prior and posterior marginal distributions are left to appendix B.5. With only a few exceptions the data is informative about the estimated parameters, with most of the marginal posterior distributions being either tighter than the prior or centered on a different value.

I start by discussing the estimates for the technological/behavioral parameters. I estimate a mean of 1.26 for $100(\beta^{-1} - 1)$, almost equal to the value set as prior. For the inverse of the elasticity of intertemporal substitution, σ, the estimated value is precisely the one set as prior, 1.5. In both these cases the standard deviation is slightly lower, though. For the inverse Frisch elasticity η the estimated mean is 0.23, lower then value 0.33 set as the prior mean. For the parameter controlling the formation of consumption habits, γ, I estimate a mean of 0.66 from a prior mean of 0.75. The posterior mean for the capital share α is estimated at 0.32, way below the prior mean of 0.4 considering that I have set a relatively tight prior with a standard deviation of

Table 2.1: Prior and Posterior

		Prior		Posterior				
	Distr.	Mean	St. Dev.	Mode	Mean	St.Dev.	5th prct.	95th prct.
$100\left(\beta^{-1}-1\right)$	Gamma	1.25	0.05	1.26	1.26	0.04	1.21	1.32
σ	Gamma	1.50	0.10	1.50	1.50	0.08	1.37	1.63
η	Gamma	0.33	0.20	0.19	0.23	0.11	0.05	0.39
γ	Beta	0.75	0.10	0.66	0.66	0.04	0.60	0.73
α	Beta	0.40	0.03	0.32	0.32	0.02	0.29	0.35
$100\delta_0$	Gamma	2.50	0.50	2.46	2.47	0.35	1.92	3.04
δ_2	Gamma	5.00	2.50	4.13	4.36	1.48	2.04	6.82
κ	Gamma	2.50	1.50	2.48	2.90	0.64	1.81	3.90
ϵ_p^*	Gamma	11.00	2.50	7.84	8.61	1.90	5.56	11.82
θ_p	Beta	0.65	0.10	0.68	0.68	0.05	0.60	0.76
ι_p	Beta	0.50	0.15	0.39	0.40	0.11	0.23	0.57
θ_w	Beta	0.65	0.10	0.70	0.70	0.05	0.62	0.77
ι_w	Beta	0.50	0.15	0.42	0.42	0.11	0.24	0.58
ϕ_r	Beta	0.75	0.10	0.76	0.75	0.04	0.69	0.82
ϕ_π	Normal	1.50	0.30	2.12	2.19	0.23	1.81	2.54
ϕ_y	Normal	0.50	0.10	0.02	0.04	0.06	-0.05	0.12
ζ	Beta	0.20	0.05	0.20	0.20	0.03	0.14	0.25
G/Y	Beta	0.20	0.05	0.21	0.21	0.03	0.17	0.25
ρ_a	Beta	0.50	0.15	0.98	0.97	0.01	0.95	0.99
ρ_z	Beta	0.50	0.15	0.37	0.38	0.08	0.23	0.51
ρ_p	Beta	0.50	0.15	0.43	0.45	0.09	0.30	0.60
ρ_w	Beta	0.50	0.15	0.51	0.51	0.12	0.32	0.72
ρ_v	Beta	0.50	0.15	0.15	0.18	0.06	0.09	0.28
ρ_g	Beta	0.50	0.15	0.54	0.54	0.08	0.42	0.66
ρ_s	Beta	0.50	0.15	0.51	0.50	0.11	0.32	0.69
σ_a	Inv.Gamma	0.10	1.00	0.01	0.02	0.00	0.01	0.02
σ_z	Inv.Gamma	0.10	1.00	0.05	0.06	0.01	0.04	0.08
σ_p	Inv.Gamma	0.10	1.00	0.19	0.23	0.10	0.09	0.38
σ_w	Inv.Gamma	0.10	1.00	0.05	0.05	0.03	0.03	0.08
σ_m	Inv.Gamma	0.10	1.00	0.01	0.01	0.00	0.01	0.01
σ_v	Inv.Gamma	0.10	1.00	0.02	0.02	0.00	0.02	0.03
σ_g	Inv.Gamma	0.10	1.00	0.01	0.01	0.00	0.01	0.02
σ_s	Inv.Gamma	0.10	1.00	0.01	0.01	0.00	0.01	0.01
σ_η^y	Inv.Gamma	0.01	0.01	0.01	0.01	0.00	0.00	0.01
σ_η^w	Inv.Gamma	0.01	0.01	0.02	0.02	0.00	0.01	0.02
σ_η^h	Inv.Gamma	0.01	0.01	0.00	0.01	0.00	0.00	0.01
ϵ_w	Fixed	11.00						
R^s	Fixed	1.00						

0.03. The posterior mean for quarterly depreciation rate $100\delta_0$ is estimated to be 2.47, very close to the prior mean. For the capacity utilization adjustment cost, δ_2, the estimated value is 4.36 down from a prior centered around 5. For the investment adjustment cost parameter κ the estimated mean 2.9 is higher than the prior mean 2.5. For the steady-state elasticity of substitution across goods, ϵ_p, I find a posterior mean of 8.6 against a prior mean of 11. The estimated mode for the transformed nominal rigidity parameters θ_p and θ_w is 0.68 and 0.70, respectively, higher but close to the prior mean of 0.65. The posterior mode for the indexation parameters ι_p and ι_w is 0.4 and 0.42, below the prior mean. Regarding the Taylor rule parameters, the estimated values for the response coefficients ϕ_π and ϕ_y is 2.19 and 0.04, respectively. For the smoothness parameter ϕ_r the estimated value is is equal to the prior mean 0.75, but the standard deviation is considerably lower. Finally, steady-state government spending is estimated to be 0.21 while the overall importance of earmarked credit is estimated to be 0.20, as in the prior.

Turning to the exogenous stochastic processes, the TFP shock is estimated to be very persistent, with an autoregressive root of 0.97. The risk-premium shock, on the other hand, is estimated to be very short lived, with an AR root of 0.18. For the other shocks the estimated persistence assumes intermediate values, closer to the prior mean.

Regarding the measurement errors real wages are estimated to be the most poorly measure variable, its noise having a volatility of 0.0164. Then comes employment measurement error, with a volatility of 0.0056, and then finally the output noise with a volatility of 0.0053.

2.3.6 Impulse response functions

In the next section only the IRF for a monetary shock is discussed, given the objectives of this paper. The interested reader may find the IRFFs for the other shocks in appendix B.6.

2.4 Results

I present two set of results. Section 2.4.1 discusses the relationship between earmarked credit and monetary policy power, which is the main focus of the paper. But the model can of course be used to discuss other topics, and section 2.4.2 exemplifies this by discussing how much the presence of earmarked credit boosts investment in the steady-state.

2.4.1 Main application: earmarked credit and monetary policy power

Here I discuss whether the presence of earmarked credit (with subsidized interest rates which are insensitive to the monetary cycle) indeed reduces monetary policy power, as usually claimed, and quantify this effect.

First (section 2.4.1.1) I show that this is not necessarily the case in the model *a priori*, using the prior distribution. For instance, power over inflation may decrease or increase, depending on the parameterization. Using the prior distribution is useful for the purpose of investigating *theoretically possible outcomes* because it is reasonably loose, allowing for a wide range of parameter combinations[23].

[23] Ideally these theoretical possibilities would assessed analytically by finding the closed-form solution for the model. This is impossible for a medium-sized DSGE model, though, so the alternative is to numerically evaluate under different calibrations. Using the prior distribution is just a convenient way of achieving this.

Then (section 2.4.1.2) I sharpen the analysis by considering the estimated posterior distribution. Because it embeds information contained in the data, and because it a tighter distribution, is it more useful for us to assess what is the *likely outcome*. It suggest that monetary policy power is indeed reduced over both aggregate output and inflation, but that these effects are small. Also, it should be noted that the effect over output's responsiveness is stronger then over inflation's, meaning that the *sacrifice-ratio* is lower when earmarked credit is present.

2.4.1.1
Prior distribution: monetary policy power is ***not necessarily*** reduced

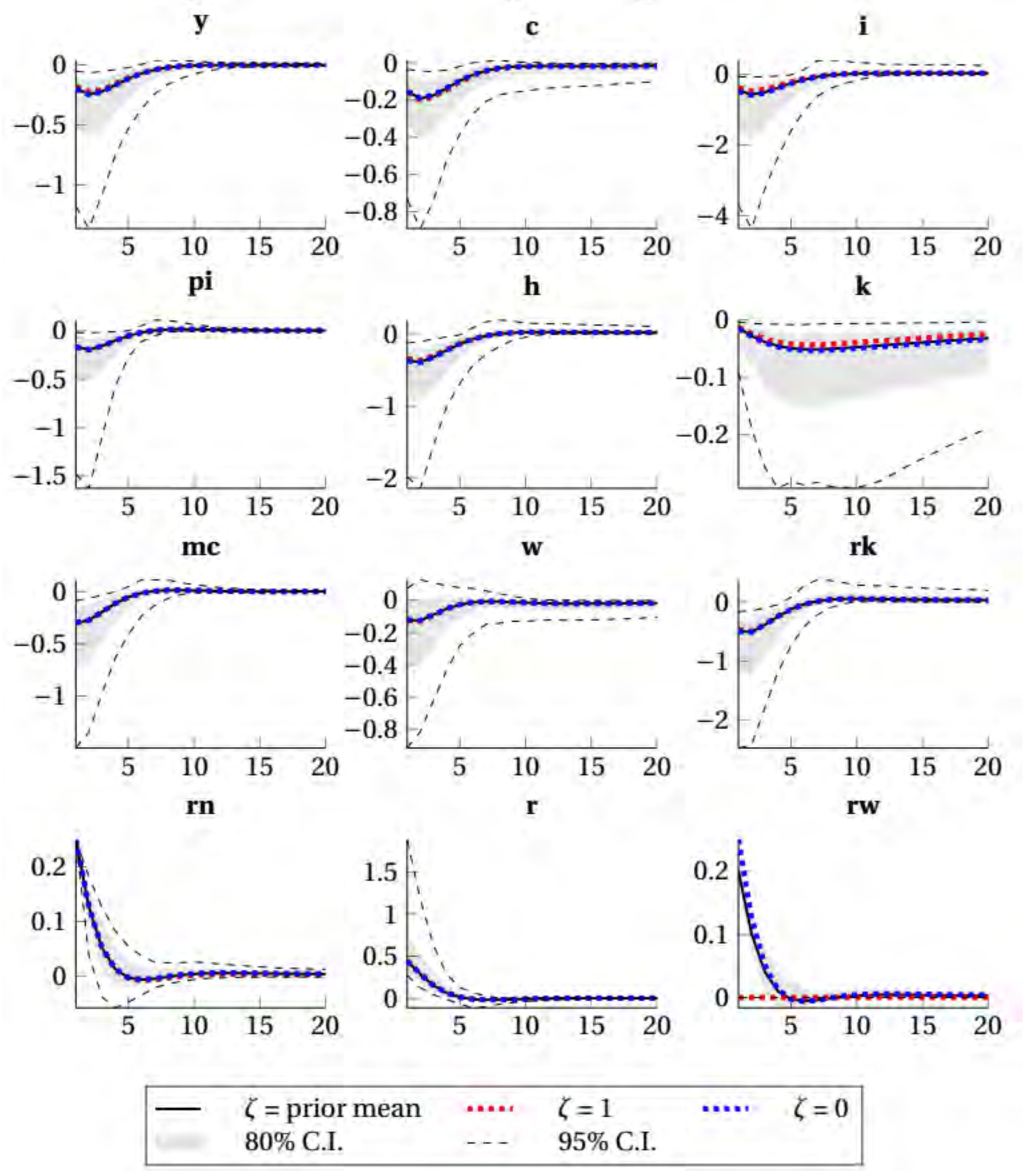
Figure 2.2 shows impulse response functions to a monetary shock that results in a 25b.p. (100b.p., annualized) increase in the nominal interest rate, for selected aggregate variables. It shows an IRF computed for parameters valued at the prior mean (solid black line), together with the 80% (shaded area) and 95% (area comprised by the dashed lines). The results are conventional: output and inflation falls on impact and the peak effect occurs about around 2 quarters after the shock. Consumption and investment falls, the later more than the former. Both real wages and the implicit capital rental rate (the marginal productivity of capital) fall, implying a reduction on real marginal costs which in fact drives inflation's response. The IRFs have these signs even taking into account the huge prior parameter uncertainty.

Figure 2.2 also shows the IRFs compute at the prior mean when the parameters controlling the overall importance of earmarked credit is fixed at the polar cases $\zeta = 0$ (blue dotted line) and $\zeta = 1$ (red dotted line). It is very hard to see any difference between these cases (except for R^w), in part because the range of the y-axis is very large in order to display the 95% credible interval. But also because the differences are themselves small in the first place. To better compare the IRFs across these cases Figure 2.3 plots the difference in the impulse response functions, i.e., *macro effects* in the terminology of section 2.2.2. Actually, for analytical convenience[24], I normalize the difference by taking into account the sign and magnitude of the impulse response function (when $\zeta = 0$) at its peak. More specifically, let t^* be the quarter after the shock where $\mathrm{IRF}_t(\theta; \zeta = 0)$ reaches its maximum absolute value. Then I compute, for a given parameter vector $\boldsymbol{\theta}$ (which excludes ζ):

$$\mathrm{NDIF}_t(\boldsymbol{\theta}) = \frac{\mathrm{IRF}_t(\boldsymbol{\theta}; \zeta = 1) - \mathrm{IRF}_t(\boldsymbol{\theta}; \zeta = 0)}{\mathrm{IRF}_{t^*}(\boldsymbol{\theta}; \zeta = 0)}$$

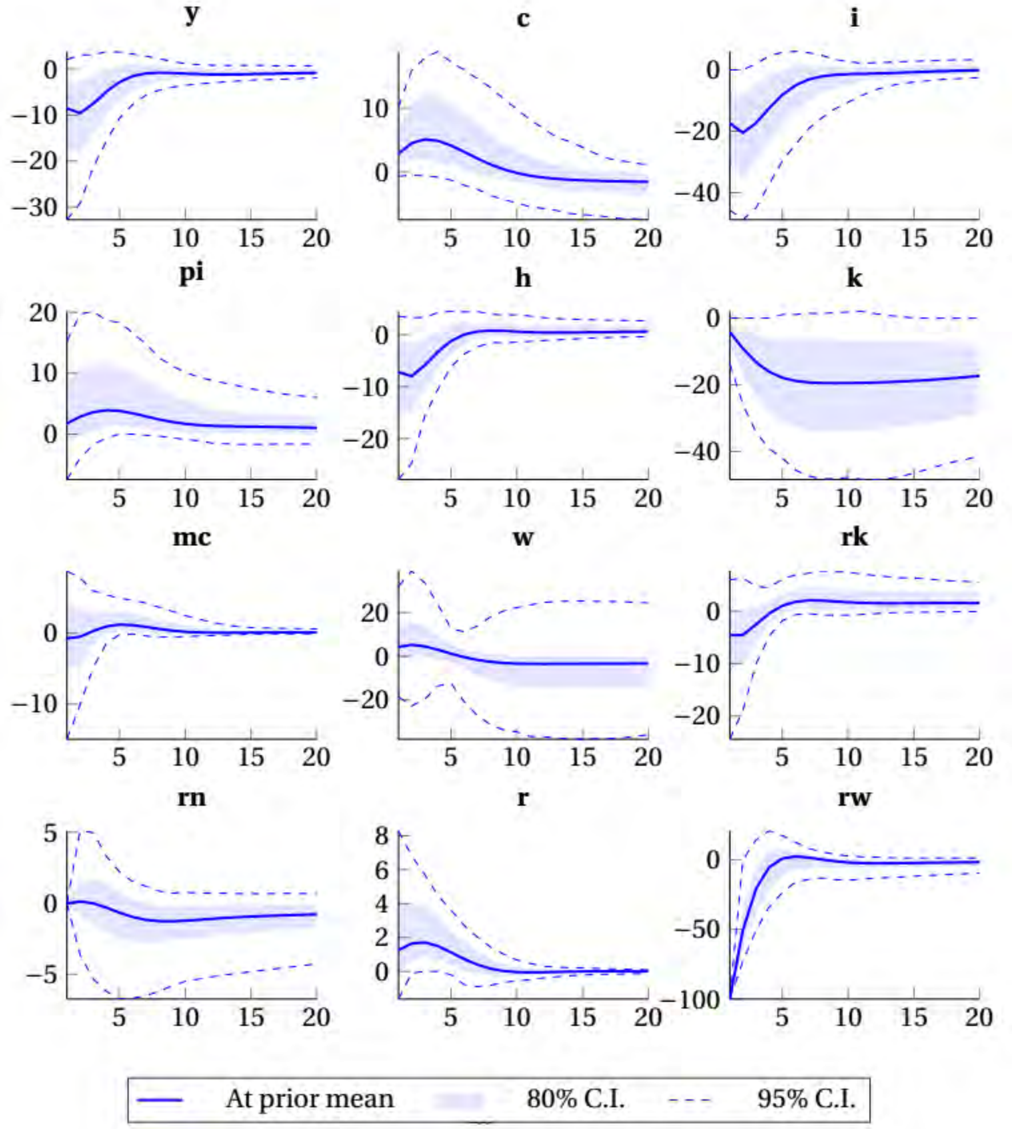
[24] The results does not hinge on the normalization.

Figure 2.2: IRFs to a monetary shock — prior distribution



In Figure 2.3 the line corresponds to the normalized difference computed at the prior mean, while the shaded area and the area comprised by the dashed lines correspond to 80% and 95% credible intervals, respectively. A positive (negative) value means that monetary policy power increased (decreased) over that variable. It is shown that investment's response to a monetary policy shock is indeed reduced when earmarked credit is present, just as we would expect from common sense reasoning. This holds for virtually any parameter vector in the support of the prior distribution, at least for the first few quarters after the shock. What is truly remarkable, thought, is that the same result does not necessarily apply to aggregate output and inflation, as the 95% credible interval shows. While it is true that most of the prior probability mass support the idea that output's responsiveness to a monetary shock falls when earmarked credit

Figure 2.3: Normalized difference in IRFs to a monetary shock — prior distribution



is present, there is some prior probability mass on the opposite relationship. For inflation the result is even more surprising: most of the prior probability mass support the idea that inflation becomes *more* responsive when earmarked credit is present.

Why is it theoretically possible for output and inflation macro effects to go both ways? To shed some light into this question I decompose the macro effects into micro and external effects, in the spirit of section 2.2.2[25]. These effects are normalized by both the sign and magnitude of the peak impulse response of the respective *aggregate variable*, consistent with Figure 2.3. Normalized micro and external effects still add up to the normalized macro

[25] I do not actually compute the macro effect at the margin, as in that section, but by considering a discrete change from $\zeta_j = 0$ to $\zeta, \zeta_j = 1$. Appendix A.4 details how it is done.

effect because I also use the same scale for all. Due to the sign normalization, negative values contribute to (in the case of micro and external effects) or outright denote (for macro effect) a decrease in the responsiveness of the aggregate variable to a monetary shock, when earmarked credit is present. Of course, this procedure is only meaningful for variables which are firm specific, and here I focus on investment, output and inflation.

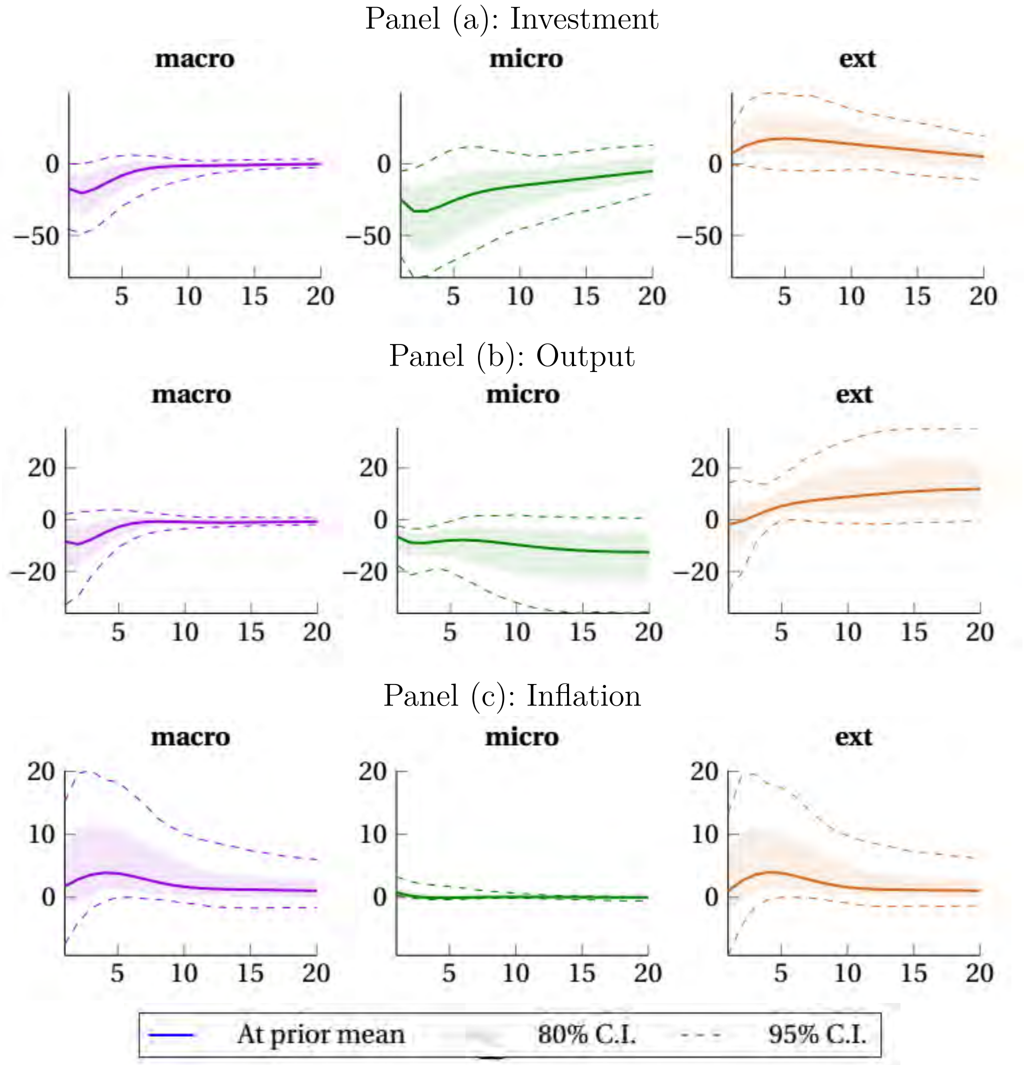
Figure 2.4 shows the decomposition result. The micro effects all have the expected sign: investment and output decisions of a individual firm are less sensitive to monetary conditions when the firm has higher access to earmarked credit. The nominal price change chosen by these firms always become more sensitive, however, because by disinvesting less they end up with a lower production costs in comparison to their competitors[26]. Turning to the external effects, the figure shows that following a contractionary monetary shock a firm cuts investment by more when *other firms* have more access to earmarked credit. But the effect over its output and pricing decisions are not as clearcut. By the nature of this external effect this must be the result of complex general equilibrium forces, which might be very strong and dominate the more well-behaved micro effect.

For aggregate output one general equilibrium force that may lead to a increased response is related to consumption. As figure 2.3 shows, most of the prior distribution probability mass points to that aggregate consumption falls *more* when earmarked credit is present. Fist, this might be related to a direct crowding-out of consumption for investment in the aggregate demand. Not only that, if inflation falls more and the nominal interest rate path is somewhat unchanged, then the real interest rate will increase by more and thus give support for the bigger fall in consumption. For inflation, there are both demand and supply forces at play and they have different implications. The *demand force* is straightforward: the fall in aggregate demand for goods leads to a fall in aggregate demand for labor which, in turn, puts downward pressure on real wages and, hence, real marginal costs and inflation. Because aggregate demand falls less when earmarked credit is present this force is associated with a smaller decrease in inflation. This demand mechanism is the one economists have in mind when arguing that the presence of earmarked credit decreases monetary policy power over inflation. But there are also supply forces at play, and they cannot be dismissed. In particular, we should take into account labor supply's response. When aggregate consumption falls the labor supply schedule shifts outward, as households try to smooth their consumption path,

[26] In fact, the demand condition for a firm's goods makes clear how micro effects for the relative price and for output must have opposite signs.

putting a downward pressure on real wages. If consumption's responsiveness rises when earmarked credit is present, it means that real wage's and, hence, inflation's responsiveness must also rise. Interestingly, note how inflation's and consumption's responsiveness support each other.

Figure 2.4: Normalized macro, micro and external effects — prior distribution

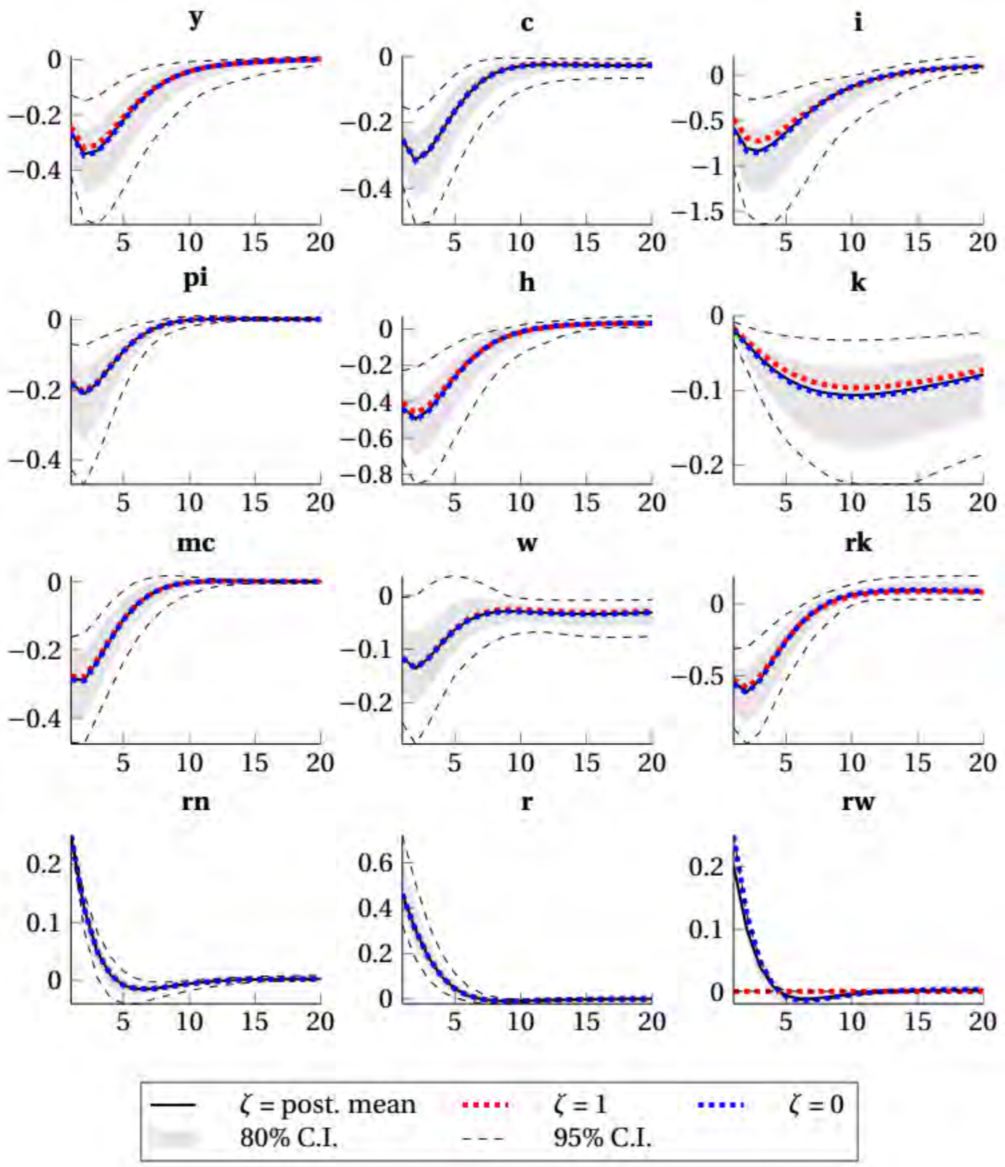


2.4.1.2
Posterior distribution: support for the hypothesis of power reduction, but...

I now redo the analysis considering the posterior distribution, which allows for a more precise assessment of the *likely* relationship between monetary policy power and the presence of earmarked credit, among the theoretical possibilities.

Again I start by showing (Figure 2.5) the impulse response functions for a monetary shock that raises the nominal rate by 25 b.p.. The overall picture stays the same, the IRFs displaying the expected qualitative result, but

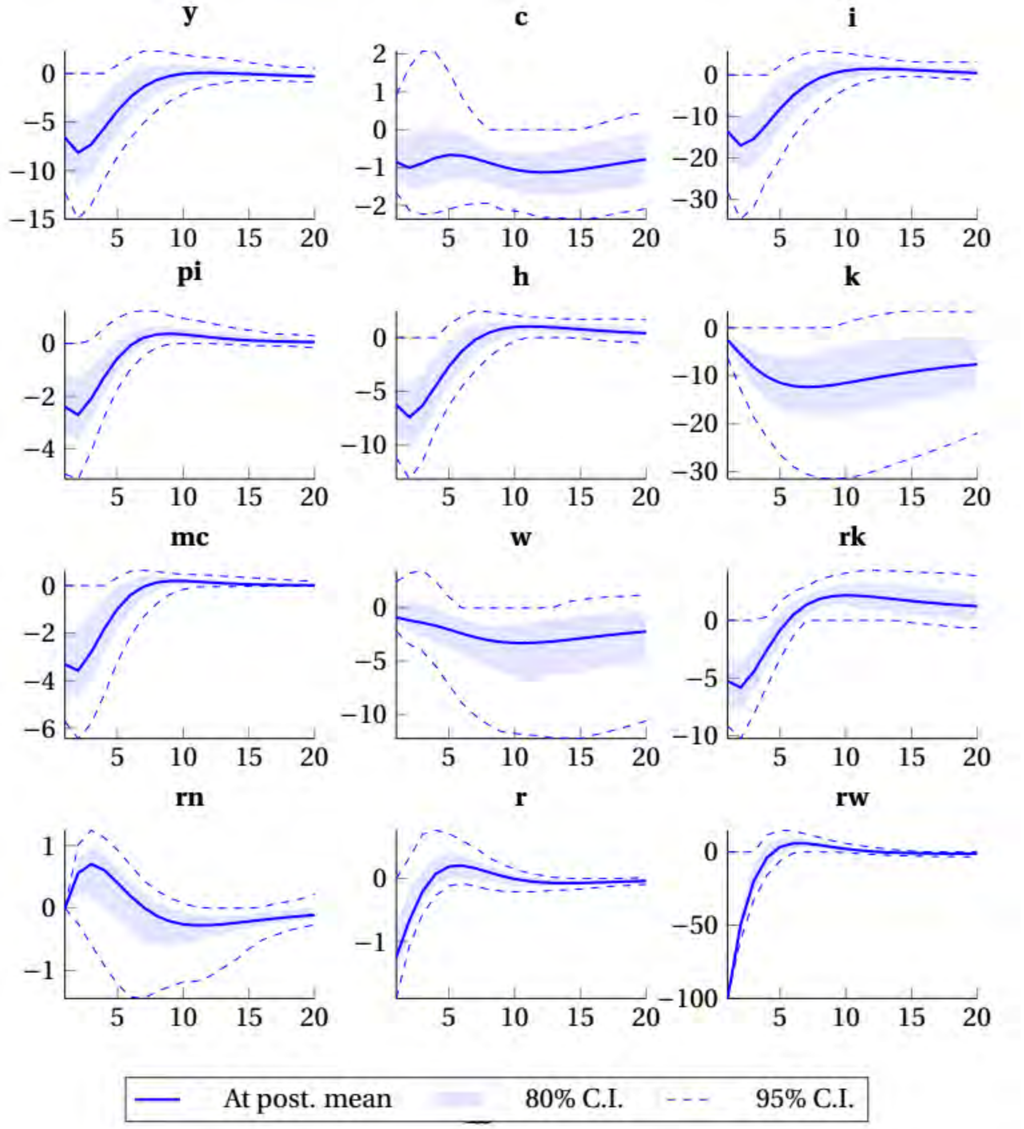
Figure 2.5: IRFs to a monetary shock — posterior distribution



the credible intervals are now much tighter. Looking at the IRFs computed at posterior mean we see that following a contractionary monetary shock aggregate output falls 0.34% below the steady-state value two quarters after the shock, consumption and investment falling 0.32% and 0.81%, respectively. Inflation falls 0.21 p.p..

Cutting to the main point, Figure 2.6 shows that the posterior distribution of normalized macro effects is also tighter. The solid line displays the normalized difference in IRFs computed at the posterior mean, the shaded area corresponds to a 80% credible interval while the are comprised by dashed lines denotes the 95% credible interval. For most variables the sign of the macro effect is well determined for short horizons, and they have the expected sign. In particular: now we do not see any probability mass for an increase in output's and inflation's responsiveness to monetary shocks when earmarked credit present.

Figure 2.6: Normalized difference in IRFs to a monetary shock — posterior distribution



For aggregate investment the normalized difference computed at the posterior mean is of -17.0% at the second quarter, indicating a significant fall in monetary policy's influence. When earmarked credit is absent investment falls 0.84% after two quarters, but only 0.69% when all investment is financed by the government. The credible 80% and 95% credible intervals are (-19,2%, -8.4%) and (-28.2%, 0%), respectively. For the aggregate capital stock the peak effect (-12.4% at the posterior mean) takes more periods to be achieved (around 7 quarters), because capital stock differentials is slowly accumulated by investment differentials through time.

For aggregate output, on the other hand, normalized difference at the posterior mean is -8.2% at the second quarter, the 80% and 95% credible intervals being (-9.5%, -4.0%) and (-12%, 0%), respectively. It also should be noted that output's baseline impulse response itself is of a smaller magnitude than investment's, so the absolute change is even smaller: a 0.35% fall when earmarked credit is absent becomes a 0.32% fall when it completely present. Hence, even though monetary policy looses traction over aggregate output, the magnitude of this effect is not as significant. This is related to the fact that investment corresponds to approximately 20% of aggregate demand. Also, it is not as clear how is aggregate consumption's response to a monetary shock affected. We still can not rule that consumption becomes more responsive when earmarked credit is present — the 95% credible interval being (-1.7%, 0.9%) on the first period — but the posterior probability mass does not favor this hypothesis as much as the prior distribution did. The response of aggregate employment is also less pronounced when earmarked credit is present, the effect resembling the one observed for aggregate output. In fact, because capital is very slow moving most there is a big correlation between output and employment, conditional on a monetary shock.

Real wage's responsiveness to a monetary shock may either increase or decrease when earmarked credit is present on the first few periods: the 95% credible interval is (-2.2%, 2,5%). There are two forces at play. On one hand, demand for labor falls less when earmarked credit is present because it is closely linked to the aggregate demand for goods (through firms' production function), and this is a force in the direction of reducing monetary policy power. On the other hand, households' labor supply schedule may shift downwards more or less, depending on consumption's responsiveness. If consumption falls even more then the downward shift will be bigger, this being associated with a larger real wage fall. On the medium run the real wage becomes unambiguously (given the posterior credible interval) less responsive, along with aggregate consumption.

The normalized difference on the IRFs of the marginal productivity of capital (R^k) is negative in the first few horizons and positive later on. This is due to the interplay between aggregate demand, which increases the marginal productivity of capital, and the capital stock, which decreases it. Both are positively affected by investment, but investments' effect on the aggregate demand is shorter-lived than its effect on the capital stock, which depreciates slowly. Weighting the effects over the real wage and the marginal productivity of capital, we find that the fall in the real marginal production cost is smaller when earmarked credit is present, at least in the first few periods. It may become more responsive after eight quarters following the real marginal production cost, but the estimative for this effect is small.

Inflation's responsiveness to a monetary shock also becomes weaker in the presence of earmarked credit, under the posterior distribution. That monetary policy power over inflation is reduced comes at no surprise given the previous discussion on the effect over real marginal costs — since inflation, as in most New Keynesian models, is mostly a discounted sum of future expected real marginal costs[27]. The peak effect is of -2.7% and occurs in the second quarter, if computed at the posterior mean. This magnitude is very small: inflation's IRFs in the second quarter are -0.212% and -0.206% when $\zeta = 0$ and $\zeta = 1$, respectively, barely distinguishable. The 80% and 95% credible intervals are respectively (-3.4%, -1.2%) and (-5.0%, 0%).

Figure 2.7 shows the decomposition of the (normalized) macro effects into micro and external effects, for investment, output and inflation. Appendix B.7 shows the decomposition for other firm-specific variables. Note the the plots for the micro effects are qualitatively similar to the ones obtained with the prior distribution, but are now tighter. The same is also true for investment's and output's external effects. For inflation's external effect we now see a very different distribution, though: it now completely assumes negative values (at least on the first few periods), whereas the prior put most of the probability mass on positive values. This change is associated with the change in output's and, hence, labor supply's responsiveness, as already discussed.

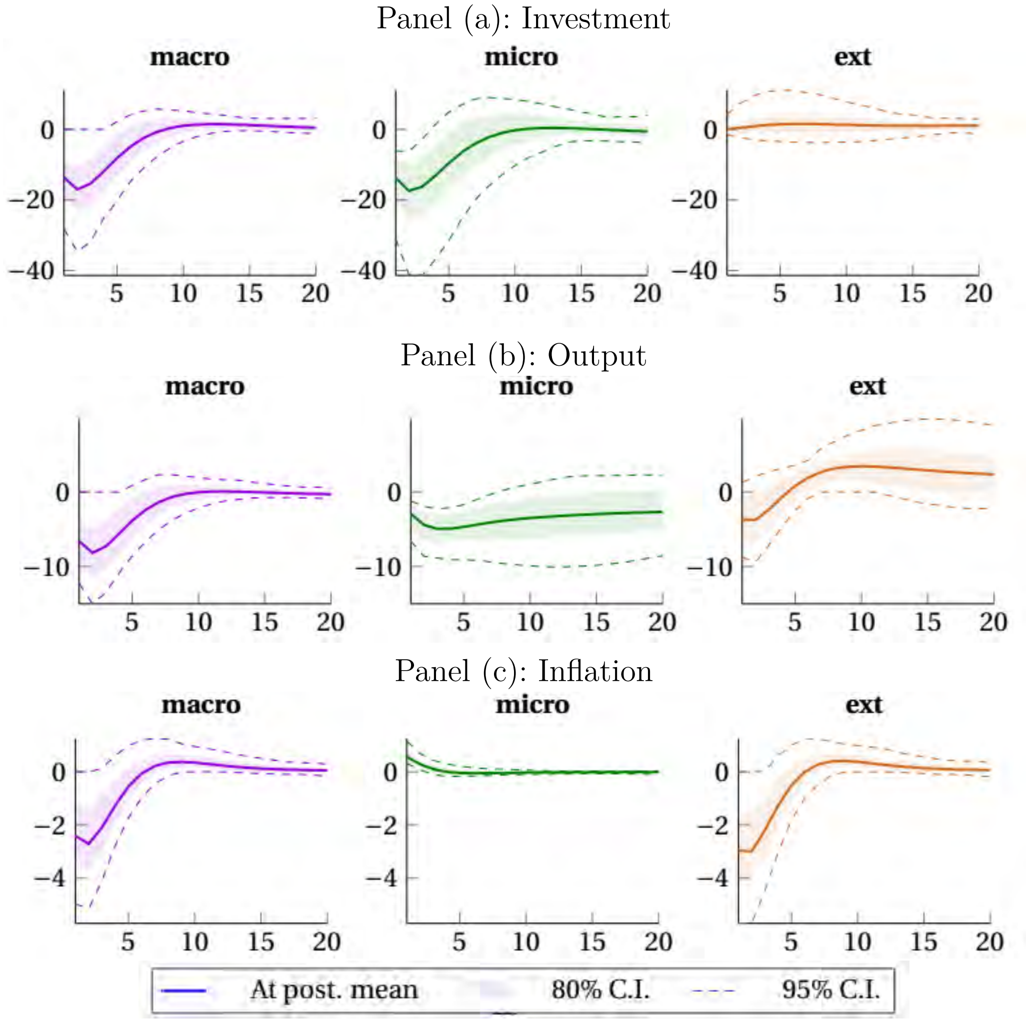
Hence, through the lens of the model the data seems to support the hypothesis that monetary policy power is reduced in the presence of earmarked credit, for variables policy mostly cares about: output and inflation. Indeed this is what we find not only considering the posterior mean, but also the whole posterior distribution.

This finding does not necessarily means that the Central Bank's job becomes harder, however. In fact it may actually be a good notice for the Central Bank, provided the power loss is bigger for output than for inflation. In this case, to achieve a given (short-term) fall in inflation the economy pays a lower cost in terms of output loss — in the profession' jargon, the economy faces a lower *sacrifice ratio*[28]. Figure 2.8 shows that this seems to be the case. Panel (a) shows the cumulative output and inflation loss and the sacrifice

[27] As can be seen by iterating forward the log-linearized Phillips curve A.40, in the appendix.

[28] This is actually a modification of the original concept, which is usually applied in the context of disinflations to mean the cumulative output loss, during a transition period, that is necessary in order for a 1p.p. *permanent fall* in inflation to happen. Because I am not studying a disinflation episode, I adapt the concept and measure the cumulative output loss necessary to bring a 1% drop to the price level. The same adaptation of the concept can be found in (38).

Figure 2.7: Normalized macro, micro and external effects — posterior distribution

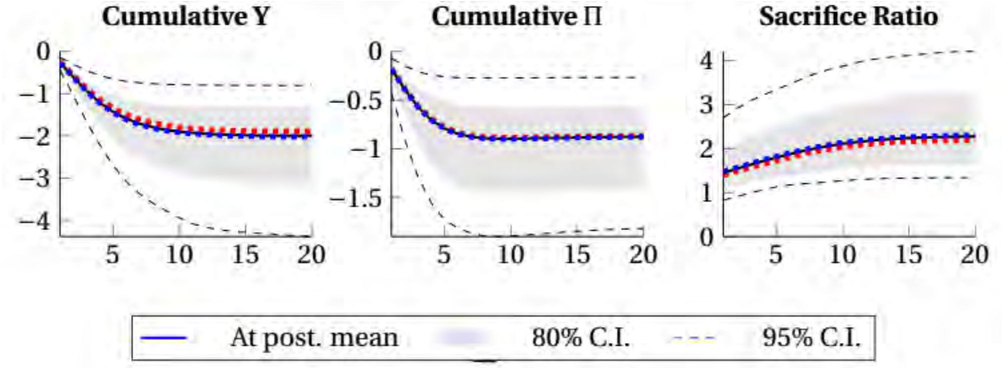


ratio, computed by dividing the former by the later, for both cases: $\zeta = 0$ and $\zeta = 1$. Panel (b) shows the normalized differences computed at the posterior mean. After 20 quarters the cumulated output loss amounts to 2% of output's steady-state when $\zeta = 0$, and to 1.9% when $\zeta = 1$, at the posterior mean, for a normalized difference of -6.4%. The cumulated reduction in inflation is -0.88% and -0.86%, respectively, for a normalized difference of -1.6%. At the same horizon, the sacrifice ratio computed at the posterior mean is 2.3 when earmarked credit is absent, and 2.2 when it is fully present: a reduction of -4.9%. The 80% and 95% credible intervals for the change in the percent change in the sacrifice ratio are respectively (-8.6%, -1.5%) and (-15.5%, 0.3%).

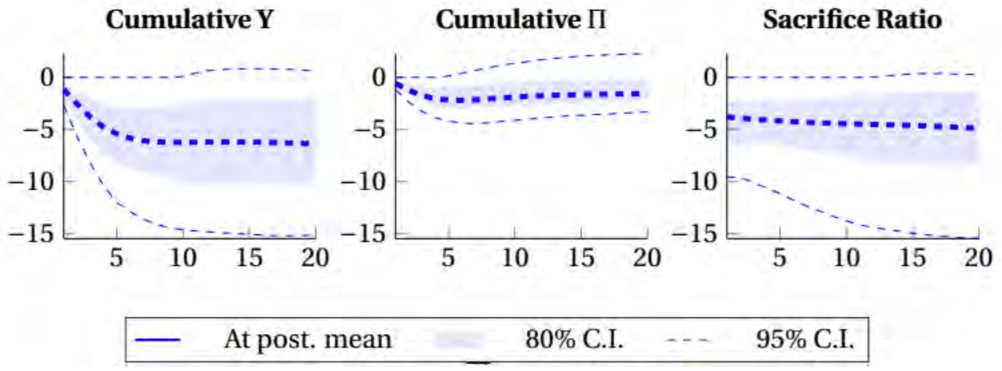
Finally, I can not stress enough that all the estimated effects are relatively small, even considering a huge policy change of moving of a world without earmarked credit ($\zeta = 0$) to a world where all investment is financed by the government ($\zeta = 1$). Considering that the estimated posterior mean for ζ is 0.20, we should not expect a noticeable change in monetary policy power by

Figure 2.8: Earmarked credit and the sacrifice ratio

Panel (a): Cumulative output, cumulative inflation and sacrifice ratio



Panel (b): Normalized difference



linking the credit policy rate to the policy rate.

2.4.2 Another application: earmarked credit and the steady-state investment

The model was mainly built with the purpose of studying the relationship between monetary policy power and the presence of earmarked credit with subsidized interest rates which are insensitive to the monetary cycle. But the model can also be used to cast some light on other topics. Here I discuss how effectively the presence of this subsidized credit increases aggregate investment in the steady-state.

Tables 2.2 and 2.3 show, in its first row, the steady-state values for output investment, computed for parameters valued at their posterior mean, under the two extreme scenarios for earmarked involvement in the credit market: when all investment is market financed ($\zeta = 0$, first column); and when all investment is government financed ($\zeta = 1$, second column). Also, it includes steady-state values for for these same variables at the firm level, for zero-measure firms without any access to government credit ($\zeta_j = 0$, second row) and fully financed by the government ($\zeta_j = 1$, third row). With those I decompose the aggregate variation into micro and external effects.

Moving from an economy without government credit to an economy

Table 2.2: Mean output steady-state

	$\zeta = 0$	$\zeta = 1$	$\Delta(\zeta)$
Agg.	1.0000	1.0075	0.0075
Firm($\zeta_j = 0$)	1.0000	0.9736	-0.0264
Firm($\zeta_j = 1$)	1.0348	1.0075	-0.0273
Δ(Firm)	0.0348	0.0339	-

Table 2.3: Mean investment steady-state

	$\zeta = 0$	$\zeta = 1$	$\Delta(\zeta)$
Agg.	0.1864	0.1901	0.0038
Firm($\zeta_j = 0$)	0.1864	0.1822	-0.0042
Firm($\zeta_j = 1$)	0.1945	0.1901	-0.0044
Δ(Firm)	0.0081	0.0080	-

where all investment is subsidized increases the mean steady-state output level by 0.75%, only. Investment raises by 2%, and the investment rate by 0,23 p.p. (from 18.64% to 18.87%). This is a very modest increase, considering the fact that government support for investment, measured in terms of output, raises from 0% to 19%.

We observe, hence, a big crowding out effect. Cheaper earmarked credit not only finances projects that would not be financed in its absence, but also finances projects that would be anyway financed at market rates anyway. A simple calculation based on the results above points to a crowding out of roughly 98% — 1 – (0.1901 – 0.1864)/0.1901. The remaining 2% represents projects that became viable because the real interest rate firms face when investment is reduced from approximately 5% to 0% *per annum*.

Such big crowding-out effect arises in the model because it is implicitly assumed that the government is unable or unwilling to sort out projects that are viable at market rates from those which are not[29]. This assumption does not seems unreasonable in the light of recent empirical findings (e.g., (23) and (24)) pointing out that subsidized credit is preferentially accessed by large and profitable firms, having little effect investment in spite of reducing recipients' financial expenses. In fact, despite the modeling assumption, firm-level investment seems to be more positively affected by the subsidy in the model than these papers report: I find an average micro effect of 4.3% (the average of the last row, divided by 0.1864). earmarked credit effect on investment as measured at the firm level is bigger than as measure at the aggregate level because the average external effect is negative, -2.3% (the

[29] Remember the modeling device: the government commits itself to finance a fixed fraction ζ_j of firm's j total investments.

average of the last column for firms' rows, divided by 0.1864)[30]. The reason this negative sign is straightforward: when competitors have more access to subsidized credit a firm will have a harder time competing and will lose market-share. The optimal response for the firm in this setting is to cut back investment. For output micro and external effects are given by 3.4% and -2.7% and the same logic applies.

2.5 Conclusion

This paper investigates the relationship between monetary policy power and the presence of earmarked credit with subsidized interest rates which are insensitive to monetary policy. Differently form previous macroeconomic models on this issue, which rely on a cost channel in order to introduce earmarked credit to firms, in this model such credit is channeled to firms' investment. This makes the model more useful for the recent policy discussion on the TJLP, now substituted by the TLP, since BNDES's main activity is investment financing.

I show that in the model the presence of such type of earmarked credit does not necessarily decrease the power of monetary policy over macroeconomic variables we mostly care about, such as aggregate output and inflation. For some parameter values it does, but there are others for which it does not. This is related to general equilibrium effects which are the resultant of forces in different directions. On one hand, the more muted response of aggregate investment implies a less responsive aggregate demand and a less responsive inflation. On the other hand, one should note that over time the milder investment response translates into a more muted capital stock response, which operates through a supply channel. Even though the capital stock moves slowly, inflation's forward-looking nature contributes to bring this effect to the present to some extent. Finally, it is also important to take into account the fact that general equilibrium effects over the labor supply can go both ways.

By estimating the model with Brazilian data, however, I recover a posterior distribution for the parameters that allows for a sharper analysis. The estimated posterior distribution supports the hypothesis that monetary policy power is reduced when earmarked credit is present, but the estimated effect is very small. Moreover, power over aggregate output is reduced more than the power over aggregate inflation, implying that the sacrifice ratio (i.e., the output loss necessary to induce a given fall in inflation) is actually smaller when earmarked credit is present. Hence, it is not obvious whether it really

[30] Note how the average micro and external effects add up to the macro effect of 2%.

creates a handicap for the monetary authority to pursue its stabilization goals, and even if it does, the handicap seems to be small.

Finally I also use the model to understand how steady-state investment is affected by the presence of earmarked credit. I show that massively increasing subsidized government credit's share leads only to a very modest increase in investment. While some projects do become viable with the subsidy, the majority of funds go to projects that would be viable anyway, even without subsidies.

3
Capital inflow shocks: do their type matter

Do capital inflow effects on a small open economy's business cycles depend on the inflow type — for instance, whether it is bond or equity flow, liability or asset flow? If so, how important are these differences? I build a medium-sized, small-open economy DSGE model with financial frictions to investigate these questions. I identify mechanisms that makes different types of inflows to have different effects, emphasizing the role of the portfolio-balance channel. But this heterogeneity in effects are quantitatively small when the model is calibrated with reasonable values.

Keywords: Capital flows; business cycles; portfolio-balance channel; sterilized interventions.

JEL Classification: E44, E52, F31, F32, F41

3.1
Introduction

"*Are capital inflows expansionary or contractionary?*", ask (39). They remark that such simple question has not yet been settled and that, in fact, two opposite views on this issue are quite common. The first emphasizes the exchange rate appreciation that follows capital inflows and predicts contraction of aggregate demand through net exports, in line with Mundell-Flemming. The second, most popular among emerging market policymakers, emphasizes that capital inflows may cause credit booms, overheating, and may even end up in a sudden-stop episode.

(39) attempt to reconcile these views with a simple explanation, that capital flows are not all alike: some may be expansionary, others may be contractionary. And, they argue, the *type* of the inflow (for instance, whether it is foreign direct investment, equity portfolio, bond portfolio, bank loans) may have something to do with these differences. To support this view they provide some econometric evidence along with a very simple model of asset returns that features a portfolio-balance channel. The mechanisms is that imperfect substitutability across assets implies that demand shocks for different assets

require different equilibrium asset prices (i.e., returns) to clear the markets, with allocative implications for the real economy. However, their model is static and of partial equilibrium (only the asset market is modeled), and one may wonder how well the results hold in fully-fledged general equilibrium model. This paper aim to fill this gap.

I take a standard small open economy, medium-sized DSGE model (with habits formation in consumption, investment adjustment costs, nominal price rigidity, price indexation and local currency pricing)[1] and add financial frictions following (40)'s approach. I allow intermediaries to trade both stock (claims over the economy's capital stock) and bonds, domestic and foreign. The friction plagues both domestic and external financial intermediation, allowing for time-varying spreads between the expected returns of stocks and bonds (equity-premium), and of domestic and foreign assets (UIP failure). Following (41), I model capital inflows through noisy-trading shocks, i.e., an exogenous demand by some agents. This is particularly convenient here because of the goal is differentiating between *capital flow types*. The considered demand shocks differ by the asset being purchased — stocks or bonds, domestic or foreign — and the identity of buyer — whether a domestic or a foreign household. With this I can generate *gross inflows*[2] and *gross outflows*[3] for both stocks and bonds.

I show that looking at the Jacobian of the model with respect to the shocks is sufficient for answering whether the different inflow shocks are isomorphic to each other (i.e., if they have the same effects) and for understanding the related mechanisms. Because the analysis does not depend on the Jacobian of the model with respect to the endogenous variables, the answer to this question is somewhat general, robust to many details of the model. I identify three *direct mechanisms* associated with the capital inflows, two of them (*static balance of payments mechanism* and *capital demand mechanism*) similar to those appearing in (39) and another that arises in my model (*dynamic balance of payments mechanism*), but not in theirs, because mine is a dynamic model. I show how these mechanisms imply that shocks are indeed not isomorphic to one another in general, provided that a portfolio-balance channel is operative on global financial intermediation.

That these inflow shocks are not isomorphic to each other does not

[1] The reason for building upon a medium-sized model, instead of a simpler vanilla model, is because these models are considered more realistic — in the sense that they better fit the data. This is useful for a more quantitative discussion.

[2] Also know as *liability flows*, when a foreigner buys a domestic asset. This can be negative, meaning that the foreigner is selling a domestic asset. Note that a negative gross inflow is a capital outflow.

[3] Also known as *asset flows*, when a resident buys a foreign asset. This can be negative, when the resident sells its foreign asset and return the proceeds home. Note that a negative gross outflow is a capital inflow.

mean these differences are quantitatively important, however. In fact, for a reasonable calibration of the model it turns out that the different inflow shocks look much similar one to another. I show that the strength of the portfolio-channel in domestic financial intermediation is key to understand the size of the difference between stock and bond inflow shocks, as it is related to the capacity of the domestic financial sector to channel foreign resources to the most valuable use.

Finally, I also use the model and the analytical framework to study three other questions. First, I show that the analyzing the consequences of gross outflow shocks is more complicated than the analysis of gross inflow shocks, as it required information not only on the foreign asset being purchased or sold, but also on the funding of such operation. Second, I show that in the model usual FX interventions by the Central Bank are mechanically equivalent to gross bond outflows, but not to other flow types, raising questions about the capacity of this policy instrument to stabilize the economy following certain inflow episodes. Third, I briefly compare my main results with those that would be obtained had I followed the more conventional approach of modeling capital inflows, through changes in foreign assets returns. Around the appropriate steady-state for the comparison (one without noisy-trading) the results are the same.

Literature review. On the empirical front there are quite a few papers that *distinguish between inflow types* in order to understand how capital inflows affect the economy. (42) study how different types of net capital flows affect the real effective exchange rate (REER), finding evidence that the appreciation of the REER is higher with portfolio flows than with FDI and bank flows. They argue that this is the case because portfolio flows are more volatile and less directed to investment. (43) focus on the effects of gross inflows (non-residents induced) on the REER, and find that the evidence that inflows appreciate the REER is less clear cut in the case of FDI than in the case of other inflows. (39) look at the effect of gross inflows on output growth and find the non-bond inflows (which includes FDI, portfolio equity and bank loans) are expansionary while the effect of bond inflows are statistically insignificant. (44) study whether the type and source of the inflow matters for macroeconomic imbalances. As measures of macro imbalances, they look at REER overvaluation, output growth and output gap. They find that asset and liability flows are similar to one another, but that FDI flows are associated with a less appreciated REER and to higher output growth/gap, in comparison to portfolio and other flows. I would argue that some caution is needed when interpreting this body of empirical evidence, however. Capital inflows are clearly endogenous to the outcomes variables, and it the different empirical strategies used in the literature to overcome this problem do not undoubtedly address this problem.

On the theoretical front not much can be found regarding how different types of inflow shocks may have different consequences for the business cycle. As already mentioned, there is (39) static, partial equilibrium model, which I build upon. Perhaps the closest to my paper is (45): also motivated by (39), they build a DSGE model where exogenous cuts in both borrowing and lending foreign interest rates lead to contractionary inflows, while cuts only in the lending rate (i.e., a decrease in the spread) lead to expansionary inflows.

Also close to my paper is the recent literature on sterilized interventions — for instance, (46), (47), (48), (41), (49). By including frictions in agents' portfolio choice problems this literature features a portfolio-balance channel[4]. and thus allows sterilized interventions to have an effect in general equilibrium models, thus overturning (50) irrelevance result. I show that in my model sterilized interventions are not mechanically different from domestic noisy-traders demand for foreign bonds. This literature studies what happens when there is an inflow shock (due to changes in foreign interest rates or because of a noisy-trading shock) and whether and how should the government respond with sterilized interventions. But these models only consider the existence of one type of inflow. In principle, it is possible that the intervention prescription may depend on the inflow type, if their effects are heterogeneous. For instance, in my model FX interventions can fully offset a foreign demand shock for domestic bonds, but not a foreign demand shock for domestic stocks.

Guideline. The model is presented in section 3.3. Section 3.2 presents the main analytical tool applied to the model, which is presented in section 3.3. Results are presented in section 3.4, and section 3.5 concludes.

3.2 Analyzing the (non) isomorphism of shocks

This paper investigates whether and how the effects of capital inflows shocks depend on the type of the inflow. In this section I present the main tool I later employ for such analysis.

Throughout this section let $\boldsymbol{y}$ be the vector of relevant endogenous variables and $\boldsymbol{x}$ the vector of exogenous variables. The endogenous variables must be relevant, as will become clearer ahead. Also, let $\boldsymbol{g}(\boldsymbol{y}, \boldsymbol{x}) = \boldsymbol{0}$ represent the set of equilibrium conditions of the model. Assume that $\boldsymbol{g}(\boldsymbol{y}, \boldsymbol{x}) = \boldsymbol{0}$ is

[4] How channel is included in the model varies, the existence of financial frictions being one possibility. It is also common to include adjustment cost on the representative agent's portfolio-choice problem.

differentiable and that an equilibrium exists and is unique. Hence, there is a function such that $\boldsymbol{y} = \boldsymbol{f}\big(\boldsymbol{x}\big)$.

Definition: Two shocks (say, x_1 and x_2) are isomorphic if and only if

$$\left.\frac{\partial \boldsymbol{y}}{\partial x_1}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} = \lambda \left.\frac{\partial \boldsymbol{y}}{\partial x_2}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} \qquad \forall \text{ valid } \left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right) \tag{3-1}$$

i.e., iff both shocks affect the endogenous variables of the model in the same way, up to a normalization coefficient λ which assures that the analysis does not depend on the scale of the variables.[5]

This definition of isomorphism is intuitive, but requires us to first find the solution of the model. It turns out that equilibrium unicity allows for a more straightforward way of checking whether the shocks are isomorphic.

Proposition: Shocks are isomorphic if and only if

$$\left.\frac{\partial \boldsymbol{g}}{\partial x_1}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} = \lambda \left.\frac{\partial \boldsymbol{g}}{\partial x_2}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} \qquad \forall \text{ valid } \left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right) \tag{3-2}$$

Proof: This is only an application of the implicit function and inverse function theorems. Existence and uniqueness of the equilibrium implies that the model $\boldsymbol{g}(\boldsymbol{y},\boldsymbol{x}) = \boldsymbol{0}$ is a one-to-one map from the space of shocks to the space of endogenous variables and as such the matrix $\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}$ has full rank. Hence:

$$\begin{aligned}
&\left.\frac{\partial \boldsymbol{y}}{\partial x_1}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} = \lambda \left.\frac{\partial \boldsymbol{y}}{\partial x_2}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} \\
&\quad\Longleftrightarrow \left.\left(-\left[\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\right]^{-1}\frac{\partial \boldsymbol{g}}{\partial x_1}\right)\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} = \lambda \left.\left(-\left[\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\right]^{-1}\frac{\partial \boldsymbol{g}}{\partial x_2}\right)\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} \\
&\quad\Longleftrightarrow \left.\frac{\partial \boldsymbol{g}}{\partial x_1}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)} = \lambda \left.\frac{\partial \boldsymbol{g}}{\partial x_2}\right|_{\left(\bar{\boldsymbol{y}},\bar{\boldsymbol{x}}\right)}
\end{aligned}$$

[5]For example, to assure the a \$1000 million dollar gross bond inflow to have the same impact as a \$1 billion dollar gross bond inflow. Also, shocks can be positive or negative, and if one shock is defined to be the negative of the other we still want to classify them as isomorphic.

3.2.1
A few technicalities

What if the implicit function theorem can not be used? In this case the simpler condition (3-2) ceases to be sufficient for isomorphism, but continues to be necessary. To see this, not that if shocks are isomorphic (as defined in as defined in (3-1)) then we can write

$$\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}\left(\frac{\partial \boldsymbol{y}}{\partial x_1}-\lambda\frac{\partial \boldsymbol{y}}{\partial x_2}\right)=-\left(\frac{\partial \boldsymbol{g}}{\partial x_1}-\lambda\frac{\partial \boldsymbol{g}}{\partial x_2}\right)$$

The inapplicability of the implicit function theorem manifests itself through the rank-deficiency of the matrix $\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}$. If $\left(\frac{\partial \boldsymbol{y}}{\partial x_1}-\lambda\frac{\partial \boldsymbol{y}}{\partial x_2}\right)\neq 0$ is in the null-space of $\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}$ we may have $\left(\frac{\partial \boldsymbol{g}}{\partial x_1}-\lambda\frac{\partial \boldsymbol{g}}{\partial x_2}\right)=0$. But $\left(\frac{\partial \boldsymbol{y}}{\partial x_1}-\lambda\frac{\partial \boldsymbol{y}}{\partial x_2}\right)=0$ does imply $\left(\frac{\partial \boldsymbol{g}}{\partial x_1}-\lambda\frac{\partial \boldsymbol{g}}{\partial x_2}\right)=0$.

Relevancy of endogenous variables. Note that in the definition of shocks' isomorphism I have emphasized that endogenous variables in $\boldsymbol{y}$ must be relevant. To understand why, consider an example.

Suppose x_1 and x_2 are isomorphic in the basic model $\boldsymbol{g}(\boldsymbol{y},\boldsymbol{x})=\boldsymbol{0}$, and now extend the model to include a new variable y_{new} and a new equation, $g_{\text{new}}(y_{\text{new}},x_1)=0$. In the extended model the shocks are not isomorphic anymore, since y_{new} is affected by x_1 but not by x_2. Of course, the basic model and the extended one are identical in terms of economic substance, the difference being only that the later include an irrelevant additional variable and equation.

A question that naturally arises, then, is how to determine whether a variable is relevant. My take here is that this must be determined *a priori* by the researcher, based on the phenomena he is interested in. It is possible that not all the endogenous variables in the model are relevant for the analysis, even in the case where they all have economic meaning. The best example I can come up with — and one that also applies to the model to be presented in section 3.3 — is that of understanding the effects of lump-sum taxation shocks in a model where Ricardian equivalence holds. In particular, consider taxing today (τ_t) or tomorrow (τ_{t+1}). It is well known that which is chosen doesn't really matter for output, consumption, etc, and one may be tempted to say that these taxation shocks are isomorphic. But this is true only if governmental debt is excluded from the vevctor of endogenous variables, because the taxation path do have implications for the debt path.

Linear models. Condition (3-2) looks for isomorphism in the non-linear model of the economy, but the researcher may be interested in a first order approximations of the model around a steady-state. This is the case, for instance, if the goal is to understand the impulse response responses as generated by log-linearized models. The condition for isomorphism in this case is easier to be satisfied as now the elements in the model's Jacobian matrix would not be time-varying. At most they would depend on the steady-state values of endogenous and exogenous variables. One thing to have in mind is that it is possible for the isomorphism analysis for the non-linear model and for the linear model to differ, whenever differences between the effects of the shocks are of second order.

3.2.2
More substantial significance

Robustness to model details. Condition (3-2) is not only more computationally convenient than condition (3-1), but it also is much more insightful: it means that the results of the isomorphism analysis do not depends on the Jacobian of the model with respect to endogenous variables, $\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}$. It does not matter how the endogenous variables interact with each other in general equilibrium, but only on how the exogenous variables shock the system. Hence, the analysis so far is very general an is robust to many changes one can make in the model $\boldsymbol{g}(\boldsymbol{y}, \boldsymbol{x})$. In fact, the conclusion may be the same for two different models, provided they have essentially the same Jacobian with respect to the shocks. Many of the results discuss in section 3.4 are, thus, robust to other settings.

Understanding mechanisms. The Jacobian of the model with respect to the shocks also allows us to understand why isomorphism is present or not. Rows of the Jacobian represent equations of the model, while columns represent shocks. By comparing the rows we can check which equation generates non-isomorphism and, thus, understand the forces at play.

(Insufficiency for) Quantitative assessments. Checking the Jacobian of the model with respect to the shocks allows us to answer a binary question: "*Are the shocks isomorphic, yes or no?*". It the case the shocks are not isomorphic it does not answer *how much different their effects are*. For this we have to solve the model and check both $\frac{\partial \boldsymbol{g}}{\partial x_1}$ and $\frac{\partial \boldsymbol{g}}{\partial x_2}$. Such an assessment depends not only on $\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}$ but also on $\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{y}}$. General equilibrium effects may operate to magnify or mitigate the effects of the shocks, and to making them more alike or more different. Thus, for a quantitative assessment the details of the model are very important. However, because $\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}$ still affects the analysis it also provide information for understanding quantitative results.

3.3
Model

This is a small open economy, New-Keynesian medium-sized model. In addition to the usual nominal price rigidity which defines the New-Keynesian literature, and the financial frictions that give rise to the portfolio balance channel which is central to the analysis, I include in the model the following set of frictions: (i) habit formation in consumption; (ii) convex investment adjustment costs; (iii) price indexation; and (iv) local currency pricing.[6] As discussed in the last section, much of these features are only really important for a more quantitative discussion and do not change other qualitative results.

Importantly, the model is consist with the main features found in (39)'s static and partial equilibrium model. For a very brief exposition of their model, see appendix C.1.

3.3.1
Description

The world. There is a unit-mass continuum of countries, indexed by i. The *home* (or domestic) economy ($i = H$), the one we focus on, is just one of them. Countries are all symmetrical to each other, and the structure of the home economy is representative of others'. I aggregate all other countries in an entity called *rest of the world* (ROW), and make reference to variables like "world output", "world price level", "world bonds", etc. Such variables should be understood as the appropriate aggregations of all other countries' respective variables[7]. To give concreteness to the writing I will name "BRL" the currency of the home economy, and "USD" the world currency.

[6] At first my intention was to just include investment adjustment costs, in order to allow for variation in the real-price of capital (*Tobin's-Q*). But then I realized that in such a setting consumption becomes much more volatile than investment, explaining almost all output variation. A fix would also be to include consumption habits, which is common DSGE models. But then net exports starts to explain most of the variation in output. This is because, in a simple producer currency pricing setting where the law of one price holds, exchange rate changes are rapidly transmitted to prices, affecting import and export volumes. A fix for that would be to use local currency pricing, which leads to imperfect exchange rate pass-through and more smooth net-export responses. In the end I concluded that I should either have none of these frictions, or all of them. I chose the later to give a more quantitative grip to the model, but chose not to include other features (except for the price indexation, which is more standard.)

[7] See (51) for how it is done (e.g. aggregate bilateral exchange rates into an effective interest rate of the domestic currency against the world currency).

Households. Households maximize

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\ln \left(C_t - \psi C_{t-1} \right) - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \right\}$$

subject to the following nominal budget constraint:

$$\begin{aligned} P_t C_t + D_t^n + F_{S,t}^n + \mathcal{E}_t^n F_{B^*,t}^n + \mathcal{E}_t^n F_{S^*,t}^n &= P_t W_t L_t + R_{t-1}^n D_{t-1}^n \\ &+ R_t^{ns} F_{S,t-1}^n + \mathcal{E}_t^n R_{t-1}^{n*} F_{B^*,t-1}^n + \mathcal{E}_t^n R_t^{ns*} F_{S^*,t-1}^n + P_t \Omega_t + P_t T_t \end{aligned}$$

First of all, some words on notation: C_t denotes households' consumption of the final good, whose unit price is P_t. The amount of labor supplied is denoted by L_t, and real wages by W_t. Ω_t represents real profits earned by households on their non-negotiable ownership of firms, while T_t represents real lump-sum net-transfers from the government. R_{t-1}^n and R_t^{ns} are ex-post nominal BRL returns of domestic bond and stocks[8], respectively, while starred-variables R_{t-1}^{n*} and R_t^{ns*} denote ex-post nominal USD returns of the corresponding foreign assets. The nominal BRL/USD exchange rate is denoted by $\mathcal{E}_t^n$ — hence, an increase in $\mathcal{E}_t^n$ means depreciation of the home currency. D_t^n denote 1-period BRL nominal bonds, $F_{S,t}^n$ the nominal BRL value of domestic stocks held by the households, $F_{B^*,t}^n$ and $F_{S^*,t}^n$ the nominal USD values of foreign bonds and stocks held, respectively.

A very important assumption I make in this model is that households can freely trade nominal 1-period domestic bonds, but that their demand for other types of assets (local stocks and foreign assets) is exogenous[9], thus allowing for *noisy-trading* shocks. This is a convenient way of modeling exogenous capital flows. After all, capital flows are in fact endogenous variables that respond to domestic (pull) and external (push) factors, a feature that complicates empirical analysis of their effects. Because this is a theoretical work, however, I am able to directly impose exogenous variation in capital flows. Also, this approach is particularly useful in this paper given its goal of understanding the effects of different types of inflows. This noisy-trading approach was also pursed by (41), who argues that these shocks "*...can be thought of as an inelastic demand coming from noise family members*", or "*motivated as liquidity shocks or the result of time-varying portfolio constraints*".

I have presented the budget constraint in terms of nominal flows in just

[8] In the case of bonds note that the ex-post nominal return is equal to the ex-ante nominal return, and that this is captured by the time-index, which captures the time period where the value of the random variable is realized.

[9] The case where this exogenous demand is equal to zero is the typical case of a segmented market, which is common in models with financial frictions. For instance, in (52) households are allowed to trade bonds, but not capital.

to emphasize that the assets in the model are nominal. But as a matter of personal preference I rather derive the model in terms of real variables. The real-flows budget constraint, in terms of the domestic final good, is given by:

$$C_t + D_t + F_{S,t} + \mathcal{E}_t F_{B^*,t} + \mathcal{E}_t F_{S^*,t} = W_t L_t + R_t D_{t-1} + R_t^s F_{S,t-1} + \mathcal{E}_t R_t^* F_{B^*,t-1} + \mathcal{E}_t R_t^{s*} F_{S^*,t-1} + \Omega_t + T_t$$

where a variables without the n superscript is the real counterpart of the respective nominal variable. In particular, we have the ex-post real interest rate

$$R_t = \frac{R_{t-1}^n}{\Pi_t} \tag{3-3}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is inflation measured in terms of the final good. $\mathcal{E}_t = \mathcal{E}_t^n \frac{P_t^*}{P_t}$ is the real exchange rate; $F_{S,t} = \frac{F_{S,t}^n}{P_t}$ denotes households real stock holdings; $F_{B^*,t} = \frac{F_{B^*,t}^n}{P_t^*}$ and $F_{S^*,t} = \frac{F_{S^*,t}^n}{P_t^*}$ denote households real holdings of foreign assets in terms of the foreign price level.

Because of the asset trading constraints, households' optimization program has only C_t, L_t and D_t as choice variables. The first order conditions can be rearranged as:

$$\Xi_t = \beta \left(\frac{C_t - \psi C_{t-1}}{C_{t-1} - \psi C_{t-2}} \right)^{-1} \tag{3-4}$$

$$W_t = \chi \Big(C_t - \psi C_{t-1} \Big) L_t^{\varphi} \tag{3-5}$$

$$\mathbb{E}_t \left\{ \Xi_{t+1} R_{t+1} \right\} = 1 \tag{3-6}$$

Particular attention should be given to equation (3-6), *because* it is just an ordinary non-arbitrage condition found in frictionless models of macro-finance. This is to remark that in this model the market for domestic bonds is frictionless, at least from the point of view of the domestic household. This feature should always be taken into account when analyzing this paper's results. Also, note that similar pricing conditions for the other assets (domestic stocks and foreign assets) are not present, since the representative household is not allowed to optimally trade in those markets. For instance, if the household was allowed to trade foreign bonds we would be able to recover the usual non-arbitrage condition $\mathbb{E}_t \left\{ \Xi_{t+1} \Big(R_{t+1} - \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} R_{t+1}^* \Big) \right\}$, i.e., the uncovered interest parity condition that allows for a risk premium related to the covariance of the excess return with the household's stochastic discount factor. Instead, the model features condition (3-39), which comes from intermediaries portfolio problem, and such equilibrium condition is a proper downward sloping demand schedule, not a non-arbitrage condition.

Final, domestic and imported goods. The final good utilized in the home economy is produced by combining domestically produced inputs (Z_H) with imported ones (Z_F). The transformation function is:

$$Y_t = \frac{Z_{F,t}^{\gamma} Z_{H,t}^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}$$

where γ measures the relative importance of the imported input and can be interpreted as the degree of commercial openness. Equivalently, $1-\gamma$ measure home bias in consumption. Final good producers operate in a perfectly competitive market. The optimal use of inputs imply the following conditional demands:

$$Z_{F,t} = \gamma \left(p_{F,t}\right)^{-1} Y_t \tag{3-7}$$

$$Z_{H,t} = (1-\gamma) \left(p_{H,t}\right)^{-1} Y_t \tag{3-8}$$

where $p_{H,t} = \frac{P_{H,t}}{P_t}$ and $p_{F,t} = \frac{P_{F,t}}{P_t}$ are respectively the relative prices of home and foreign inputs, in terms of the domestic final good[10]. There is free-entry in the market for final goods and these producers have zero-profit in equilibrium. Hence their prices satisfy:

$$1 = p_{F,t}^{\gamma} p_{H,t}^{1-\gamma} \tag{3-9}$$

Domestically produced inputs are used both in the production of the domestic final goods (as above) and (by symmetry) in the production of foreign final goods, for which they are exported. Its total supply is given by $Y_{H,t}$, and is given by a constant elasticity of substitution (CES) composite of a continuum of input varieties, index by j:

$$Y_{H,t} = \left(\int_0^1 Y_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j\right)^{\frac{\epsilon}{\epsilon-1}}$$

Firms who aggregate these inputs into the composite basket also operate in a perfectly competitive market. Cost minimization imply the following

[10] A word on notation: nominal prices are representative by capital P's while relative prices are represented by lowercase p's.

conditional demand curve for each variety:

$$Y_{H,j,t} = \left(\frac{P_{H,j,t}}{P_{H,t}}\right)^{-\epsilon} Y_{H,t}$$

Zero-profit condition imply that the nominal price for domestic inputs must satisfy:

$$P_{H,t} = \left(\int_0^1 P_{H,t}^{1-\epsilon}(j)\mathrm{d}j\right)^{\frac{1}{1-\epsilon}}$$

Also, imported inputs are themselves a CES composite basket of a variety of imported goods. One source of variety if the fact that the rest of the world in comprised of a continuum of small open economies, as in (51), and that the good of each country is an imperfect substitute to each other. I assume that the foreign composite (i.e. the "world basket") is given by:

$$Z_{F,t} = \left(\int_0^1 Z_{F,t}(i)^{\frac{\epsilon_f - 1}{\epsilon_f}} \mathrm{d}j\right)^{\frac{\epsilon_f}{\epsilon_f - 1}}$$

As before, this implies the following conditional demand for a given variety:

$$Z_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\epsilon_f} Z_F$$

a condition which, vy symmetry, will be important when analyzing home exports.

Domestic input variety producers. In each economy there is a continuum of monopolistically competitive firms, indexed by j, each producing a different input variety with the following technology:

$$Y_{H,j,t} = K_{jt}^{\alpha} L_{jt}^{1-\alpha}$$

Cost minimization problem, for a given desired production level Y, is:

$$\begin{aligned} \min_{K_{jt},L_{jt}} \quad & W_t L_{jt} + R_t^r K_{jt} \\ \text{s.a.} \quad & K_{jt}^{\alpha} L_{jt}^{1-\alpha} = Y \end{aligned}$$

where W_t and R_t^r are the real wages and capital rental rate, respectively. The Lagrange multiplier of this problem, μ_t, is the real marginal cost of production.

First order conditions are:

$$R_t^r = \mu_t \alpha \frac{Y}{K_{jt}}$$
$$W_t = \mu_t (1-\alpha) \frac{Y}{L_{jt}}$$
$$\mu_t = \left(\frac{R_t^r}{\alpha}\right)^{\alpha} \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \tag{3-10}$$

Note that the real marginal cost (in terms of the numeráire P_t) is equal across firms. Total real costs are $\text{Cost}_{jt} = \mu_t Y_{jt}$ (there are no fixed-costs), and real profit is $\Omega_{j,t} = (p_{j,t} - \mu_t)\, Y_{H,j,t}$.

These firms are subject to nominal price rigidity *a la* Calvo. In each period only a fraction θ of them, randomly chosen, are allowed to optimally reset their prices. Those not allowed to do so only apply a mechanical adjustment index X_t that takes into account past domestic inputs price inflation (PPI):

$$X_t = \left(\Pi_{H,t-1}\right)^{\iota} \tag{3-11}$$

The parameter ι captures the importance of price indexation mechanism. When allowed to reset its price a domestic input firm solves the following problem:

$$\max_{\hat{p}_t} \qquad \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \theta^s \Xi_{t,t+s} \left(\hat{p}_t \frac{X_{t,t+s}}{\Pi_{t,t+s}} - \mu_{t+s} \right) \left(\frac{\hat{p}_t}{p_{H,t+s}} \frac{X_{t,t+s}}{\Pi_{t,t+s}} \right)^{-\varepsilon} Y_{H,t+s} \right\}$$

where $\Xi_{t,t+s}$ and $\Pi_{t,t+s}$ are, respectively, the real stochastic discount factor and realized inflation between t e $t+s$. The choice variable is the relative (to the numéraire) reset price $\hat{p}_t$. Optimality requires:

$$\hat{p}_t = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \theta^s \Xi_{t,t+s} \left(p_{H,t+s}\right)^{\varepsilon} \left(\Pi_{t,t+s} / X_{t,t+s}\right)^{\varepsilon} Y_{H,t+s}\, \mu_{t+s} \right\}}{\mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \theta^s \Xi_{t,t+s} \left(p_{H,t+s}\right)^{\varepsilon} \left(\Pi_{t,t+s} / X_{t,t+s}\right)^{\varepsilon - 1} Y_{H,t+s} \right\}}$$

a condition that can be replaced by the following set of recursive equations:

$$\hat{p}_t = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{u_{1,t}}{u_{2,t}} \tag{3-12}$$
$$u_{1,t} = \left(p_{H,t}\right)^{\varepsilon} Y_{H,t}\, \mu_t + \theta \mathbb{E}_t \left\{ \Xi_{t+1} \left(\Pi_{t+1} / X_{t+1}\right)^{\varepsilon} u_{t+1} \right\} \tag{3-13}$$
$$u_{2,t} = \left(p_{H,t}\right)^{\varepsilon} Y_{H,t} + \theta \mathbb{E}_t \left\{ \Xi_{t+1} \left(\Pi_{t+1} / X_{t+1}\right)^{\varepsilon - 1} u_{2,t+1} \right\} \tag{3-14}$$

Producers price index $p_{H,t}$ evolves accordingly to:

$$p_{H,t} = \left((1-\theta)\hat{p}_t^{1-\varepsilon} + \theta \left(p_{H,t-1} \frac{X_t}{\Pi_t} \right)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}} \tag{3-15}$$

Aggregate real profit of these firms is given by:

$$\Omega_{p,t} = \Big[p_{H,t} - \emptyset_t \mu_t \Big] Y_{H,t}$$

where $\emptyset_t = \int_0^1 \left(\frac{P_{j,t}}{P_{H,t}} \right)^{-\varepsilon} \mathrm{d}j$, a measure of price dispersion, follows:

$$\emptyset_t = (1-\theta) \left(\frac{\hat{p}_t}{p_{H,t}} \right)^{-\varepsilon} + \theta \left(\frac{\Pi_{H,t}}{X_t} \right)^{\varepsilon} \emptyset_{t-1} \tag{3-16}$$

and $\Pi_{H,t}$ is inflation measured by the producer price index (PPI):

$$\Pi_{H,t} = \left(\frac{p_{H,t}}{p_{H,t-1}} \right) \Pi_t \tag{3-17}$$

Imports. I allow for incomplete exchange-rate pass-through and short run deviations of the law of one price, by assuming *local currency pricing*. There is a continuum of importers, each with the monopoly over imports of a given foreign country. Importers are also subject to Calvo-type nominal rigidities, in such a way that their price-setting problem is analogous to the one of domestic variety producers. The differences lie in the conditional demand they face (see subsection 3.3); in the indexation rule they follow, which is tied to past *foreign goods* inflation:

$$X_{F,t} = \Big(\Pi_{F,t} \Big)^{\iota_f} \quad ; \tag{3-18}$$

and in their real marginal cost:

$$\mu_{F,t} = (1-\tau)\mathcal{E}_t p^*_{F,t} \tag{3-19}$$

Two comments about this marginal cost. First, note that in principle it should be linked to the producer price index in the exporting country i, $p^*_{F,t}(i)$, and not on the world's producer price index, $p^*_{F,t}$. Linking it to the world price is not a problem for all the purposes of this paper, though, because I assume that the rest of the world is in steady-state and, hence, that $p^*_{F,t}(i) = p^*_{F,t}$ for all i. Second, note that I have introduced a subsidy τ on importers marginal cost (in relation to the price they would pay had the law of one price held), a subsidy that is paid by the exporting country and set as $(1-\tau)=\frac{\epsilon_f-1}{\epsilon_f}$, for convenience[11].

Now, following the steps of the last subsection, the following set of equilibrium conditions can be derived:

$$\hat{p}_{F,t}=\left(\frac{\epsilon_f}{\epsilon_f-1}\right)\frac{u_{F,1,t}}{u_{F,2,t}} \tag{3-20}$$

$$u_{F,1,t}=\left(p_{F,t}\right)^{\epsilon_f}Z_{F,t}\,\mu_{F,t}+\theta_f\mathbb{E}_t\left\{\Xi_{t+1}\left(\Pi_{t+1}\big/X_{F,t+1}\right)^{\epsilon_f}u_{F,t+1}\right\} \tag{3-21}$$

$$u_{F,2,t}=\left(p_{F,t}\right)^{\epsilon_f}Z_{F,t}+\theta_f\mathbb{E}_t\left\{\Xi_{t+1}\left(\Pi_{t+1}\big/X_{F,t+1}\right)^{\epsilon_f-1}u_{F,2,t+1}\right\} \tag{3-22}$$

The price index of imported goods evolves accordingly to:

$$p_{F,t}=\left((1-\theta_f)\hat{p}_{F,t}^{1-\epsilon_f}+\theta_f\left(p_{F,t-1}\frac{X_{F,t}}{\Pi_t}\right)^{1-\epsilon_f}\right)^{\frac{1}{1-\epsilon_f}} \tag{3-23}$$

Aggregate real profit of importers is given by:

$$\Omega_{F,t}=\left[p_{F,t}-\emptyset_{F,t}\mu_{F,t}\right]Z_{F,t}$$

where $\emptyset_{F,t}=\int_0^1\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\epsilon_f}\mathrm{d}i$, a measure of price dispersion, follows:

$$\emptyset_{F,t}=(1-\theta_f)\left(\frac{\hat{p}_{F,t}}{p_{F,t}}\right)^{-\epsilon_f}+\theta\left(\frac{\Pi_{F,t}}{X_{F,t}}\right)^{\epsilon_f}\emptyset_{F,t-1} \tag{3-24}$$

and $\Pi_{F,t}$ is imported goods inflation:

$$\Pi_{F,t}=\left(\frac{p_{F,t}}{p_{F,t-1}}\right)\Pi_t \tag{3-25}$$

Finally, note that by setting $\theta_f=0$ one recovers a version of the model where the law of one price holds in the short-run as well.

Exports. For symmetry there are firms in each of the other countries responsible for importing goods from the home economy. The problem of these firms are analogous to the one in the last subsection, and again I fast forward the derivation. Parameterization is the same. Conditional demand for exports

[11] This subsidy plays no important role in the analysis, except that it allows for the law of one price to hold in the steady-state, and for us to recover the more traditional producer currency pricing, perfect exchange-rate pass-through model, by setting $\theta_f=0$.

of our domestic economy is given by[12]:

$$Z_H^* = \left(p_{H,t}^*\right)^{-\epsilon_f}\left(\gamma Y^*\right) \tag{3-26}$$

The real marginal cost (in terms of the domestic good) is given by:

$$\mu_{H,t}^* = (1-\tau)p_{H,t} \tag{3-27}$$

and, hence, the real marginal cost is terms of the foreign good is $\frac{1}{\mathcal{E}_t}\mu_{H,t}^*$. As before, these firms face Calvo-style nominal rigidity and when not allowed to optimally reset their prices are adjusted by an indexed that takes into account past inflation of similar goods around the world:

$$X_{H,t}^* = \left(\Pi_{H,t}^*\right)^{\iota_f} \tag{3-28}$$

The optimality condition for firms reseting their price is given by the following set of equations:

$$\hat{p}_{H,t}^* = \left(\frac{\epsilon_f}{\epsilon_f - 1}\right)\frac{u_{H,1,t}^*}{u_{H,2,t}^*} \tag{3-29}$$

$$u_{H,1,t}^* = \left(p_{H,t}^*\right)^{\epsilon_f} Z_{H,t}^* \frac{\mu_{H,t}^*}{\mathcal{E}_t} + \beta\theta_f \mathbb{E}_t\left\{\left(1\big/X_{H,t+1}^*\right)^{\epsilon_f} u_{H,1,t+1}^*\right\} \tag{3-30}$$

$$u_{H,2,t}^* = \left(p_{H,t}^*\right)^{\epsilon_f} Z_{H,t}^* + \beta\theta_f \mathbb{E}_t\left\{\left(1\big/X_{H,t+1}^*\right)^{\epsilon_f - 1} u_{H,2,t+1}^*\right\} \tag{3-31}$$

Note that I have already imposed the fact that the rest of the world is in steady-state, so that the stochastic discount factor is β and the inflation is equal to one. Imported goods price index evolves accordingly to:

$$p_{H,t}^* = \left((1-\theta_f)\left(\hat{p}_{H,t}^*\right)^{1-\epsilon_f} + \theta_f\left(p_{H,t-1}^* X_{H,t}^*\right)^{1-\epsilon_f}\right)^{\frac{1}{1-\epsilon_f}} \tag{3-32}$$

where $\Pi_{H,t}^*$ denotes the average inflation of home goods in the rest of the world and is given by:

$$\Pi_{H,t}^* = \left(\frac{p_{H,t}^*}{p_{H,t-1}^*}\right) \tag{3-33}$$

[12] The $\left(p_{H,t}^*\right)^{-\epsilon_f}$ comes from the demand for goods of our domestic economy, conditional on total demand for imports. The (γY_t^*) comes from the demand for imports in the rest of the world (and the fact that it is in a symmetric steady state where the index of imported prices is equal to the index of domestic prices).

Capital good producers. There is a representative firm that operates a technology that transforms the final good into capital goods. The technology is represented by:

$$K_{\text{new},t} = I_t \left(1 - f\left(\frac{I_t}{I_{t-1}}\right)\right)$$

where I_t (investment) is the quantity of final goods utilized as inputs and $K_{\text{new},t}$ is the quantity of newly produced capital. The function $f(\cdot)$ captures convex investment adjustment costs and is characterized by $f(1) = f'(1) = 0$ e $f''(1) = \kappa > 0$. [13]

Capital is traded in a competitive market with the price $P_t Q_t$; thus, Q_t denotes its relative price in terms of the final good. Period t real cash flows for capital producers is given by:

$$\Omega_{k,t} = Q_t I_t \left(1 - f\left(\frac{I_t}{I_{t-1}}\right)\right) - I_t$$

and is transfered to the owners, households, in the same period. Capital producers investment decisions are taken to maximize the discounted value of its cash-flow:

$$\max_{\{I_{t+s}\}} \quad \sum_{s=t}^{\infty} \mathbb{E}_t \left\{\Xi_{t,t+s} \Omega_{k,t+s}\right\}$$

The first order condition to this problem is given by:

$$Q_t \left[1 - f\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}} f'\left(\frac{I_t}{I_{t-1}}\right)\right] + \mathbb{E}_t \left\{\Xi_{t+1} Q_{t+1} \left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right)\right\} = 1 \tag{3-34}$$

Total amount of capital in the economy evolves according to the following law of motion:

$$K_t = (1-\delta) K_{t-1} + I_t \left[1 - f\left(\frac{I_t}{I_{t-1}}\right)\right] \tag{3-35}$$

Stocks. Stocks are negotiable claims to the economy's physical capital stock. When a foreigner buys a stock from the home economy he becomes

[13]More specifically, I assume the following functional form:

$$f(x) = \frac{1}{2}\left\{\exp\left[\sqrt{\kappa}(x-1)\right] + \exp\left[-\sqrt{\kappa}(x-1)\right] - 2\right\}$$

In a linear approximation any function with the characteristics given in the text yields the same results.

entitled to the stream of income that the corresponding units of capital produce in the home economy. In this sense, such a transaction is equivalent to a foreign direct investment or to a portfolio equity flow.

Let S_t be the real value of the issued stocks (market capitalization). This value is backed by the number of units of physical capital available and by its real price:

$$Q_t K_t = S_t \tag{3-36}$$

Capital is rented to input producing firms at the real rate R^r_{t+1} and depreciates at the rate δ. *Ex-post* real return per unity of physical capital is given by

$$R^s_t = \frac{R^r_t + Q_t(1-\delta)}{Q_{t-1}} \tag{3-37}$$

Clearly, this is also the real return received by a stock investor.

Financial intermediaries. In this model I assume the existence of three types of intermediaries:

1. **Domestic intermediary:** arbitrage returns between domestic bonds and stocks. It is the main holder of domestic economy's stocks, and performs a function that is similar to the one financial intermediaries perform on the (8).

2. **Global fixed-income intermediary:** arbitrage between domestic and foreign bonds — as the one in (41).

3. **Global variable-income intermediary:** arbitrage between domestic and foreign stocks.

Variables specific to these intermediaries are indexed by $i = \{1, 2, 3\}$, according to the numeration above.

Intermediation frictions are introduced following (40)'s framework, with minor adaptations[14]. The problem faced by each of these intermediaries is very similar, and I first present a general formulation of the problem that applies to each of them. For completeness I then briefly present each intermediary.

[14] In (40) Γ is not a parameter, but a function of the exchange rate conditional volatility: $\Gamma = \gamma_0 \operatorname{var}_t (\mathcal{E}_{t+1})^{\gamma_1}$, with $\gamma_0, \gamma_1 \geq 0$. If $\gamma_1 = 0$ then Γ becomes a parameter, just as in my model. Their more general formulation allows the generation of excess exchange rate return volatility (volatility amplification), but is not really important for many of the other interesting applications of their model. Because I solve not the original model, but the log-linearized version of it, conditional variance terms vanishes from the analysis. So, to be more transparent, I remove it from the model altogether. (41) also base his analysis in the case where $\gamma_1 \downarrow 0$.

Before continuing, a brief commentary on this modeling choice. First, note that each intermediary only trades in two markets. As I show in appendix C.2, this is a modeling trick that allows for different wedges between all possible pair of assets while sticking with (40)'s framework of the portfolio-balance channel[15]. Second, the inclusion of other types of intermediaries (for instance, one that arbitrages between domestic stocks and foreign bonds) most results in section 3.4 would not really change if the amount of friction each intermediary faces is adjusted accordingly.[16]. The exception is discussed in section 3.4.2.3.

General problem of an intermediary. Suppose an intermediary that arbitrages between returns of assets A and B, and let X_A and X_B denote how much it holds of each asset. The intermediary has no equity (i.e., zero net-worth), so in order to hold a long position in one asset it must short the other. Hence, its balance sheet is represent by[17]:

$$X_A + X_B = 0$$

Let R_A and R_B denote the ex-post gross returns on each asset. The (ex-post) arbitrage profit the intermediary makes is given by:

$$\Omega = R_A X_A + R_B X_B$$

Now comes the interesting part: the intermediary is subject to a moral hazard problem. After building its position, but before returns are realized, it may divert a fraction (ΓX) of its total assets $X = |X_A| = |X_B|$.[18] Hence, it can run away with the total amount $\left(\Gamma X^2\right)$.[19] In order not to be optimal for the

[15] (40)'s model only features two assets, so they did not have to deal with this issue. The same for (41), in a sense.

[16] This is because the overall level of friction in the economy depends not only on how much friction each intermediary faces, but on the number of intermediaries. Suppose all intermediaries' demand have the same functional form: $X_i = \frac{1}{\Gamma}\left(R_A - R_B\right)$. Total intermediaries demand is then $X = \frac{N}{\Gamma}\left(R_A - R_B\right)$, where N is the number of intermediaries. Hence, if the aggregate risk-baring capacity is $\bar{\Gamma}$ than the individual risk-baring capacity is $N\bar{\Gamma}$

[17] Variables and parameters used in this subsection are *local*, and should not be confounded with similar values used elsewhere in the model. Also, here I abstract from time subscripts, for simplicity. This is not a problem because in this model intermediaries' problem is not dynamic (i.e., there are no endogenous state-variables.), anyway.

[18] Of course the divertible fraction can not exceed unity. I assume that the value of Γ is such that this is always the case. When we turn to the quantitative assessment the adopted parameters make sure this is the steady-state and for all shocks within a (unspecified) bound.

[19] The square (in X^w) crucial in (40)'s framework. In a setting where the total divertible amount is only linearly increasing in X the equilibrium condition would imply a fixed expected excess return in disfavor of the shorted asset. The square is what makes the expected excess return time-varying.

intermediary to run away with the assets the expected profit of the position, $E[\Omega]$ must be at least as big as the divertible amount of assets. And, because agents would not lend to the intermediary if they didn't expect them to pay back, the following incentive-compatibility constraint holds:

$$\mathbb{E}[\Omega] \geq \Gamma X^2$$

Finally, the intermediary is assumed to be risk neutral and to maximize expected returns. Hence, its problem can be formally stated as:

$$\begin{aligned} \max \quad & \mathbb{E}[\Omega] \\ \text{s.t.} \quad & 0 = X_A + X_B \\ & \Omega = R_A X_A + R_B X_B \\ & \mathbb{E}[\Omega] \geq \Gamma \Big(X_A \Big)^2 \end{aligned}$$

The incentive compatibility constraint always binds in equilibrium, because the objective function is linear and the constraint is convex. In more substantial economics, if there is an expected return differential the intermediary would like to take advantage of it purchasing the high return asset while shorting the one with low return, increasing its leverage until unable to do: when the constraint binds. The solution can thus be found just by using the IC with equality. After simplification:

$$X_A = \frac{1}{\Gamma} \mathbb{E}\Big[R_A - R_B \Big]$$

The parameter Γ determines how much the intermediary is capable of leveraging and, in this sense it captures a financial friction. Absent the friction ($\Gamma = 0$) the intermediaries would leverage infinitely whenever a return differential exists, and the equilibrium would be characterized by a no-arbitrage condition $\mathbb{E}[R_A] = \mathbb{E}[R_B]$. On the other hand, if the friction is too extreme ($\Gamma \to \infty$) then no intermediation happens at all: $X_A, X_B \to 0$.

Intermediaries have a role in this model only because households are not allowed to directly arbitrage away these returns (they only trade domestic bonds). Again, this market segmentation is a common feature of many DSGE models with financial frictions — e.g., (8). Note that (40)'s formulation is simpler than many of other models with financial frictions by ignoring the financial accelerator mechanism associated with the endogenous variation if intermediaries' net-worth.

Domestic intermediary. The domestic financial intermediary raises deposits from households to finance its stock (capital) holdings. Differently from (8), but in accordance to the framework advanced by (40), the intermediary has no equity. Its balance sheet in nominal BRL is given by $S^n_{1,t} + B^n_{1,t} = 0$, but I rather write it in real terms:

$$S_{1,t} + B_{1,t} = 0$$

where $S_{1,t} = \frac{S^n_{1,t}}{P_t}$ — the same goes for $B_{1,t}$. Realized real profit is given by

$$\Omega_{1,t} = R^s_t S_{1,t-1} + R_t B_{1,t-1}$$

and the incentive compatibility constraint:

$$\mathbb{E}_t \left\{\Omega_{1,t+1}\right\} \geq \Gamma_H \Big(S_{1,t}\Big)^2$$

Note that the level of friction in domestic intermediation is captured by the parameter Γ_H. Following the steps used to solve the general problem, we arrive at the following demand equation:

$$S_{1,t} = \frac{1}{\Gamma_H} \mathbb{E}_t \left\{R^s_{t+1} - R_{t+1}\right\} \tag{3-38}$$

Finally, it is assumed that realized profits from domestic intermediaries are integrally transferred to the domestic households.

Global fixed-income intermediary. The balance-sheet of this intermediary, in nominal USD, is given by $\frac{1}{\mathcal{E}^n_t} B^n_{2,t} + B^{n*}_{2,t} = 0$, where again I work with real variables. Divide each nominal asset holding for the price level of the respective issuer country we can write the balance sheet in terms of the foreign final good:

$$\frac{1}{\mathcal{E}_t} B_{2,t} + B^*_{2,t} = 0$$

Realized real profits (again, in terms of the foreign final good) is:

$$\Omega^*_{2,t} = \frac{1}{\mathcal{E}_t} R_t B_{2,t-1} + R^*_t B^*_{2,t-1}$$

The incentive compatibility constraint is

$$\mathbb{E}_t \left\{\Omega^*_{2,t+1}\right\} \geq \Gamma_F \left(\frac{B_{2,t}}{\mathcal{E}_t}\right)^2$$

Note that the level of friction in fixed-income international intermediation is captured by the parameter Γ_F, potentially different from Γ_H. Following the steps used to solve the general problem, we arrive at the following demand equation:

$$B_{2,t} = \frac{\mathcal{E}_t}{\Gamma_F}\mathbb{E}_t\left\{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}R_{t+1} - R^*_{t+1}\right\} \tag{3-39}$$

The realized profits received by this intermediary are integrally transferred to the foreign household. Because the domestic economy is small, however, these profits are infinitesimal from the point of view of the ROW.

Global variable-income intermediary. The real balance-sheet of this intermediary, in terms of foreign final good, is given by:

$$\frac{1}{\mathcal{E}_t}S_{3,t} + S^*_{3,t} = 0$$

Realized real profits (of course, in terms of foreign final good):

$$\Omega^*_{3,t} = \frac{1}{\mathcal{E}_t}R^s_t S_{3,t-1} + R^{s*}_t S^*_{3,t-1}$$

Incentive compatibility constraint:

$$\mathbb{E}_t\left\{\Omega_{3,t+1}\right\} \geq \Gamma_F\left(\frac{S_{3,t}}{\mathcal{E}_t}\right)^2$$

Note that I have assumed that the amount of friction in variable-income intermediation is also Γ_F, the same as in the fixed-income intermediation. This is just for simplicity as the results do not hinge on this equality.

Again, following the steps of the general problem:

$$S_{3,t} = \frac{\mathcal{E}_t}{\Gamma_F}\mathbb{E}_t\left\{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}R^s_{t+1} - R^{s*}_{t+1}\right\} \tag{3-40}$$

The realized profits received by this intermediary are integrally transferred to the foreign household. Because the domestic economy is small, however, this profits are infinitesimal from the point of view of the ROW.

Balance of payments. Let $\mathcal{A}_t$ denote the real value of home economy's net international investment position (a.k.a, net foreign assets), in terms of its own final good. By definition:

$$\mathcal{A}_t = \mathcal{E}_t\left(F^{\text{CB}}_t + F_{B^*,t} + F_{S^*,t}\right) - \left(B_{2,t} + F^*_{B,t}\right) - \left(S_{3,t} + F^*_{S,t}\right) \tag{3-41}$$

where F_t^{CB}, to be formally presented in the next subsection, denotes Central Bank international reserves (in terms of the foreign good). The equation that characterizes the balance of payments is derived in appendix C.3 and can be rewritten as:

$$\underbrace{\underbrace{\text{TB}_t}_{\text{trade balance}} + \underbrace{\text{IB}_t}_{\substack{\text{income}\\\text{account}}}}_{\text{current account}} + \underbrace{-\Delta\mathcal{A}_t}_{\substack{\text{financial}\\\text{account}}} = 0 \tag{3-42}$$

where the trade balance is:

$$\text{TB}_t = \mu^*_{H,t} Z^*_{H,t} - \emptyset_{F,t}\mu_{F,t} Z_{F,t} \tag{3-43}$$

and the the income account balance (including revaluation effects) is:

$$\begin{aligned}\text{IB}_t = {} & \mathcal{E}_t R^*_t \left(F^{\text{CB}}_{t-1} + F_{B^*,t-1}\right) + \mathcal{E}_t R^{s*}_t F_{S^*,t-1} - R_t\left(B_{2,t-1} + F^*_{B,t-1}\right) \\ & - R^s_t\left(S_{3,t-1} + F^*_{S,t-1}\right) - \mathcal{A}_{t-1}\end{aligned} \tag{3-44}$$

Note that acquisition of reserves by the Central Bank is here included in the financial account.

Central bank, and the government. The Central Bank is assumed to have two instruments at its disposal: nominal interest rates and foreign exchange (FX) interventions. The first is set following a a simple Taylor rule with smoothing:

$$R^n_t = R^{\rho_m}_{t-1}\left[\frac{1}{\beta}\Pi^{\phi_m}_t\right]^{1-\rho_m} \tag{3-45}$$

FX interventions consists of buying, or selling, foreign bonds. Let F_t^{CB} denote the holdings of foreign bonds by the central bank. For the purposes of this paper it is treated as exogenous. Of course I could have specified some rule that relates the intervention to outcome variables such as the level of exchange rate, the level of foreign reserves, or deviations of these variables to a given target. Some papers have pursued this avenue[20], while others have looked for optimal policy[21]. Instead, I opt to treat it as exogenous in order to better isolate the mechanics of the intervention effects.

The FX interventions I consider here are sterilized interventions, in

[20] (46), (53), (47)
[21] (48), (41), (49)

a specific sense: the intervention does not directly affect the interest rate policy. But in equilibrium the FX intervention may have an indirect effect on the nominal interest rate — for instance, a depreciation induced by the acquisition of foreign bonds may lead to inflation and, hence, higher interest rates. As in (41) I could also characterize the sterilized intervention by requiring that purchases of foreign bonds by the government to be financed by issuing domestic bonds of equal value:

$$\mathcal{E}_t F_t^{\mathrm{CB}} + B_{G,t} = 0$$

This condition does seem to define a sterilized intervention, as we know it, but in this model it is immaterial. Because Ricardian Equivalence holds it does not matter whether the acquisition of foreign bonds is financed by debt issuance or by an increase in (lump-sum) taxes.[22]. I assume that interventions are financed by home bond issuance anyway, though, without loss of generality.

Central Bank's interventions may result in surpluses or deficits, depending on the return differential on domestic and foreign bonds. Also, the government must collect lump-sum taxes to finance exports subsidy. The net result is integrally transfered to domestic household in the same period, in a lump-sum fashion, the transfer being given by:

$$T_t = \left(\mathcal{E}_t R_t^* F_{t-1}^{\mathrm{CB}} + R_t B_{G,t-1}\right) - (\tau p_{H,t})$$

Market clearing. The final good is used for consumption or investment:

$$Y_t = C_t + I_t \tag{3-46}$$

The domestically produced aggregate input is used in the production of the domestic final good or exported for the production of final goods in other countries:

$$Y_{H,t} = Z_{H,t} + Z_{H,t}^* \tag{3-47}$$

[22] Also, I have not included money in the model. Had we done it, though, we would have a (log-linearized) money demand equation such as:

$$m_t = \nu_y y_t - \nu_i i_t$$

which makes clear that as long the nominal interest rate is being fixed by the Taylor Rule the Central Bank has no space to finance the acquisition of foreign bonds by issuing money.

In the labor market, households' labor supply must equal firms' demand:

$$\begin{aligned} L_t &= \int_0^1 L_{jt} \mathrm{d}j \\ &= (1-\alpha)\frac{\mu_t}{W_t} Y_{H,t} \emptyset_t \end{aligned} \tag{3-48}$$

In the market for capital rental services, the capital stock of the economy must be fully used by firms:

$$\begin{aligned} K_{t-1} &= \int_0^1 K_{jt} \mathrm{d}j \\ &= \alpha \frac{\mu_t}{R_t^r} Y_{H,t} \emptyset_t \end{aligned} \tag{3-49}$$

Demand for domestic bonds must match its net supply, assumed to be zero (w.l.g):

$$D_t + B_{1,t} + B_{2,t} + B_{G,t} + F^*_{B,t} = 0$$

Demand for domestic stocks must equal its supply:

$$S_{1,t} + S_{3,t} + F_{S,t} + F^*_{S,t} = S_t \tag{3-50}$$

Exogenous processes and the ROW, again.

I am particularly interested in the effects of the portfolio flows (F^*_B, F^*_S, F_{B^*}, F_{S^*} and F_S) and sterilized interventions (F^{CB}_t). All these shocks are assumed to follow independent univariate autoregressive processes. For instance, for gross stock inflows the process is given by:

$$F^*_{S,t} = (1-\rho)\bar{F}^*_S + \rho F^*_{S,t-1} + \xi^*_{B,t}$$

Outcomes in the rest of the world do not depend on outcomes of the infinitesimally small home economy. Hence, Y^*, p^*_F, R^* and R^{s*} are for all our purposes exogenous.

In order to understand the effects of capital inflows endogenously determined by returns on the rest of the world I also allow R^* and R^{s*} to follow independent univariate auto-regressive processes. Shocks to the rate of return of foreign assets are another form of inducing capital flows in a open economy model — actually, a much more common modeling form.

Regarding Y^* and p^*_F, I fix them constant at their steady-state level. They are not of direct interest here because they do not affect the home economy through capital flows directly, but through the current account.

Of course, in a fully specified model for the ROW economy the variables (Y^*, p^*_F, R^*, R^{s*}) would be jointly determined as a function of world-level fundamentals, including productivity and monetary shocks. But for our purpose it is convenient to make each world-level variables independent of the others so to isolate their effect. The effects of truly exogenous world fundamentals on the home economy would operate only through their effects on each of the world endogenous variables.

For a similar reason — understand specifically the role of capital flows — I opt not to focus on traditional fundamentals for the home economy. Hence the model does not feature productivity shocks, monetary shocks, etc.

Equilibrium. Equilibrium is defined as a sequence for the endogenous variables that satisfies economic agents' optimality conditions and market clearing conditions, simultaneously, given the realized sequence of the exogenous stochastic processes. The full set of equilibrium conditions, together with the associated endogenous variables, is presented in the appendix C.4. The log-linearized model is presented in appendix C.5.

3.3.2 Steady-State

When log-linearizing the model I do it around a symmetric steady-state. The computation is shown in appendix C.6. In this steady-state all countries are equally rich and, hence, one does not finance the other. Net foreign asset positions are zero, as is the trade balance. Global financial intermediaries, of both fixed and variable income, hold a zeroed position due to the absence of arbitrage opportunities to explore. The domestic intermediaries are active, however, holding all stocks of its own country. That stocks must be held by intermediaries imply that in the steady state there must be an equity premium, which arises not through risk considerations (the intermediary is risk neutral, after all) but through the existence of financial frictions, limiting arbitrage.

3.3.3 Calibration

Recall that I have already assumed a unitary intertemporal elasticity of substitution (log-utility) and unitary elasticity of substitution between home and foreign goods (Cobb-Douglas transformation function in the final goods sector). This is sometimes called Cole-Obstfeld parameterization, and is common in many papers on international economics.

Parameters take values that are in line with the literature. I set $\beta = 0.99$ to target an annualized real interest rate of 4%. A capital share of one-third is targeted by setting $\alpha = 0.33$, and $\delta = 0.025$ sets an annualized depreciation rate of 10%. $\gamma = 0.3$ targets an imports/GDP ratio of 30%, close to the world's average. $\varepsilon = 6$ implies an average mark-up of 20%. The parameter that governs nominal rigidity is set $\theta = 0.75$, implying that prices are optimally chosen once a year, on average, and for price indexation I set $\iota = 0.2$ — both values are close to the ones in (25). For the parameters related to the problem of importers (ϵ_f, θ_f and ι_f) I use the same values from the corresponding parameters of domestic firms' problem. Regarding consumption habits, $\psi = 0.7$ is very close to the weighted average of values found in macro models, as reported in (54)'s meta-study. $\phi = 0.33$ implies a Frisch labor supply elasticity of 3, in the mid range of the values reported by (36). For investment adjustment costs, $\kappa = 2.5$ as in (13) implies an elasticity of investment to the price of capital 0.4 — the median among the values reported by (37). For Taylor rule parameters I set the commons $\rho_m = 0.75$ and $\phi_m = 1.5$. Finally, for all exogenous process I set an autoregressive coefficient $\rho = 0.8$, implying a half-life of roughly three quarters.

More challenging is calibrating the gamma-parameters, related to amount of financial friction in the economy. Only Γ_H, domestic intermediaries risk-bearing capacity, is reflected on the symmetric steady-state values of the model. Following closed economy models with financial frictions (e.g., (52), (8)) I calibrate this parameter to target a given value for the equity-premium ($\mathcal{R}$)— in this case, of 400 b.p. in annual terms. As I show in appendix C.6, Γ_H and $\mathcal{R}$ are related through the expression $\Gamma_H = \mathcal{R}\big/\left(K - F_S^* - F_S\right)$, so I also need to set values for the steady-state (noisy) holdings of domestic stocks by both foreign and domestic households. The usual approach with segmented markets is to just set $F_S^* = F_S = 0$, which implies a fairly low level of financial friction[23]: $\Gamma_H = 9.2138 \times 10^{-4}$, roughly 0.001. This level can be increased by allowing households to hold some capital in the steady-state, but not that much. For instance, in order to have $\Gamma_H = 0.1$ the steady-state exogenous demand for stocks must amount to 99% of the market capitalization. The conclusion, here, is that in this model it is very hard to get big numerical levels for Γ_H. As a benchmark I take $\Gamma_H = 0.001$.

It is less straightforward to calibrate Γ_F as this parameter does not affect the deterministic symmetric steady-state. The related literature consider values considerably larger than those discussed for Γ_H in the previous paragraph.

[23] Of course, high and low are relative concepts. We say it is low in comparison to usual values for Γ_F.

For instance, (41) estimates for his model an aggregate risk bearing capacity close to 0.2. Considering that my model features two types of global financial intermediaries, whereas his included only one, his finding is consistent with each of my global intermediary having $\Gamma_F = 0.4$. (49) also acknowledge the difficulty of calibrating Γ, and consider the values of 1, 10 in order for FX interventions to have an effect size "*in the ballpark of empirical estimates*". The takeaway is that it is hard to consider low numerical values for Γ_F. As a benchmark I take $\Gamma_F = 1$.

Table 3.1: Basic Calibration

Parameter	Meaning	Value
Households		
β	Discount factor	0.99
ψ	Habit formation	0.7
φ	Inverse of Frisch Elasticity	0.33
Absorption		
γ	Openness to trade	0.3
ϵ	Elasticity of substitution among domestic varieties	6
ϵ_f	Elasticity of substitution among foreign varieties	6
Domestic firms		
α	Capital share	0.33
θ	Calvo pricing rigidity	0.75
ι	Price indexation	0.2
Importers		
θ_f	Calvo pricing rigidity	0.75
ι_f	Price indexation	0.2
Investment		
κ	Curvature of investment adjustment cost function	2.5
δ	Depreciation rate	0.025
Central Bank		
ρ_m	Smoothing coefficient	0.75
ϕ_m	Responsiveness to inflation	1.5
Exogenous process		
ρ	AR(1) coefficient of all processes	0.8
Financial friction on	**domestic intermediation**	
$\mathcal{R}$	S.S. Equity premium (quarterly)	0.01
Γ_H	Local intermediary risk-baring capacity	0.001
Financial friction on	**global intermediation**	
Γ_F	Global intermediary's risk-baring capacity	1

The benchmark calibration is shown in Table 3.1, and used mostly for the more quantitative assessment. To illustrate theoretical results it will be convenient to consider other values of Γ_F and Γ_H, at times.

3.4 Results

This section is divided in two. Subsection 3.4.1 contain the main results of this paper, regarding the effects of capital inflows in the economy's business cycle. It discusses whether or not the different inflow shocks are isomorphic, the quantitative importance of a eventual non-isomorphism, and the mechanisms involved. Subsection 3.4.2 contain other results, about (i) the importance of taking into account a "hidden transaction leg", absent in balance of payments statistics; (ii) FX interventions and their effectiveness in stabilizing the economy following different types of inflow shocks; and (iii) the relation between noisy-trading shocks and foreign return shock, for the purpose of modeling capital inflow.

3.4.1 Main results

3.4.1.1 Relevant endogenous variables, and some normalizations

In this paper I am interested in the production and allocation of goods, prices and returns. I exclude the amount of domestic risk-free debt held by the household (D) since Ricardian equivalence holds in the model and the household is able to frictionlessly trade this asset. Also it will prove useful to exclude the balance-sheet position of the financial intermediaries, both domestic and foreign (S_1, B_2 and S_3), when discussing the relationship between exogenous capital flow shocks and foreign assets return shocks (which are important *push* factors in the literature of capital flows).

The model includes four exogenous capital flows shocks: (i) gross bond inflows, F_B^*; (ii) gross stock inflows, F_S^*; (iii) gross bond outflows F_{B^*}; and (iv) gross stock outflows F_{S^*}. Remember that gross inflows are defined as acquisition of domestic assets (bonds or stocks) by foreigners, while gross outflows are defined as acquisition of foreign assets by domestic residents. These flows can be positive, when the asset is being purchased, or negative, when it is being sold. Each flows is measured in the currency denomination of the asset being purchased; i.e., gross inflows are measured in domestic currency (BRL) and gross outflows are measured in foreign currency (USD). In order to control for this I normalize all shocks to be measured in terms of domestic currency. Also, in order to focus on inflows to the domestic economy, I consider positive gross inflows shocks and negative gross outflows shocks.

3.4.1.2 Are the different capital inflow shocks isomorphic? Why?

Capital inflow shocks appear directly in the following equations of the model: (3-41), (3-42) and (3-50). Substituting intermediaries optimality conditions — in order to eliminate the irrelevant variables S_1, B_2 and S_3 —

they can be respectively written as:

$$\Gamma_F\,\mathcal{A}_t = \Gamma_F\left[\mathcal{E}_t\Big(F_t^{\mathrm{CB}} + F_{B^*,t} + F_{S^*,t}\Big) - F^*_{B,t} - F^*_{S,t}\right] - \mathcal{E}_t\left[\mathbb{E}_t\left\{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}R_{t+1} - R^*_{t+1}\right\} + \mathbb{E}_t\left\{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}R^s_{t+1} - R^{s*}_{t+1}\right\}\right] \quad (3\text{-}51)$$

$$\Gamma_F\,\mathrm{IB}_t = \Gamma_F\left[\mathcal{E}_tR^*_t\Big(F^{\mathrm{CB}}_{t-1} + F_{B^*,t-1}\Big) + \mathcal{E}_tR^{s*}_tF_{S^*,t-1} - R_tF^*_{B,t-1} - R^s_tF^*_{S,t-1} - \mathcal{A}_{t-1}\right] - \mathcal{E}_{t-1}\left[R_t\mathbb{E}_{t-1}\left\{\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}R_t - R^*_t\right\} + R^s_t\mathbb{E}_{t-1}\left\{\frac{\mathcal{E}_{t-1}}{\mathcal{E}_t}R^s_t - R^{s*}_t\right\}\right] \quad (3\text{-}52)$$

$$\Gamma_H\Gamma_F\Big(Q_tK_t\Big) = \Gamma_H\Gamma_F\left(F_{S,t} + F^*_{S,t}\right) + \Gamma_F\left[\mathbb{E}_t\left\{R^s_{t+1} - R_{t+1}\right\}\right] + \Gamma_H\left[\mathcal{E}_t\mathbb{E}_t\left\{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}R^s_{t+1} - R^{s*}_{t+1}\right\}\right] \quad (3\text{-}53)$$

As argued (section 3.2), in order to understand whether the different capital inflow shocks are isomorphic to each other we can focus on the Jacobian of the model with respect to the shocks. Table 3.2 presents its relevant rows (i.e. the rows related to the equations where the shocks appear), considering the original nonlinear model. One can easily see that in general the columns are not pair-wise collinear and, hence, that these shocks are not isomorphic. An exceptional case is when global financial intermediation is frictionless ($\Gamma_F = 0$).

Table 3.2: Isomorphism analysis: jacobian of the non-linear model

Mechanism	$F^*_{B,t}$	$F^*_{S,t}$	$-\mathcal{E}_tF_{B^*,t}$	$-\mathcal{E}_tF_{S^*,t}$
Eq. (3-51)	$-\Gamma_F$	$-\Gamma_F$	$-\Gamma_F$	$-\Gamma_F$
Eq. (3-52)	$-\Gamma_F\mathbb{E}_t\left\{R_{t+1}\right\}$	$-\Gamma_F\mathbb{E}_t\left\{R^s_{t+1}\right\}$	$-\Gamma_F\mathbb{E}_t\left\{R^*_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right\}$	$-\Gamma_F\mathbb{E}_t\left\{R^{s*}_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right\}$
Eq. (3-53)	0	$\Gamma_F\Gamma_H$	0	0

The first row of the table represents the immediate effect that capital flows have on the balance of payments and, hence, on a country's net investment position. I call it *static balance of payments mechanism*. These shocks affect this equation in the same way (after due normalizations), which is trivial: one dollar inflow reduces the home country net foreign asset position in one dollar, *ceteris paribus*, independently of the asset being transacted and the residency of agent.

The second row of table 3.2 represents what I call a *dynamic balance of payments mechanism*, and is a general force for non-isomorphism between different capital flow shocks. Each asset in the economy (domestic/foreign

×bond/stock) has its own, potentially idiosyncratic, return. Then, the income paid (received) over time by the country due to the inflow (outflow) will depend on what asset was transacted. Because agents are forward looking this income differential impacts the real economy even in the period of the shock, before incomes are effectively paid. It is important to note that (39)'s static model does not feature this mechanism.

As an example, consider a case where stocks pay a premium over bonds on average (as is the model), and suppose that foreigners short domestic bonds ($F_B^* = -f$) to buy domestic stocks ($F_S^* = +f$). This transaction has no impact on the balance of payments in the instant it happens. But since stocks pay more than bonds on average, the home country net income payments to the rest of the world will increase, at least in expectation. This reduces home country's *overall wealth* (which includes holdings of assets as well as the income stream), i.e., makes it poorer. This wealth effect is likely to make households in the home economy work and produce more, to smooth the consumption loss.

Finally, Table 3.2's third row represents an *asset-demand mechanism* — in the case, demand for domestic stocks. Of all the four assets considered (domestic/foreign ×bond/stock) domestic stocks is the only one whose market clearing condition appears for equilibrium determination[24]. By definition only gross stock inflows have a direct exert demand pressure on the stock market, directly leading to higher stock prices which, in turn, stimulates capital producers to produce more capital units. This is similar, in a sense, to the mechanism (39) have in mind when arguing that non-bond inflows are likely to be less contractionary than bond inflows.

In sum, capital flows of different types may have different business cycles implications because (i) they may have different expected rate of returns, triggering wealth effects, and (ii) they may put pressure on different market-clearing conditions, thus requiring different asset price (return) vectors to clear the markets, with allocative implications.

Analysis in the log-linearized model. When solving the model and computing impulse response functions I consider a log-linearized version of the model around a symmetric steady-state. In principle it is possible for the conclusions about shocks' isomorphism to change in this setting, if the differences between the effects of the shocks are of second order.

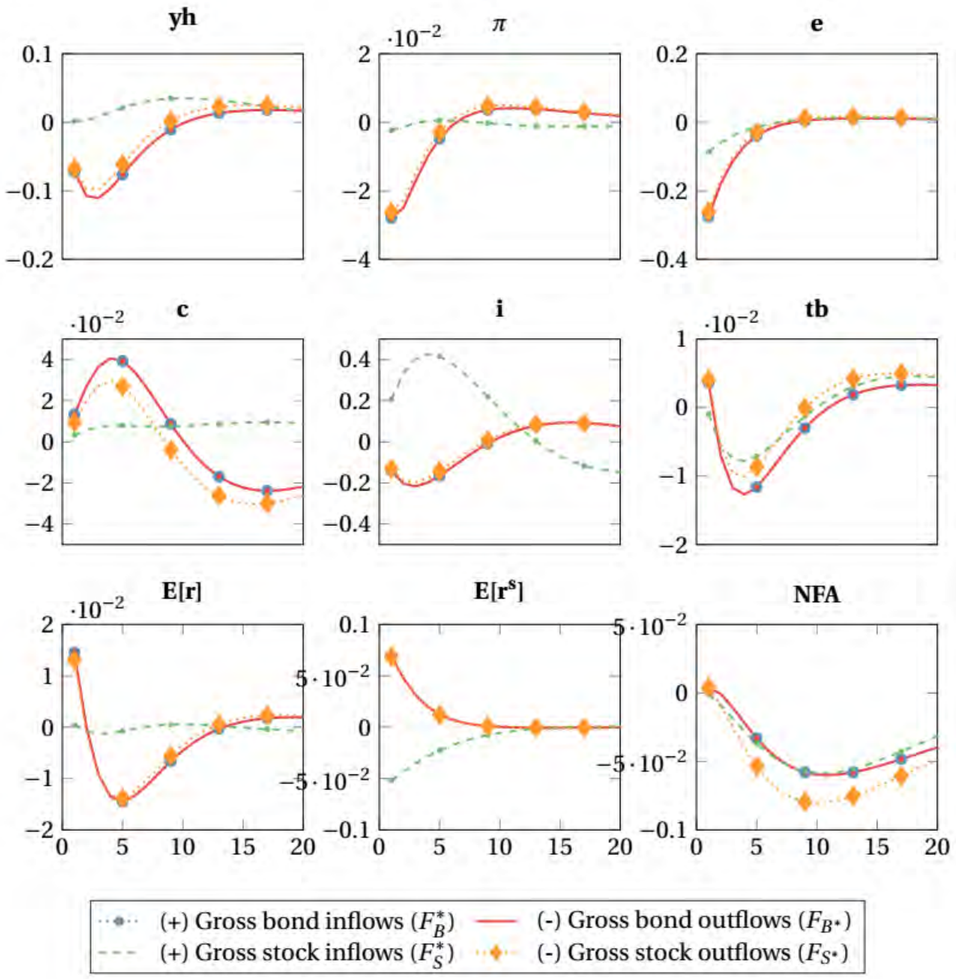
[24] Market clearing conditions for foreign assets are not necessary because the home economy is small and, hence, is not able to influence the global market for these assets. The market clearing condition for domestic bonds is not necessary because the domestic household is able to frictionlessly trade in this market and because Ricardian Equivalence holds, as already argued.

Table 3.3: Isomorphism analysis: jacobian of the log-linearized model

Mechanism	$F^*_{B,t}$	$F^*_{S,t}$	$-F_{B^*,t}$	$-F_{S^*,t}$
Eq. (3-51)	$-\Gamma_F$	$-\Gamma_F$	$-\Gamma_F$	$-\Gamma_F$
Eq. (3-52)	$-\Gamma_F R^*$	$-\Gamma_F R^{s*}$	$-\Gamma_F R$	$-\Gamma_F R^s$
Eq. (3-53)	0	$-\Gamma_F \Gamma_H$	0	0

Table 3.3 shows the Jacobian of the log-linearized model — *before* imposing the steady-state relations. This Jacobian is much simpler then the one associated with the non-linear model as its coefficients are not time-varying. We still conclude that the four shocks are not isomorphic to each other, in general, if we do not impose any specific steady-state relation.

Figure 3.1: IRFs w.r.t. exogenous capital flow shocks, illustrative



Note: Computed for a Table 3.1's benchmark calibration, except for $\Gamma_H = 0.1$ and $\Gamma_F = 0.01$. This is for illustrative purpose. Computed for a shock size of 1% of s.s. capital stock.

Under the assumption of a symmetric steady-state, however, where $R = R^*$ and $R^a = R^{s*}$, the conclusion is that gross bond inflows and gross bond outflows are isomorphic to each other. Bond and stock flows still have different implications in this case whenever there is an equity premium, which triggers the *dynamic blance of payment mechanism* (3-52). Also, the *asset-demand mechanism* (3-50) still implies that gross stock inflows and (negative) gross stock outflows are non-isomorphic. Figure 3.1 illustrates these results.

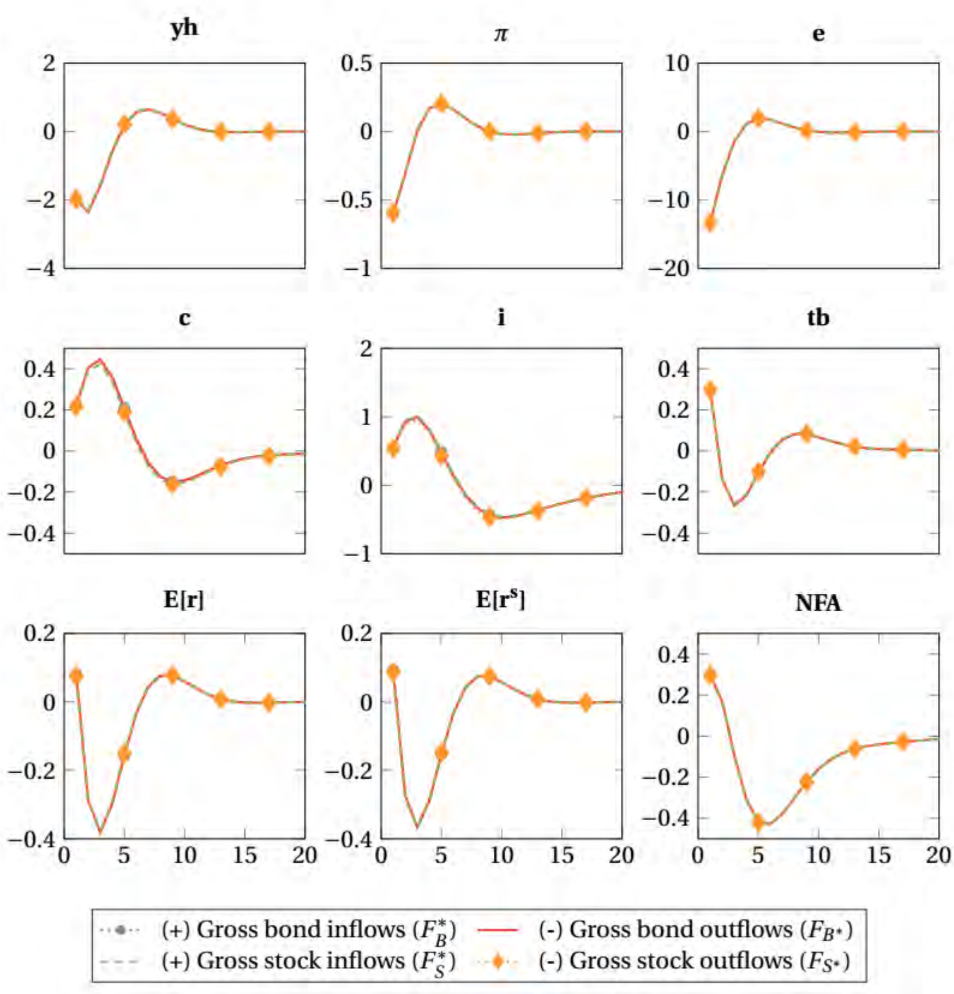
Figure 3.1 also illustrates that gross stock inflows are generally less contractionary (for GDP, Y_H) than other inflow types, in particular bond inflows, and maybe even expansionary as (39) claim. From a demand perspective, this is because exogenous demand shock for home stocks directly stimulates investment, unlike the others, suggesting that the asset-demand mechanism is particularly important. Negative gross stock outflows are also less contractionary than bond flows, albeit marginally, because of the wealth effect associated with the dynamic balance of payment mechanism.

3.4.1.3
Is the non-isomorphism quantitatively important?

In the last subsection I have focused on whether the shocks are isomorphic to each other, and on understanding the related mechanisms, and I showed that the different capital inflow indeed have different consequences for the relevant endogenous variables. However, it is possible that this theoretical non-isomorphism to be very small, irrelevant in practice. Indeed, this is what I find when I parameterize my model with Table 3.1's values, which seem reasonable in the light of the related literature.

Figure 3.2 shows impulse response functions os selected variables to the four capital flow shocks, computed for a log-linearized version of the model with a symmetric steady-state. The remarkable thing to notice is that the impulse response functions of the four inflow shocks are almost perfectly juxtaposed. Differences in the impulse response functions do exist, as the last subsection shows, but in comparison to the IRFs scale they are so small that they become imperceptible. Note that the calibration used for Figure 3.1 and Figure 3.2 computations differ only on the values of Γ_F and Γ_F. In fact, in the next subsection I show how they are key to understand this quantitative irrelevance.

Figure 3.2: IRFs w.r.t. exogenous capital flow shocks, more standard parameterization

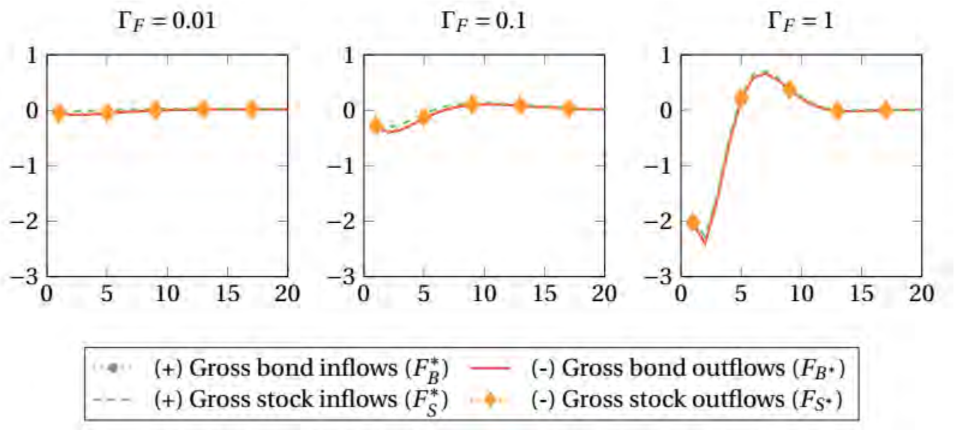


Note: Computed for a Table 3.1's benchmark calibration and a shock size of 1% of s.s. capital stock.

3.4.1.4
The role of the portfolio-balance channel: sensitivity to Γ_H and Γ_H

The conclusion that different capital flow shocks are non-isomorphic in general holds when a portfolio balance channel is active for global financial intermediaries ($\Gamma_F > 0$), as Table 3.3 makes clear. In this case the total demands for a home economy asset well-behaved schedules, decreasing (increasing) in the asset's price (expected excess return). Given this negatively sloped demand schedule, changes in net supply (total supply minus exogenous demand) imply changes in equilibrium prices, hence in expected returns and real allocation. Because all the entries in Table 3.3 are multiples of Γ_F it must be that the total effect of noisy-trading shocks is increasing in the amount of friction. Intuitively, the higher the friction then the more inelastic is total demand, and the higher are price and allocation consequences of noisy-trading shocks. This is illustrated by Figure 3.3.

Figure 3.3: Output (Y_H)'s IRF w.r.t. inflow shocks: sensitivity to Γ_F



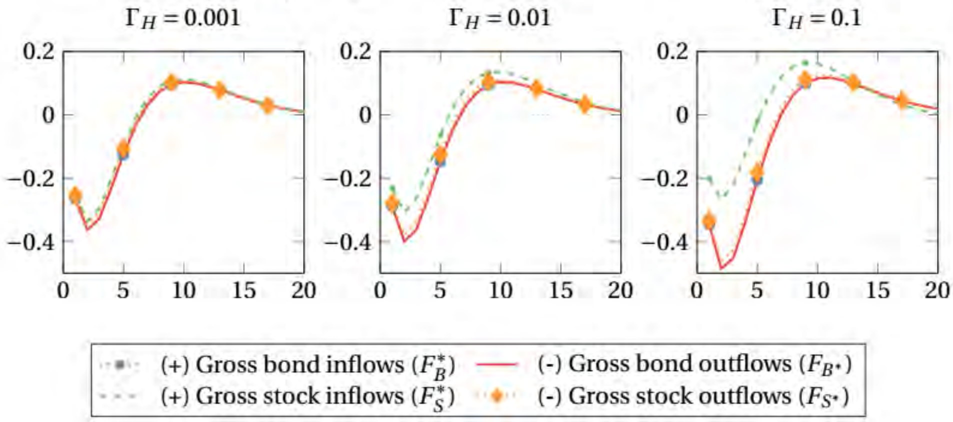
Note: Computed for a Table 3.1's benchmark calibration, except for $\Gamma_H = 0.1$ and the respective Γ_F. This is for illustrative purpose. Computed for a shock size of 1% of s.s. capital stock.

The polar case of frictionless global financial intermediation ($\Gamma_F = 0$) is a special one where all capital flow shocks become isomorphic. In such case intermediaries optimality conditions are no longer well-behaved demand schedules. Instead, demand is such that it may assume any finite value as long the excess return is equal to zero, and explodes to $\pm\infty$ otherwise. As any equilibrium must have finite supply excess return must equal zero. Financiers optimality conditions are thus better interpreted as a no-arbitrage conditions: as long they hold intermediaries are willing to hold any finite asset position. In particular, they are willing to hold a position that is exactly contrarian to noisy-traders' — e.g., $B_2 = -F_B^*$, $S_3 = -F_S^*$. This is in fact what happens in equilibrium following a noisy-trading shock. Total demand is perfectly elastic, and these shocks have no consequences for prices and allocation (except, of course, intermediaries portfolio).

Now, what are the effects of the domestic financial friction, captured by Γ_H? From Table 3.3 we see that this parameter only influences the asset demand mechanism (3-53), having nothing to do with the two balance of payment mechanism. This is because these local intermediaries do not directly engage in international financial trading. But engage in arbitrage between domestic assets and, hence, in how the different capital inflow shocks are *effectively channeled* once inside the home economy. For instance, if $\Gamma_H = 0$

the local intermediary will end-up channeling the foreign resources to their most valuable use independent of how they entered the economy, through the bond or the stock market.[25]. Hence, the size of the non-isomorphism between gross bond and stock inflows depend on Γ_H, as in Figure 3.4 illustrates.

Figure 3.4: Output (Y_H)'s IRF w.r.t. inflow shocks: sensitivity to Γ_H



Note: Computed for a Table 3.1's benchmark calibration, except for $\Gamma_F = 0.01$ and the respective Γ_H. This is for illustrative purpose. Computed for a shock size of 1% of s.s. capital stock.

This discussion allows us to conclude why the non-isomorphism of capital flow shocks do no show up as quantitatively important in Figure 3.2. The calibration in which it was based attach a relatively low value for the domestic financial friction in comparison to the cross-country financial friction. Noisy-trading inflow shocks thus have a relatively large effect on the economy, but the type of the inflow does not matter much as the local intermediary is able to channel it towards more valuable uses.

3.4.2
Other results

3.4.2.1
Gross outflow shocks: the importance of the *hidden transaction leg*

Remember the assumption that households are not allowed to optimally trade assets *except* for their own country bonds. By implication, their purchases of foreign assets (following a noisy-trading shock) must be funded by selling domestic bonds it owns, given an optimal level for their savings. For the same reason, when they sell foreign assets the proceeds must be allocated in domestic bonds.

[25] In a broad sense, an application of (29)'s theorem.

This second transaction — which funds the acquisition of foreign assets or allocates the proceeds of selling them — is what I call *hidden transaction leg*. This transaction lag is "hidden" in the sense that it is not captured by balance of payments statistics[26]. It turns out that the exact nature of this hidden transaction leg matters a great deal for the analysis, however, and the results of the last subsections must take this into account. They are conditional on the fact that agents in the model are always using the bond market as the adjustment margin for the portfolio shocks.

To see how results could have changed had I assumed other adjustment margins, remember that in the model I have allowed for a noisy-trading shock not yet considered (because it is not an inflow shock): exogenous demand for domestic stocks by domestic households $\left(F_S\right)$. Now consider the case where noisy trading of foreign stocks by home residents are funded by taking the opposite position in the domestic stock market. As Table 3.4 shows, turns out that this set of transaction becomes more similar to a gross stock inflow shock (when a foreigner buys domestic stock). And becomes exactly isomorphic when the approximating around a symmetric steady-state, which Figure 3.5 illustrates.

Table 3.4: Isomorphism analysis: gross stock inflows × outflows

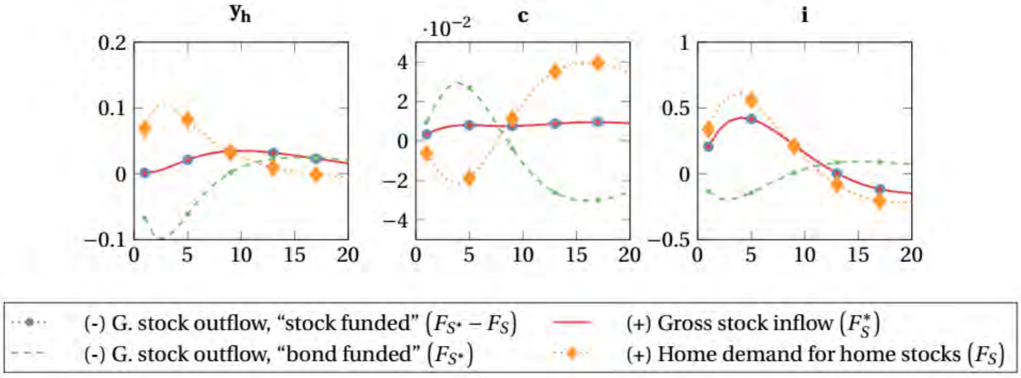
Mechanism	$F_{S^*,t}$	$F_{S,t}$	$F_{S,t} - F_{S^*,t}$	$F^*_{S,t}$
Eq. (3-51)	Γ_F	0	$-\Gamma_F$	$-\Gamma_F$
Eq. (3-52)	$\Gamma_F R^{s*}$	0	$-\Gamma_F R^{s*}$	$-\Gamma_F R^s$
Eq. (3-53)	0	$\Gamma_F \Gamma_H$	$\Gamma_F \Gamma_H$	$\Gamma_F \Gamma_H$

In the case of gross inflow shocks it is not necessary to consider what the hidden transaction leg is. Because the home economy is small, any inflow is small in comparison to the world market which finances it. It does not matter (for the home economy) whether the funding comes from world stock or bond market as, in any case, the flow will be of negligible in relation to the corresponding market size.

These considerations are important not only to understand this paper's results, but also any empirical result on the consequences of gross capital flows. Because it is hard to control for the hidden transaction leg studies focusing on gross inflows are likely to be more reliable than the ones focusing on gross outflows, or making no distinction between them.

[26] This would only appear directly if the second transaction has a non-resident as a counterpart. This does not have to be the case.

Figure 3.5: IRFs comparing different stock noisy-trading shocks



Note: Computed for a Table 3.1's benchmark calibration, except for $\Gamma_H = 0.1$ and $\Gamma_F = 0.01$. This is for illustrative purpose. Computed for a shock size of 1% of s.s. capital stock.

3.4.2.2
FX intervention's effectiveness against different inflow shocks

Until now I have considered how the economy responds to capital inflow shocks under the hypothesis that the Central Bank does not intervene in the FX market (although it endogenously change the interest rate following a Taylor rule). If the Central Bank intervenes by purchasing foreign bonds, what are the effects?

Remember that FX interventions as treated as purely exogenous in the model. By looking at the Jacobian of the model with respect to FX intervention shocks we can check what mechanisms it triggers, and even compare them with those associated with other capital inflow/outflow shocks. Table 3.5 compares it with exogenous gross bond outflow, and shows that they are isomorphic even in the original non-liner model.

Table 3.5: Isomorphism analysis: jacobian of the non-linear model

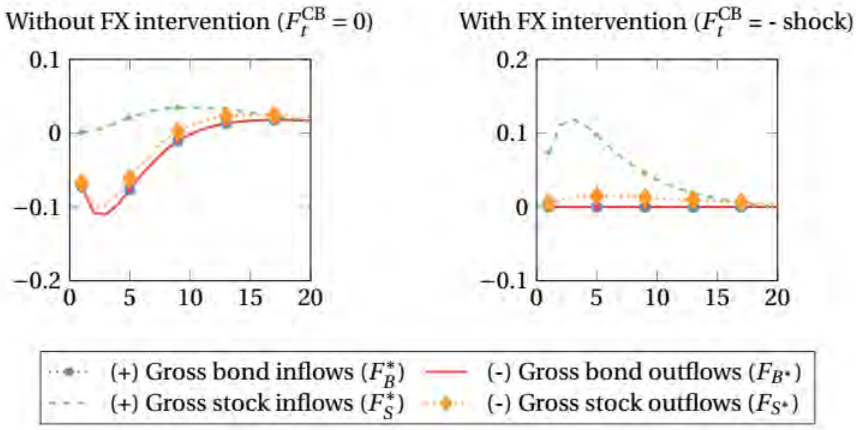
Mechanism	$\mathcal{E}_t F_{B^*,t}$	$\mathcal{E}_t F_t^{\text{CB}}$
Eq. (3-51)	Γ_F	Γ_F
Eq. (3-52)	$\Gamma_F \mathbb{E}_t \left\{ R_{t+1}^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}$	$\Gamma_F \mathbb{E}_t \left\{ R_{t+1}^* \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right\}$
Eq. (3-53)	0	0

This isomorphism means that it does not matter whether the acquisition of foreign bonds is done by the government or by the households. This happens in the model because government revenues are always rebated to households, eventually, through lump-sum transfers. In a sense, the government is just a veil, an investment vehicle for the households. Also, because this is a model of full-information rational expectations it precludes the possibility that interventions contain signalling effects, which is actually one of the preferred explanations for the effectiveness of FX interventions[27].

An important corollary of the fact that gross bond outflows shocks and FX intervention are isomorphic is that the later can be used to perfectly offset the former. In other words, in the model FX intervention can be fine-tuned to prevent any fluctuations which is caused by the noisy-trading of foreign bonds by domestic residents. All that is needed is that the government buy those bonds.

Another corollary is that FX intervention can not perfectly offset other types of capital inflow shocks if those are not isomorphic to gross bond outflow shocks. For instance, consider the case where foreign investors purchase 100 USD in domestic stocks and the central bank responds by purchasing buy 100 USD in foreign bonds. The central bank response matches in size the inflow shock and maintain the capital account unaffected in the first period, *ceteris paribus*. But it does not tackles the "dynamic balance of payment" and the "asset demand" mechanisms, and thus business cycle fluctuation may occur. Figure 3.6 illustrates this point. Note that, because the FX intervention is isomorphic to the bond flows, the second set of plots is obtained from the first by just adding the (negative) of the red solid line (or, equivalently, the blue dotted line with a circle marker). The effect of a gross stock inflow is magnified when the CB intervenes in response because the intervention depreciates the exchange-rate.

Figure 3.6: Y_H IRFs: with and without FX intervention



Note: Computed for a Table 3.1's benchmark calibration, except for $\Gamma_H = 0.1$ and $\Gamma_F = 0.01$. This is for illustrative purpose. Computed for a shock size of 1% of s.s. capital stock.

[27] See, for instance, (55).

3.4.2.3
Noisy trading shocks × foreign returns shocks

How should a researcher model capital inflow shocks? Should he consider exogenous changes the foreign interest rates, as do (48), (49), (45) and others? Or is it better to consider exogenous demand shocks by noisy traders, as I have done so far following (41)?

Here I compare both approaches, taking the Jacobian of the log-linearizing model with respect to both gross outflows shocks and return shocks. Table 3.6 shows the result. In general both approaches are not equivalent, as the columns are not pair-wise collinear. A deeper inspection shows that the difference lies in the dynamic balance of payments effect. To see why, set the steady-state level of the noisy trading demand and of central bank interventions to zero ($F^{CB} = F_{B^*} = F_{S^*} = 0$), which affects only the second the second row of the table. Now the first and second columns become collinear — with a collinearity vector $\left(\frac{R^*}{\Gamma_F} \;\; 1\right)$ —, and also do the third and fourth — with a collinearity vector $\left(\frac{R^{s*}}{\Gamma_F} \;\; 1\right)$[28]. Hence it is in the second row, capturing the dynamic balance of payment mechanism, that the difference lies.

Table 3.6: Isomorphism: gross inflow shocks × Return shocks

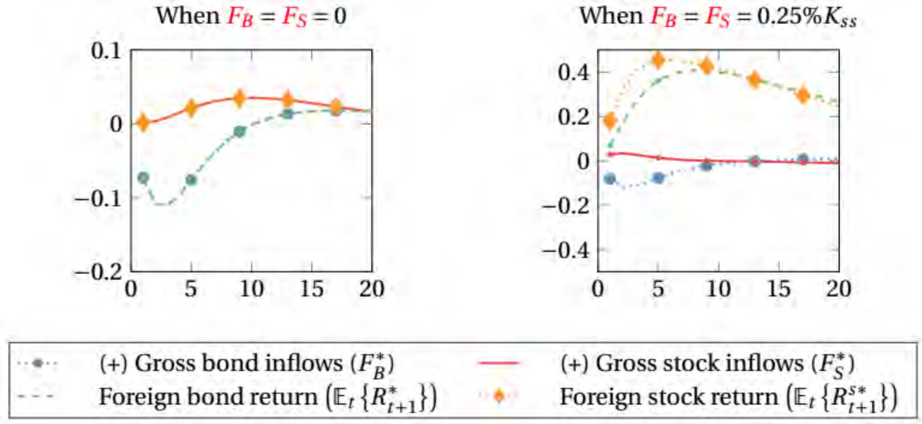
Mechanism	Bond		Stock	
	$F^*_{B,t}$	$\mathbb{E}_t\left\{R^*_{t+1}\right\}$	$F^*_{S,t}$	$\mathbb{E}_t\left\{R^{s*}_{t+1}\right\}$
Eq. (3-51)	$-\Gamma_F$	R^*	$-\Gamma_F$	R^{s*}
Eq. (3-52)	$-\Gamma_F R$	$R^*\left[R + \Gamma_F\left(F^{CB} + F_{B^*}\right)\right]$	$-\Gamma_F R^s$	$R^{s*}\left[R^s + \Gamma_F F_{S^*}\right]$
Eq. (3-53)	0	0	$\Gamma_F \Gamma_H$	$-\Gamma_H R^{s*}$

The reason for this is that return shocks not only induce capital flows but also affect income payments (per amount invested) to noisy-traders, *whose demand is inelastic to return considerations*. On the other hand the effect of noisy-trading shocks on noisy-traders income payments (per amount invested) is of second order: it is a $\mathrm{d}R\mathrm{d}F$ term. The only first order term is related to inducing capital flows. When noisy-traders steady-state position is zero the difference between the approaches vanishes, as now only the flow induction mechanism are active. This is illustrated by Figure $fig3 : return$.

Now, models that follows the return shock approach does not usually feature noisy-traders, i.e., they already impose $F_{B^*} = F_{S^*} = 0$. If it also assumes that $F^{CB} = 0$ then their results are essentially the same one can find with noisy-trading shocks. This result is interesting because it connects two modeling choices used previously in the literature.

[28] About these collinearity vectors: the coefficient on the exogenous demand shock is decreasing in Γ_F, meaning that the higher is the amount of friction in the economy the smaller the exogenous demand shock needs to be in order to mimic the effect of a return shock.

Figure 3.7: Y_H IRFs: gross inflow shocks × expected foreign return shocks



Note: Computed for a Table 3.1's benchmark calibration, except for $\Gamma_H = 0.1$ and $\Gamma_F = 0.01$. This is for illustrative purpose. Computed for a shock size of 1% of s.s. capital stock.

Two caveats are in order, however. First, the above reasoning is only true for $\Gamma_F \in (0, \infty)$, breaking for the polar cases of $\Gamma_F = 0$ and $\Gamma_F = \infty$. When external trading is frictionless noisy trading shocks have no effect on the domestic economy (since financiers fully offset the shock by being contrarian), while return shocks have their effects magnified. On the other hand, noisy trading have strong effects and return shocks have no effect when global intermediaries are not able to trade.

Second, that result that bond (stock) return shocks are isomorphic to exogenous gross bond (stock) inflows shocks, when noisy-traders demand is zero in the steady state, also depends on how the financial intermediaries were specified in the model. Remember that my model features two types of global intermediaries, one that buys domestic bonds when foreign bonds pay little, and other that buys domestic stocks when foreign stocks pay little. This formulation mechanically links bond (stock) inflows to foreign bond (stock) returns. Had I introduced other types of intermediaries, for instance, one that buy domestic stocks when foreign bond returns are low, the links would be messier. This is one reason why, in order to understand the consequences of different type of inflow shocks, it is more straightforward to consider the noisy-trading approach as I do in this paper.

3.5 Conclusion

In this paper I build a general equilibrium model to show that the effect of capital inflows on a small open economy's business cycle depends on the type of inflow, if a portfolio-balance channel is present. This non-isomorphism between the different capital inflow shocks is due to two mechanisms: one that I call *dynamic balance of payments* mechanism, which operates through difference in expected returns associated with each inflow type; and the other that I call *asset-demand* mechanism, which operates through asset-specific market clearing conditions. I argue that both goes in the direction of making stock inflows less contractionary than bond inflows, as (39) claims. Nonetheless, for a reasonable calibration of the model this non-isomorphism does not arise as quantitatively important.

I also discuss a few other topics with the model: (i) how the analysis of the consequences of gross capital outflows is more complicated than the analysis of the consequences of gross capital inflows; (ii) how FX interventions are mechanically similar to gross bond outflows, in the absence of a signaling channel at least; and (iii) the extent to which modeling capital inflows though exogenous noisy-trading shocks, as I do, is similar to the more traditional approach of varying the rate of return on foreign assets.

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A
Appendix to Chapter 1

A.1
Decomposition with an infinite but countable number of firms

Take the derivation with a finite number N of firms:

$$\frac{\partial Y}{\partial \zeta} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial Y_i}{\partial \zeta_i} + \frac{1}{N}\sum_{i=1}^{N}\left(\sum_{j\neq i}\frac{\partial Y_i}{\partial \zeta_j}\right)$$

and now let $N \to \infty$. Assume that

$$\lim_{N\to\infty}\frac{\partial Y_i}{\partial \zeta_j} = 0 \qquad \text{and} \qquad \lim_{N\to\infty}(N-1)\frac{\partial Y_i}{\partial \zeta_j} \text{ is bounded}$$

for all i and $j \neq i$. I am just asking for a case where firm j's ability to influence firm i's decision to vanish, in an appropriate velocity, when the number of firms in the economy grows large. This assumption is reasonable — when the number of firms in the economy are very big and the distribution of size in the economy is well-behaved (meaning that no firm in the economy is too big), no firm is supposed to affect the aggregate and, through that, other firms. This assumption gives us sufficient regularity to apply the law of the large numbers and define:

$$\frac{\partial Y_i}{\partial \zeta_{-i}} = \lim_{N\to\infty}\frac{1}{N-1}\sum_{j\neq i}\left[(N-1)\frac{\partial Y_i}{\partial \zeta_j}\right]$$

With this, we can write:

$$\frac{\partial Y}{\partial \zeta} = \mathbb{E}_i\left[\frac{\partial Y_i}{\partial \zeta_i}\right] + \mathbb{E}_i\left[\frac{\partial Y_i}{\partial \zeta_{-i}}\right]$$

A.2
Decomposition of aggregate effect with an uncountable number of firms

I assume that there is a unitary measure of firms. Firm i's access to government credit is denoted by $\zeta_i = f(i)$, where $f : [0,1] \to [0,1]$ is a function that describes how much access each firm has. In equilibrium firm i's output is a function of R and a functional of $f(\cdot)$:

$$Y_i = Y_i\Big(R\,; f\Big)$$

Total differentiation of firm i's output in which respect to government access variables yields:

$$\begin{aligned}\mathrm{d}Y_i &= \int_0^1 \frac{\partial Y_i}{\partial \zeta_j}\mathrm{d}f(j) \\ &= \frac{\partial Y_i}{\partial \zeta_i}\mathrm{d}f(i) + \int_{\mathcal{C}_i} \frac{\partial Y_i}{\partial \zeta_j}\mathrm{d}f(j)\end{aligned}$$

where $\mathcal{C}_i = [0,1] \setminus [i]$. We have separated ζ_i from the other ζ_j, to make clear that the "own effect" of ζ_i over Y_i has a very different nature than the "external effect" of ζ_i, $j \neq i$. In fact, as discussed on the main text, it is likely that, by itself, the access of any given firm $j \neq i$ has negligible effect over the product of firm i. But when aggregated with all other firms the total external effect might be relevant.

Aggregate government credit is defined as $\zeta = \int_0^1 \zeta_i \mathrm{d}i$, and we again consider the policy change $\mathrm{d}\zeta_i = \mathrm{d}\zeta$ for all i. Hence:

$$\frac{\partial Y_i}{\partial \zeta} = \frac{\partial Y_i}{\partial \zeta_i} + \int_{\mathcal{C}_i} \left(\frac{\partial Y_i}{\partial \zeta_j}\frac{1}{\mathrm{d}j}\right)\mathrm{d}j$$

Note that in the second right-hand side term we are integrating $\frac{\partial Y_i}{\partial \zeta_j}\frac{1}{\mathrm{d}j}$, not $\frac{\partial Y_i}{\partial \zeta_j}$. This is consistent with the case with an infinite but countable number of firms (appendix A.1), where the "external effect" was the average of $(N-1)\frac{\partial Y_i}{\partial \zeta_j}$, not of $\frac{\partial Y_i}{\partial \zeta_j}$[1]. Again, we assume that:

$$\frac{\partial Y_i}{\partial \zeta_j} \text{ is infinitesimal and } \mathcal{O}\Big(\mathrm{d}i\Big)$$

so that $\frac{1}{\mathrm{d}i}\frac{\partial Y_i}{\partial \zeta_j}$ is bounded. Then, define

$$\frac{\delta Y_i}{\delta \zeta_{-i}} = \int_{\mathcal{C}_i} \left[\frac{1}{\mathrm{d}j}\frac{\partial Y_i}{\partial \zeta_j}\right]\mathrm{d}j$$

Note that we have used the operator δ of functional derivative, instead of ∂ of the partial derivative, respecting the fact that ζ_{-i} is in fact a functional of a function $\mathcal{C}_i \to \mathbb{R}$.

[1] Remember that $\mathrm{d}i$ is the limit of $\Delta i = N^{-1}$.

The decomposition of the effects over firm i can then be written as:

$$\frac{\partial Y_i}{\partial \zeta} = \frac{\partial Y_i}{\partial \zeta_i} + \frac{\delta Y_i}{\delta \zeta_{-i}}$$

Now we define aggregate output as $Y = \int_0^1 Y_i \mathrm{d}i$. We then have:

$$\begin{aligned} \frac{\partial Y}{\partial \zeta} &= \int_0^1 \frac{\partial Y_i}{\partial \zeta_i} \mathrm{d}i + \int_0^1 \frac{\delta Y_i}{\delta \zeta_{-i}} \mathrm{d}i \\ &= \mathbb{E}_i \left[\frac{\partial Y_i}{\partial \zeta_i} \right] + \mathbb{E}_i \left[\frac{\delta Y_i}{\delta \zeta_{-i}} \right] \end{aligned}$$

which is the same expression we derived for the case with a countable number of firms.

A.3 Decomposition with sectors and firms

Consider an economy with S sectors and N firms in each sector $s \in \{1, \ldots, S\}$. Assume that the output firm i, who operates in sector s, is, in equilibrium $Y_{si} = Y_{si}\Big(\ R \ ; \ \{\zeta_{s'i'}\} \ \Big)$, where $\{\zeta_{s'i'}\}$ lists the access of all firms in all sectors: $\zeta_{11}, \zeta_{12}, \cdots, \zeta_{1N}, \zeta_{21}, \cdots, \zeta_{SN}$.

Sectoral output is given by:

$$Y_s = \frac{1}{N} \sum_{i=1}^{N} Y_{si}$$

And aggregate output is given by:

$$\begin{aligned} Y &= \frac{1}{S} \sum_{s=1}^{S} Y_s \\ &= \frac{1}{NS} \sum_{s=1}^{S} \sum_{i=1}^{N} Y_{si} \end{aligned}$$

Aggregate and sectoral levels of access to government credit — ζ and ζ_s — are defined accordingly. Total differentiation of the aggregate output in respect to all sector-firms levels of access imply:

$$\mathrm{d}Y = \frac{1}{NS} \sum_{s=1}^{S} \sum_{i=1}^{N} \sum_{\tilde{s}=1}^{S} \sum_{\tilde{i}=1}^{N} \frac{\partial Y_{si}}{\partial \zeta_{\tilde{s}\tilde{i}}} \mathrm{d}\zeta_{\tilde{s}\tilde{i}}$$

Again, we consider a change in credit policy such that $\mathrm{d}\zeta_{si} - \mathrm{d}\zeta$, for all sectors and firms. We then separate the sums:

$$\frac{\mathrm{d}Y}{\mathrm{d}\zeta} = \frac{1}{NS}\left[\sum_{s=1}^{S}\sum_{i=1}^{N}\frac{\partial Y_{si}}{\partial \zeta_{si}}\right] + \frac{1}{NS}\left[\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{\tilde{i}\neq i}\frac{\partial Y_{si}}{\partial \zeta_{s\tilde{i}}}\right] + \frac{1}{NS}\left[\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{\tilde{s}\neq s}\sum_{\tilde{i}}\frac{\partial Y_{si}}{\partial \zeta_{\tilde{s}\tilde{i}}}\right]$$

The first term on the right-hand side accounts for "own-effects", i.e., the effect over one firm's output when we change their own access to government credit. The second and third terms account for "external-effects", with the difference that the former considers same-sector externalities while the latter considers cross-sector externalities.

In order to make make the notation easier, define:

$$\frac{\partial Y_{si}}{\partial \zeta_{s,-i}} = \sum_{\tilde{i}\neq i}\frac{\partial Y_{si}}{\partial \zeta_{s\tilde{i}}}$$

to denote the effect on sector s-firm i's output when all other firms in the same sector have their levels of access changed. Also, define:

$$\frac{\partial Y_{si}}{\partial \zeta_{-s}} = \sum_{\tilde{s}\neq s}\sum_{\tilde{i}}\frac{\partial Y_{si}}{\partial \zeta_{\tilde{s}\tilde{i}}}$$

to denote the effect on sector s-firm i's output when the levels of firms in all other sectors change. With this investment in notation, we can finally write:

$$\underbrace{\frac{\partial Y}{\partial \zeta}}_{\text{Macro effect}} = \underbrace{\underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{si}}\right]}_{\text{Average of micro effects}} + \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{s,-i}}\right]}_{\substack{\text{Average of external effects}\\ \text{same sector}}}}_{\text{Sectoral micro effect}} + \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{-s}}\right]}_{\substack{\text{Average of external effects}\\ \text{other sectors}}}$$

Now, note that at the sectoral level we can do the following decomposition:

$$\underbrace{\frac{\partial Y_s}{\partial \zeta_s}}_{\text{Sectoral micro effect}} = \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{si}}\right]}_{\text{Average of micro effects}} + \underbrace{\mathbb{E}_i\left[\frac{\partial Y_{si}}{\partial \zeta_{s,-i}}\right]}_{\substack{\text{Average of external effects}\\ \text{same sector}}}$$

A.4 Decomposition for discrete changes in accessibility

The decomposition still holds if one consider discrete variations in access to government credit. For simplicity, here we show that this is the case for a simple case with two firms.

Again, in equilibrium each firm's variable of interest is a function of the monetary policy instance and of all firms' access to government credit:

$$Y_j = Y_j\Big(u^m\,; \zeta_1, \zeta_2\Big) \qquad j = \{1, 2\}$$

and we define the aggregate variable of interest to be $Y = \frac{1}{2}(Y_1 + Y_2)$. Now we consider the effect from moving the economy from a state of no government financing ($\zeta_1 = \zeta_2 = 0$) to a state of total government financing ($\zeta_1 = \zeta_2 = 1$). By definition, the macroeconomic effect of such a move is given by:

$$\text{macro effect} = Y(1,1) - Y(0,0)$$

where we have omitted the monetary policy argument in order to save on notation. Using the definition of the aggregate:

$$\text{macro effect} \quad = \quad \frac{1}{2}\Big[Y_1(1,1) - Y_1(0,0)\Big] + \frac{1}{2}\Big[Y_2(1,1) - Y_2(0,0)\Big]$$

Now, add and subtract both $\frac{1}{2}Y_0(0,1)$ and $\frac{1}{2}Y_1(1,0)$. Rearranging the terms in a convenient way, we get:

$$\begin{aligned} \text{macro effect} \quad &= \quad \underbrace{\frac{1}{2}\Bigg\{\Big[Y_1(1,1) - Y_1(0,1)\Big] + \Big[Y_2(1,1) - Y_2(1,0)\Big]\Bigg\}}_{\text{Avg. micro effect}} \\ &+ \quad \underbrace{\frac{1}{2}\Bigg\{\Big[Y_1(0,1) - Y_1(0,0)\Big] + \Big[Y_2(1,0) - Y_2(0,0)\Big]\Bigg\}}_{\text{Avg. external effect}} \end{aligned}$$

Note that the first line indeed captures the micro effect: we consider that happens to firm 1 when we change its access from $\zeta_1 = 0$ to $\zeta_1 = 1$ while keeping $\zeta_2 = 1$. And the second line captures the external effect: for firm 1 we change *firm 2's access* from $\zeta_2 = 0$ to $\zeta_2 = 1$, while keeping $\zeta_1 = 0$.

Because variation is discrete the decomposition is sensitive to the baseline chosen for comparison. In the decomposition above we have calculated the micro effect holding the other firms' access equal to 1, but we could have proceeded by holding other firms' access equal to 0, if we had added and subtracted $\frac{1}{2}Y_0(1,0)$ and $\frac{1}{2}Y_1(0,1)$ instead. Such problems are very common with decompositions. A reasonable compromise in this case is to take the average of both computations.

A.5 Summary of model's equations

Households

$$C_t^{-\sigma} = \beta R_t \mathbb{E}_t \left\{ C_{t+1}^{-\sigma} \right\} \tag{A-1}$$

$$C_t^{\sigma} H_t^{\eta} = W_t \tag{A-2}$$

$$R_t^n = R_t \mathbb{E}_t \Big\{ \Pi_{t+1} \Big\} \tag{A-3}$$

$$\Lambda_t = \beta \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \tag{A-4}$$

Firms: $\forall i \in [0, 1]$

$$Y_{it} = \Big(p_{it}\Big)^{-\varepsilon} Y_t \tag{A-5}$$

$$Y_{it} = H_{it}^{1-\alpha} \tag{A-6}$$

$$R_{it}^w = R_t + \zeta_i \Big(R^s - R_t \Big) \tag{A-7}$$

$$\text{MC}_{it} = \left(\frac{1}{1-\alpha} \right) W_t \bigg(1 + \psi \left(R_{it}^w - 1 \right) \bigg) \Big(Y_{it} \Big)^{\frac{\alpha}{1-\alpha}} \tag{A-8}$$

$$p_{it}^* = \left[\left(\frac{\varepsilon}{\varepsilon - 1} \right) \frac{u_{it}}{z_{it}} \right]^{\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}} \tag{A-9}$$

$$u_{it} = Y_t \text{MC}_{it} + \theta \mathbb{E}_t \left\{ \Pi_{t+1}^{\frac{\varepsilon}{1-\alpha}} \Lambda_{t+1} u_{i,t+1} \right\} \tag{A-10}$$

$$z_{it} = Y_t + \theta \mathbb{E}_t \left\{ \Pi_{t+1}^{\varepsilon-1} \Lambda_{t+1} z_{i,t+1} \right\} \tag{A-11}$$

$$\Pi_{it} = \Pi_t \frac{p_{it}}{p_{i,t-1}} \tag{A-12}$$

$$1 = \theta \Pi_{i,t}^{\varepsilon-1} + (1-\theta) \left(\frac{p_{it}^*}{p_{it}} \right)^{1-\varepsilon} \tag{A-13}$$

Monetary policy

$$R_t^n = \Big(R^n \Big) \Pi_t^{\phi} U_t^m \tag{A-14}$$

$$U_t^m = \Big(U_{t-1}^m \Big)^{\rho} \exp \Big(\varepsilon_t^m \Big) \tag{A-15}$$

Market clearing and price-level

$$Y_t = C_t \tag{A-16}$$

$$H_t = \int_0^1 H_{jt} \mathrm{d}j \tag{A-17}$$

$$1 = \int_0^1 p_{it}^{1-\varepsilon} \mathrm{d}i \tag{A-18}$$

Note that we have 9 "aggregate equations" (A.1-A.4, A.14-A.18) and 9 aggregate variables (C, H, Λ, Π, R, R^n, Y, W, U^m). Also, we have 9 "firm equations" (A.5 - A.13) for 9 firm variables (Y_i, H_i, R_i^w, MC_i, p_i^*, p_i, Π_i, u_i, z_i).

A.6 The log-linearized model

The set of equations below represent the log-linearized version of the set of equations presented in appendix A.5. The only adaptation is that we have eliminated the auxiliary variables u_i and z_i, together with the respective equations, in order to write down a Phillips curve for each type of firm.

Households

$$c_t = \mathbb{E}_t\Big\{c_{t+1}\Big\} - \sigma^{-1} r_t \tag{A-19}$$

$$w_t = \sigma c_t + \eta h_t \tag{A-20}$$

$$r_t^n = r_t + \mathbb{E}_t\Big\{\pi_{t+1}\Big\} \tag{A-21}$$

Firms: $\forall i \in [0,1]$

$$y_{it} = y_t - \varepsilon p_{it} \tag{A-22}$$

$$y_{it} = (1-\alpha) h_{it} \tag{A-23}$$

$$r_{it}^w = \left[\frac{(1-\zeta_i)\beta^{-1}}{R_i^w}\right] r_t \tag{A-24}$$

$$\mathrm{mc}_{it} = w_t + \left[\frac{\alpha}{1-\alpha}\right] y_{it} + \left[\frac{\psi R_i^w}{1+\psi(R_i^w - 1)}\right] r_{it}^w \tag{A-25}$$

$$\pi_{it} = \beta \mathbb{E}_t\Big\{\pi_{i,t+1}\Big\} + \left[\frac{(1-\theta)(1-\beta\theta)}{\theta}\right]\left[\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\right)\mathrm{mc}_{it} - p_{it}\right] \tag{A-26}$$

$$\pi_{it} = \pi_t + p_{it} - p_{i,t-1} \tag{A-27}$$

$$\pi_{it} = \left[\frac{1-\theta}{\theta}\right]\Big(p_{it}^* - p_{it}\Big) \tag{A-28}$$

Monetary policy

$$r_t^n = \phi \pi_t + u_t^m \tag{A-29}$$

$$u_t^m = \rho u_{t-1}^m + \varepsilon_t^m \tag{A-30}$$

Market clearing

$$y_t = c_t \tag{A-31}$$

$$h_t = \int \left(\frac{H_i}{H}\right) h_{it}\, \mathrm{d}i \tag{A-32}$$

$$0 = \int \Big(p_i\Big)^{1-\varepsilon} p_{it}\, \mathrm{d}i \tag{A-33}$$

A.7 A representative firm

Further simplifying the set of equations in each firm's block, we find:

$$\pi_{it} = \beta \mathbb{E}_t\Big\{\pi_{i,t+1}\Big\} + \lambda \left[w_t + \left(\frac{\alpha}{1-\alpha}\right) y_t\right] + \lambda \left[\left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)}\right)\right] r_t - \delta p_{it}$$

where, for simplicity, we have defined the following coefficient:

$$\lambda = \left(\frac{(1-\theta)(1-\beta\theta)}{\theta}\right)\left(\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\right) \qquad \delta = \lambda\left(\frac{1-\alpha+2\alpha\varepsilon}{1-\alpha}\right)$$

Note that we have two firm specific variables in this Phillips curve: p_i, π_i and y_i. And also note that:

$$0 = \int \Big(p_i\Big)^{1-\varepsilon} \hat{p}_{it} \mathrm{d}i \qquad \hat{\pi}_t = \int \Big(p_i\Big)^{1-\varepsilon} \hat{\pi}_{it} \mathrm{d}i \qquad y_t = \int \Big(p_i\Big)^{1-\varepsilon} y_{it} \mathrm{d}i$$

where the weights in the equation for aggregate output come from the conditional demand relation $\frac{Y_i}{Y} = \Big(p_i\Big)^{-\varepsilon}$. This means that if we aggregate firms' Phillips curves using weights $\Big(p_i\Big)^{1-\varepsilon}$ we can write:

$$\pi_t = \beta \mathbb{E}_t\Big\{\pi_{t+1}\Big\} + \lambda \left[w_t + \left(\frac{\alpha}{1-\alpha}\right) y_t\right] + \lambda \left[\int \Big(p_i\Big)^{1-\varepsilon} \left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)}\right) \mathrm{d}i\right] r_t$$

Now, if we define ζ — and, hence, $R^w = \beta^{-1} + \zeta\Big(R^s - \beta^{-1}\Big)$ — such that:

$$\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w - 1)}\right) = \int \Big(p_i\Big)^{1-\varepsilon} \left(\frac{\psi(1-\zeta_i)\beta^{-1}}{1+\psi(R_i^w - 1)}\right) \mathrm{d}i$$

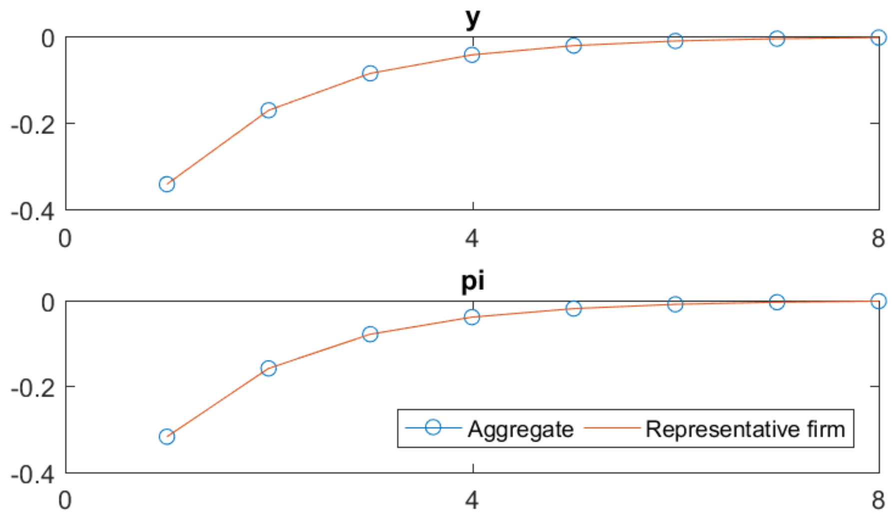
we recover an aggregate Phillips curve that is very similar to a firm-level Phillips curve — the difference is the absense of the term p_{it}:

$$\pi_t = \beta \mathbb{E}_t\Big\{\pi_{t+1}\Big\} + \lambda \left[w_t + \left(\frac{\alpha}{1-\alpha}\right) y_t\right] + \lambda \left[\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w - 1)}\right)\right] r_t$$

Now, in a model without heterogeneity where the firm has a level ζ of access to earmarked credit — ζ defined as above — its relative real price level will be equal to 1, by definition, and its log-deviation will be equal to zero. Hence, the aggregate Phillips curve we have just derived also represents the Phillips curve in this economy with a representative firm.

To illustrate the validity of our result I numerically solve a model with many firms without using this representative firm shortcut, and then compare the results with the ones I get by using a representative firm. Using 100 types of firms with ζ_i uniformly distributed over $[0, 1]$ implies that the representative firm has $\zeta = 0.501$, even though the mean of the distribution is exactly 0.5. Figure A.1 plots the impulse response functions we obtain by using both approaches, and shows that they are the same.

Figure A.1: Aggregate × representative firm's IRFs



A.8 Log-linearized model in canonical form

Starting from the aggregate Phillips curve from the last appendix section, and substituting into it the labor supply schedule together with the market-clearing condition for goods and labor, we find:

$$\pi_t = \beta\mathbb{E}_t\Big\{\pi_{t+1}\Big\} + \underbrace{\lambda\left[\sigma + \frac{\eta+\alpha}{1-\alpha}\right]}_{\equiv\kappa} y_t + \underbrace{\lambda\left[\left(\frac{\psi(1-\zeta)\beta^{-1}}{1+\psi(R^w-1)}\right)\right]}_{\equiv\gamma} r_t$$

which is the exactly the Phillips curve found in (21) textbook exposition of the New-Keynesian model (see page 49) except for the new term $\lambda\gamma r_t$, which captures the cost-channel.

Together with the usual IS curve (which combines households' Euler equation with the market-clearing condition for good):

$$y_t = \mathbb{E}_t\left\{y_{t+1}\right\} - \frac{1}{\sigma} r_t$$

and the monetary policy rule (which, here, we write in terms of the real interest rate with the help of the Fisher equation):

$$r_t = \phi_m \pi_t - \mathbb{E}_t \{\pi_{t+1}\} + u_t^m$$

this aggregate Phillips curve defines a simple three equation New-Keynesian model.

A.9
Solving for firm's relative price

Remember that the firm's relative price is determined by the following equilibrium condition:

$$p_{it} = \left(\frac{1}{1+\beta+\delta}\right) p_{i,t-1} + \left(\frac{\beta}{1+\beta+\delta}\right) \mathbb{E}_t \{p_{i,t+1}\} - \left(\frac{\gamma_i - \gamma}{1+\beta+\delta}\right) r_t$$

We solve this by the method of undetermined coefficients. Suppose a solution given by:

$$p_{i,t} = A p_{i,t-1} + B r_t$$

Substituting into the equilibrium condition, and remembering that $\mathbb{E}_t \{r_{t+1}\} = \rho r_t$, we get:

$$\left[1 - \left(\frac{\beta}{1+\beta+\delta}\right) A\right] p_t = \left(\frac{1}{1+\beta+\delta}\right) p_{i,t-1} + \left[\frac{\beta\rho B - \left(\gamma_i - \gamma\right)}{1+\beta+\delta}\right] r_t$$

Hence,

$$A = \left[1 - \left(\frac{\beta}{1+\beta+\delta}\right) A\right]^{-1} \left(\frac{1}{1+\beta+\delta}\right)$$
$$B = \left[1 - \left(\frac{\beta}{1+\beta+\delta}\right) A\right]^{-1} \left[\frac{\beta\rho B - \left(\gamma_i - \gamma\right)}{1+\beta+\delta}\right]$$

The first condition is a second degree equation in A

$$\beta A - \left(1+\beta+\delta\right) A + 1 = 0$$

with roots

$$A = \frac{\left(1+\beta+\delta\right) \pm \sqrt{\left(1+\beta+\delta\right)^2 - 4\beta}}{2\beta}$$

Let A_- and A_+ denote roots associated with the minus and plus signs, respectively. With some algebra we can place some bounds on this roots and find that $A_- \in [0,1]$ while $A_+ > 1$. Because our solution concept rules out explosive solution we then have that $A = A_-$. Substituting this into the condition for B and rearranging we get:

$$B = \frac{\left(\gamma_i - \gamma\right)}{(1 - \beta A_-) + \beta(1 - \rho) + \delta}$$

B
Appendix to Chapter 2

B.1
Summary of model's equations

Households

$$R_t = V_{t-1} R^n_{t-1} \frac{1}{\Pi_t} \tag{B-1}$$

$$\Big(C_t - \gamma C_{t-1}\Big)^{-\sigma} = \beta \mathbb{E}_t \left\{ R_{t+1} \Big(C_{t+1} - \gamma C_t\Big)^{-\sigma} \right\} \tag{B-2}$$

$$\chi \Big(C_t - \gamma C_{t-1}\Big)^{\sigma} H^{\eta}_t = W^h_t \tag{B-3}$$

$$\Lambda_t = \beta \left(\frac{C_t - \gamma C_{t-1}}{C_{t-1} - \gamma C_{t-2}} \right)^{-\sigma} \tag{B-4}$$

Firms

$$Y_{j,t} = (p_{j,t})^{-\left(1+\varepsilon^p_t\right)} Y_t \tag{B-5}$$

$$Y_{j,t} = A_t \Big(U_{j,t} K_{j,t-1}\Big)^{\alpha} \Big(H_{j,t}\Big)^{1-\alpha} \tag{B-6}$$

$$K_{j,t} = \Big[1 - \Delta(U_{j,t})\Big] K_{j,t-1} + I_{j,t} \left(1 - f\left(\frac{I_{j,t}}{I_{j,t-1}} \right) \right) \tag{B-7}$$

$$R^w_{jt} = \zeta_j R^s_t + (1 - \zeta_j) V_t R^n_t \tag{B-8}$$

$$X^p_t = \Big(\Pi_{t-1}\Big)^{\iota_p} \tag{B-9}$$

$$R^k_{j,t} = \mathcal{M}_{j,t} \alpha \frac{Y_{j,t}}{K_{j,t-1}} \tag{B-10}$$

$$\Pi_{j,t} = \frac{p_{j,t}}{p_{j,t-1}} \Pi_t \tag{B-11}$$

$$W_t = \mathcal{M}_{j,t} (1 - \alpha) \frac{Y_{j,t}}{H_{j,t}} \tag{B-12}$$

$$Q_{j,t} = \mathbb{E}_t \left\{ \Lambda_{t+1} R^k_{j,t+1} \right\} + \mathbb{E}_t \left\{ \Big[1 - \Delta(U_{j,t+1})\Big] \Lambda_{t+1} Q_{j,t+1} \right\} \tag{B-13}$$

$$R^k_{j,t} = Q_{j,t} U_{j,t} \Delta'\Big(U_{j,t}\Big) \tag{B-14}$$

$$\begin{aligned} \mathbb{E}_t \left\{ \frac{\Lambda_{t+1}}{\Pi_{t+1}} \right\} R^w_{j,t} &= Q_{j,t} Z_t \left[1 - f\left(\frac{I_{j,t}}{I_{j,t-1}} \right) - \left(\frac{I_{j,t}}{I_{j,t-1}} \right) f'\left(\frac{I_{j,t}}{I_{j,t-1}} \right) \right] \\ &+ \mathbb{E}_t \left\{ \Lambda_{t+1} Q_{t+1} Z_{t+1} \left(\frac{I_{j,t+1}}{I_{j,t}} \right)^2 f'\left(\frac{I_{j,t+1}}{I_{j,t}} \right) \right\} \end{aligned} \tag{B-15}$$

$$\begin{aligned} p_{j,t} &= \left(\frac{\varepsilon^p_t + 1}{\varepsilon^p_t} \right) \mathcal{M}_{j,t} \\ &- \left[\frac{\Big(p_{j,t}\Big)^{\varepsilon^p_t + 1}}{\varepsilon^p_t} \right] \left[\Psi'_p \left(\frac{\Pi_{j,t}}{X^p_t} \right) \frac{\Pi_{j,t}}{X^p_t} - \mathbb{E}_t \left\{ \Lambda_{t+1} \Psi'_p \left(\frac{\Pi_{j,t+1}}{X^p_{t+1}} \right) \frac{\Pi_{j,t+1}}{X^p_{t+1}} \frac{Y_{t+1}}{Y_t} \right\} \right] \end{aligned} \tag{B-16}$$

Unions

$$X_t^w = \left(\Pi_{t-1}\right)^{\iota_w} \tag{B-17}$$

$$\Pi_t^w = \frac{W_t}{W_{t-1}}\Pi_t \tag{B-18}$$

$$W_t = \left(\frac{1+\varepsilon_t^w}{\varepsilon_t^w}\right) W_t^h - \left(\frac{W_t}{\varepsilon_t^w}\right)\left[\Psi'_w\left(\frac{\Pi_t^w}{X_t^w}\right)\frac{\Pi_t^w}{X_t^w} - \mathbb{E}_t\left\{\Lambda_{t+1}\Psi'_w\left(\frac{\Pi_{t+1}^w}{X_{t+1}^w}\right)\frac{\Pi_{t+1}^w}{X_{t+1}^w}\frac{H_{t+1}W_{t+1}}{H_t W_t}\right\}\right] \tag{B-19}$$

Monetary policy

$$R_t^n = \left(R^n\right)^{1-\phi_r}\left[\left(\Pi_t\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{\phi_r} U_t^m \tag{B-20}$$

Market clearing and price level

$$Y_t = Y_t = C_t + I_t + G_t \tag{B-21}$$

$$H_t = \int_0^1 H_{j,t}\mathrm{d}j \tag{B-22}$$

$$I_t = \int_0^1 I_{j,t}\mathrm{d}j \tag{B-23}$$

$$1 = \int_0^1 \left(p_{j,t}\right)^{-\varepsilon_t^p}\mathrm{d}j \tag{B-24}$$

where:

$$f\left(x\right) = \frac{1}{2}\left(e^{\sqrt{\kappa}(x-1)} + e^{-\sqrt{\kappa}(x-1)} - 2\right)$$

$$\Delta\left(x\right) = \delta_0 + \frac{\delta_1}{1+\delta_2}\left(x^{1+\delta_2} - 1\right)$$

$$\Psi_p\left(x\right) = \frac{\psi_p}{2}\left(x-1\right)^2$$

$$\Psi_w\left(x\right) = \frac{\psi_w}{2}\left(x-1\right)^2$$

This set of equations determines the path for the following 24 variables:

$C \quad W^h \quad R \quad \Lambda$

$Y_j \quad \mathcal{M}_j \quad K_j \quad R_j^w \quad X^p \quad R_j^k \quad \Pi_j \quad H_j \quad Q_j \quad U_j \quad I_j \quad p_j$

$X^w \quad \Pi^w \quad W$

R^n

$Y \quad H \quad I \quad \Pi$

B.2
Summary of model's equations — log-linearized

Households

$$r_t = v_{t-1} + r^n_{t-1} - \pi_t \tag{B-25}$$

$$c_t - \gamma c_{t-1} = \mathbb{E}_t\Big\{c_{t+1}\Big\} - \gamma c_t - \left(\frac{1-\gamma}{\sigma}\right)\mathbb{E}_t\left\{r_{t+1}\right\} \tag{B-26}$$

$$w^h_t = \left(\frac{\sigma}{1-\gamma}\right)\Big(c_t - \gamma c_{t-1}\Big) + \eta h^h_t \tag{B-27}$$

$$\lambda_t = \left(\frac{-\sigma}{1-\gamma}\right)(c_t - (1+\gamma)c_{t-1} + \gamma c_{t-2}) \tag{B-28}$$

Firms

$$y_{jt} = y_t - \epsilon_p\Big[p_{j,t} + \ln\Big(p_j\Big)\varepsilon^p_t\Big] \tag{B-29}$$

$$y_{jt} = a_t + \alpha\Big(u_t + k_{j,t-1}\Big) + (1-\alpha)h_{jt} \tag{B-30}$$

$$k_{j,t} = (1-\delta_0)k_{j,t-1} + \delta_0\Big(i_{j,t} + z_t\Big) - \delta_1 u_{j,t} \tag{B-31}$$

$$r^w_{jt} = \left[\frac{(1-\zeta_j)\frac{1}{\beta}}{\zeta_j R^s + (1-\zeta_j)\frac{1}{\beta}}\right] r^n_t + \left[\frac{\zeta_j R^s}{\zeta_j R^s + (1-\zeta_j)\frac{1}{\beta}}\right] r^s_t \tag{B-32}$$

$$x^p_t = \iota_p\,\pi_{t-1} \tag{B-33}$$

$$r^k_{j,t} = m_{j,t} + y_{j,t} - k_{j,t-1} \tag{B-34}$$

$$\pi_{j,t} = p_{j,t} - p_{j,t-1} + \pi_t \tag{B-35}$$

$$w_t = m_{j,t} + y_{j,t} - h_{j,t} \tag{B-36}$$

$$\begin{aligned} q_{j,t} = {} & \Big[1 - \beta(1-\delta_0)\Big]\mathbb{E}_t\left\{r^k_{j,t+1} - u_{j,t+1}\right\} \\ & + \Big[\beta(1-\delta_0)\Big]\mathbb{E}_t\left\{q_{j,t+1}\right\} - \mathbb{E}_t\left\{r_{t+1}\right\} \end{aligned} \tag{B-37}$$

$$u_{j,t} = \left[\frac{1}{1+\delta_2}\right]\Big(r^k_{j,t} - q_{j,t}\Big) \tag{B-38}$$

$$\begin{aligned} i_{j,t} = {} & \left[\frac{1}{(1+\beta)\kappa}\right]\Big[q_{j,t} + z_t + v_t + r^n_t - r^w_{j,t}\Big] \\ & + \left[\frac{1}{1+\beta}\right] i_{j,t-1} + \left[\frac{\beta}{1+\beta}\right]\mathbb{E}_t\left\{i_{j,t+1}\right\} \end{aligned} \tag{B-39}$$

$$\begin{aligned} \pi_{j,t} = {} & x^p_t + \beta\mathbb{E}_t\left\{\pi_{j,t+1} - x^p_{t+1}\right\} + \\ & \left[\frac{(\epsilon_p - 1)\Big(p_j\Big)^{1-\epsilon_p}}{\psi_p}\right]\Big[m_{j,t} - p_{j,t} - (\epsilon_p)^{-1}\varepsilon^p_t\Big] \end{aligned} \tag{B-40}$$

Unions

$$x_t^w = \iota_w \, \pi_{t-1} \tag{B-41}$$

$$\pi_t^w = w_t - w_{t-1} + \pi_t \tag{B-42}$$

$$\pi_t^w = x_t^w + \beta \mathbb{E}_t \left\{ \pi_{t+1}^w - x_{t+1}^w \right\} + \left[\frac{(\epsilon_w - 1)W}{\psi_w} \right] \left[w_t^h - w_t - (\epsilon_w)^{-1} \varepsilon_t^w \right] \tag{B-43}$$

Monetary policy

$$r_t^n = \phi_r r_{t-1}^n + (1 - \phi_r) \left[\phi_\pi \pi_t + \phi_y y_t \right] + u_t^m \tag{B-44}$$

Market clearing

$$y_t = \left(\frac{C}{Y} \right) c_t + \left(\frac{I}{Y} \right) i_t + \left(\frac{G}{Y} \right) g_t \tag{B-45}$$

$$h_t = \int_0^1 \left(\frac{H_j}{H} \right) h_{j,t} \mathrm{d}j \tag{B-46}$$

$$i_t = \int_0^1 \left(\frac{I_j}{I} \right) \mathrm{d}j \tag{B-47}$$

$$0 = \int_0^1 \left(p_j \right)^{1-\epsilon_p} p_{j,t} \mathrm{d}j \tag{B-48}$$

For the relant steady-state values, check appendix B.3

B.3 Steady-state

B.3.1 An aggregation valid for the deterministic steady-state

Let start with firms' equations:

$$\begin{aligned}
Y_j &= Y\Big(p_j\Big)^{-\epsilon_p} \\
Y_j &= \Big(U_j K_j\Big)^{\alpha}\Big(H_j\Big)^{1-\alpha} \\
I_j &= \delta K_j \\
R^k_j &= \mathcal{M}_j \alpha \frac{Y_j}{K_j} \\
W &= \mathcal{M}_j (1-\alpha)\frac{Y_j}{H_j} \\
\Big(1-\beta(1-\delta)\Big)Q_j &= \beta R^k_j \\
R^k_j &= Q_j U_j \Delta'\Big(U_j\Big) \\
Q_j &= \beta R^w_j \\
p_{j,t} &= \left(\frac{\epsilon_p}{\epsilon_p - 1}\right)\mathcal{M}_j
\end{aligned}$$

First, note that the 6th equation implies that the ratio $R^k_j \Big/ Q_t$ is equalized across firms in steady-state. Hence, the 7th implies that the same is true for U_j. As will bee shown further ahead I parameterize $\Delta(\cdot)$ so that $U = 1$ in steady-state.

Substituting many of the equations into the second, we find an expression for the real marginal cost:

$$\mathcal{M}_j = \left(\frac{R^k_j}{\alpha}\right)^{\alpha}\left(\frac{W}{1-\alpha}\right)^{(1-\alpha)}$$

Since R^k_j is proportional to R^w_j (combine 6th and 8th) the ratio between two firms' marginal costs and, hence, prices, is given by:

$$\frac{p_i}{p_j} = \left(\frac{R^w_i}{R^w_j}\right)^{\alpha}$$

The ratios for output, labor and capital are then given by:

$$\frac{Y_i}{Y_j} = \left(\frac{R^w_i}{R^w_j}\right)^{-\alpha\epsilon_p} \qquad \frac{H_i}{H_j} = \left(\frac{R^w_i}{R^w_j}\right)^{\alpha(1-\epsilon_p)} \qquad \frac{K_i}{K_j} = \left(\frac{R^w_i}{R^w_j}\right)^{\alpha(1-\epsilon_p)-1}$$

such that firms with access to cheaper credit produce more, have a higher capital stock and higher more workers. Finally, note that $\frac{I_i}{I_j} = \frac{K_i}{K_j}$.

Now, lets turn to aggregation. The market clearing conditions

$$H = \int_0^1 H_i \mathrm{d}i \qquad I = \int_0^1 I_i \mathrm{d}i \qquad Y = \left(\int_0^1 Y_i^{\frac{\epsilon_p - 1}{\epsilon_p}} \mathrm{d}i \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

Can be more concisely written as:

$$H = \emptyset_H(j) H_j \qquad I = \emptyset_I(j) I_j \qquad Y = \emptyset_Y(j) Y_j \qquad 1 = \emptyset_p(j) p_j$$

by defining the quantities

$$\emptyset_H(j) = \int_0^1 \frac{H_i}{H_j} \mathrm{d}i \qquad \emptyset_I(j) = \int_0^1 \frac{I_i}{I_j} \mathrm{d}i \quad \emptyset_Y(j) = \left[\int_0^1 \left(\frac{Y_i}{Y_j} \right)^{\frac{\epsilon_p - 1}{\epsilon_p}} \mathrm{d}i \right]^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

$$\emptyset_p(j) = \int_0^1 \left(\frac{p_i}{p_j} \right)^{1 - \epsilon_p} \mathrm{d}i$$

Note that the following relations hold:

$$\emptyset_H(j) = \emptyset_p(j) = \left[\emptyset_Y(j) \right]^{\frac{\epsilon_p - 1}{\epsilon_p}}$$

Also, consider there are appropriate aggregations for other relevant variables: $K, R^k, \mathcal{M}$ and R^w. For these we are not constrained by any market clearing condition or feasibility condition, so we are somewhat free to define the aggregators that work the best. With them, I can write:

$$\begin{aligned}
\emptyset_Y(j) &= [\emptyset_p(j)]^{-\epsilon_p} \\
Y &= \left(\frac{\emptyset_Y(j)}{[\emptyset_K(j)]^{\alpha} \, [\emptyset_H(j)]^{1-\alpha}} \right) K^{\alpha} H^{1-\alpha} \\
I &= \left[\frac{\emptyset_I(j)}{\emptyset_K(j)} \right] \delta K \\
R^k &= \left[\frac{\emptyset_K(j) \, \emptyset_{R^k}(j)}{\emptyset_Y(j) \, \emptyset_{\mathcal{M}}(j)} \right] \mathcal{M} \alpha \frac{Y}{K} \\
W &= \left[\frac{\emptyset_H(j)}{\emptyset_Y(j) \, \emptyset_{\mathcal{M}}(j)} \right] M(1-\alpha) \frac{Y}{H} \\
R^k &= \left[\frac{\emptyset_{R^k}(j)}{\emptyset_{R^w}(j)} \right] \Big(1 - \beta(1-\delta) \Big) R^w \\
1 &= \left[\frac{\emptyset_p(j)}{\emptyset_{\mathcal{M}}(j)} \right] \left(\frac{\epsilon_p}{\epsilon_p - 1} \right) \mathcal{M}
\end{aligned}$$

From the first equation, and the relations that must hold, we conclude that $\emptyset_H(j) = \emptyset_p(j) = \emptyset_Y(j) = 1$. Hence, in the aggregation we should consider a representative firm j whose relative price is one and, hence, its output is equal to aggregate output. Simplifying:

$$\begin{aligned}
Y &= \Big[\emptyset_K(j)\Big]^{-\alpha} K^\alpha H^{1-\alpha} \\
I &= \left[\frac{\emptyset_I(j)}{\emptyset_K(j)}\right] \delta K \\
R^k &= [\emptyset_K(j)\, \emptyset_{R^k}(j)] \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) \alpha \frac{Y}{K} \\
W &= \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) (1-\alpha) \frac{Y}{H} \\
R^k &= \left[\frac{\emptyset_{R^k}(j)}{\emptyset_{R^w}(j)}\right] \Big(1 - \beta(1-\delta)\Big) R^w
\end{aligned}$$

Finally, note that by defining $\emptyset_K(j)$, $\emptyset_{R^k}(j)$ and $\emptyset_{R^w}(j)$ such that $\emptyset_{R^w}(j) = \emptyset_{R^k}(j) = \frac{1}{\emptyset_K(j)} = \frac{1}{\emptyset_I(j)}$, and also $\emptyset_{\mathcal{M}} = 1$, we have:

$$\begin{aligned}
Y &= \Big[\emptyset_{R^w}(j)\Big]^{\alpha} K^\alpha H^{1-\alpha} \\
I &= \delta K \\
R^k &= \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) \alpha \frac{Y}{K} \\
W &= \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) (1-\alpha) \frac{Y}{H} \\
R^k &= \Big(1 - \beta(1-\delta)\Big) R^w \\
1 &= \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) \mathcal{M}
\end{aligned}$$

which are steady-state conditions for an economy with a representative firm, but whose total factor productivity is multiplied by the factor $\Big[\emptyset_K(j)\Big]^{-\alpha}$ which takes into the potential misallocation of capital. In the analogous economy with a representative firm, the overall importance of earmarked credit, ζ, must be such that:

$$\begin{aligned}
\frac{1}{\beta} + \zeta \left(R^s - \frac{1}{\beta}\right) &= R^w \\
&= \emptyset_{R^w}(j) R^j_w \\
&= \left[\int_0^1 (R^w_i)^{\alpha(1-\epsilon_p)-1}\, \mathrm{d}i\right]^{-1} \left(R^w_j\right)^{\alpha(1-\epsilon_p)}
\end{aligned}$$

a somewhat complicated function of the whole distribution of $\{\zeta_i\}$. Finally, note that for all firms I can write the following ratios:

$$\frac{I_i}{I} = \left(\frac{R_i^w}{R^w}\right)^{\alpha(1-\epsilon_p)-1} \qquad \frac{H_i}{H} = \frac{\left(R_i^w\right)^{\alpha(1-\epsilon_p)}}{R^w \int_0^1 \left(R_j^w\right)^{\alpha(1-\epsilon_p)-1} dj}$$

$$\left(p_i\right)^{1-\epsilon_p} = \frac{\left(R_i^w\right)^{\alpha(1-\epsilon_p)}}{R^w \int_0^1 \left(R_j^w\right)^{\alpha(1-\epsilon_p)-1} dj}$$

B.3.2 Computing the steady-state

I have shown that for the computation of the steady-state we can work with an economy with a representative firm, whose production function is

$$Y = A\left(UK\right)^{\alpha} H^{1-\alpha}$$

with $A = \left[\emptyset_K(j)\right]^{-\alpha}$, and whose effective investment interest rate is given by:

$$R^w = \left[\int_0^1 \left(R_i^w\right)^{\alpha(1-\epsilon_p)-1} di\right]^{-1} \left(R_j^w\right)^{\alpha(1-\epsilon_p)}$$

where j is index of the firm whose relative price is one. I proceed from here. Isolate W in the equation for the marginal cost, $\mathcal{M} = \left(\frac{R^k}{\alpha}\right)^{\alpha} \left(\frac{W}{1-\alpha}\right)^{(1-\alpha)}$:

$$\mathcal{W} = (1-\alpha)\left[\mathcal{M}\frac{\alpha}{R^k}\right]^{\frac{1}{1-\alpha}} \left(\frac{R^k}{\alpha}\right)$$

Note that $\mathcal{M} = \frac{\epsilon_p - 1}{\epsilon_p}$ and $R^k = \left(1 - \beta(1-\delta)\right)R^w$ are straightforward functions of the parameters. From the definition of R^k note that:

$$\frac{K}{Y} = \mathcal{M}\frac{\alpha}{R^k}$$

Substituting this relation into the production function, and using $U = 1$, we have:

$$\frac{H}{Y} = \left[\mathcal{M}\frac{\alpha}{R^k}\right]^{\frac{-\alpha}{1-\alpha}} \left(A\right)^{\frac{-1}{1-\alpha}}$$

From the law of motion for capital, $I = \delta K$. The market clearing condition is then rewritten as:

$$C = \left[1 - \delta\frac{\alpha\mathcal{M}}{R^k} - \frac{G}{Y}\right] Y$$

Now, substituting into the labor supply schedule, $\chi(1-\gamma)^{\sigma} C^{\sigma} H^{\eta} =$

$W\mathcal{M}_w$ — where $\mathcal{M}_w = \frac{\epsilon_w - 1}{\epsilon_w}$:

$$\chi(1-\gamma)^{\sigma}\left[1-\delta\frac{\alpha\mathcal{M}}{R^k}-\frac{G}{Y}\right]^{\sigma}\left[\mathcal{M}\frac{\alpha}{R^k}\right]^{\frac{-\alpha\eta}{1-\alpha}}\big(A\big)^{\frac{-\eta}{1-\alpha}}\big(Y\big)^{\sigma+\eta} = \mathcal{M}_w\left[(1-\alpha)\left(\mathcal{M}\frac{\alpha}{R^k}\right)^{\frac{1}{1-\alpha}}\left(\frac{R^k}{\alpha}\right)\right]$$

Solving for Y:

$$Y = \left\{\frac{(1-\alpha)\mathcal{M}_w}{\chi(1-\gamma)^{\sigma}}\big(\mathcal{M}\big)^{\frac{1+\alpha\eta}{1-\alpha}}\left(\frac{\alpha}{R^k}\right)^{\frac{\alpha(1+\eta)}{1-\alpha}}\left[1-\delta\frac{\alpha\mathcal{M}}{R^k}-\frac{G}{Y}\right]^{-\sigma}\big(A\big)^{\frac{\eta}{1-\alpha}}\right\}^{\frac{1}{\sigma+\eta}}$$

B.4
More on the Data

National accounts data (output, private consumption, governmental consumption and investment). Raw quarterly data is obtained from the *Sistema de Contas Nacionais Trimestrais* (IBGE), already seasonally adjusted. The log of the chained series is detrended with a one-sided HP filter, and growth is computed for the cyclical component.

Employment and wages. Raw data comes from three different households surveys by IBGE: (i) the first *Pesquisa Mensal do Emprego*, which covers the period covering from 1991:01 to 2002:12; (ii) the second *Pesquisa Mensal do Emprego*, which covers the interval 2002:03- 2016:02; and (iii) the *Pesquisa Nacional por Amostra de Domicílios Contínua*, which exists since 2012:1. Hence for each variable I have three raw series (one from each survey) covering different time periods, with a small overlap. In order to build a chained series I adopt the following procedure: first I take log changes to compute growth rates; then, for overlapping periods, I take a weighted average of the values in the surveys covering the period. The weight is linearly decaying for the discontinued survey and linearly increasing for the new survey, to ensure a smooth transition[1]. For non-overlapping period the available growth rate is used. These growth rates are used to build chained level series in logs, which are then seasonally adjusted, using X-13 ARIMA-SEATS procedures. The seasonally adjusted series are then detrended with a one-sided HP filter, and the final growth rates are computed for the cyclical component.

A few additional comments: (i) although PNADC has national coverage, I restrict focus to the same metropolitan areas covered by PME; (ii) PNADC

[1] For instance, the transition from PME to PNADC has 17 quarters of overlap. The resulting PME weight is 17/18 for the first of these quarters and 1/18 for the last.

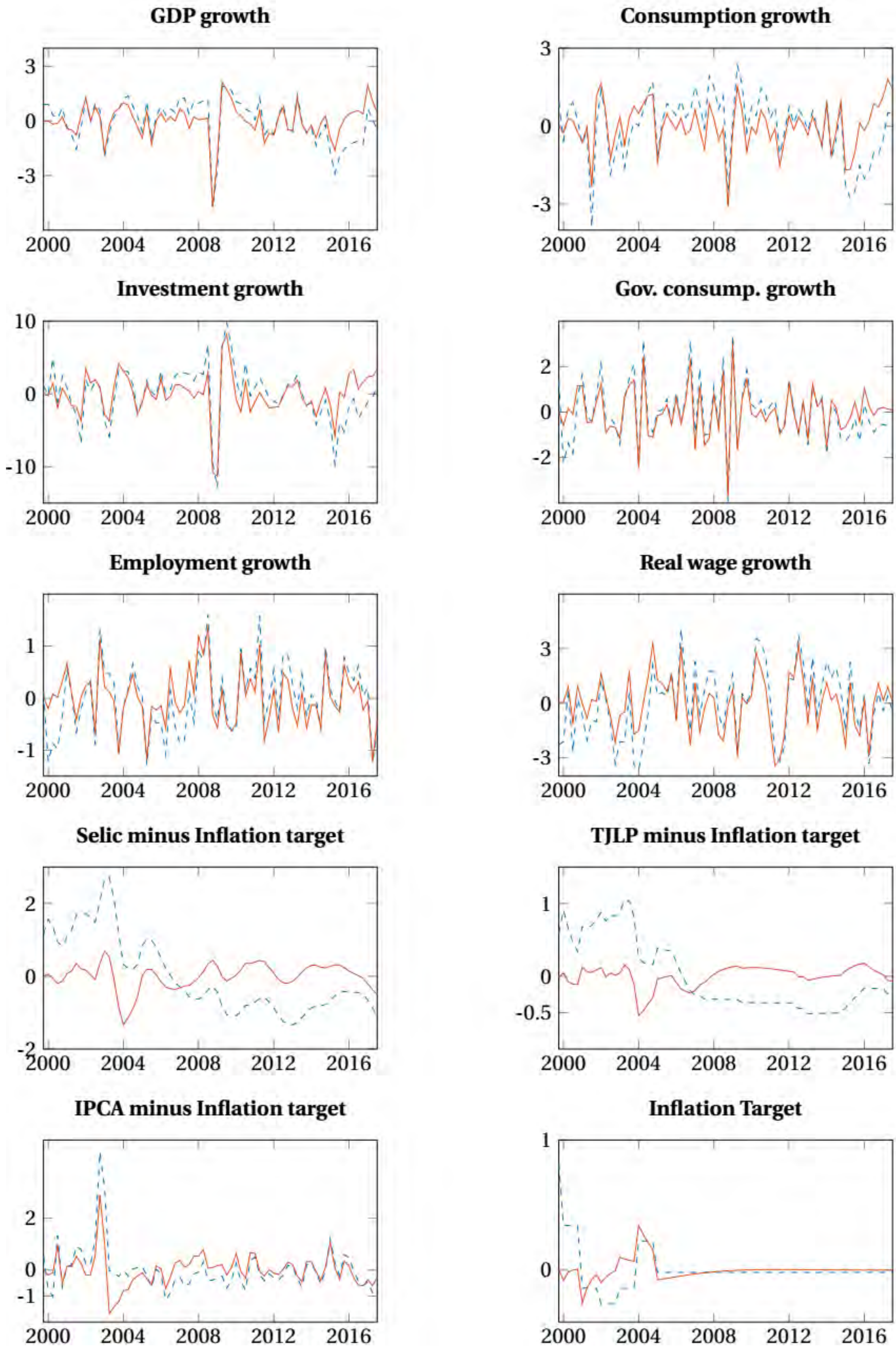
data is quarterly, and the monthly PME data is transformed to quartely data by accumulating the growth rates; (iii) for real wages I first I take the series for effectively received nominal wages and then I deflate them using the IPCA.

IPCA. Raw data comes from IBGE in monthly frequency. I transform it to quaterly frequency by accumulating inflation rates. To deal with the fact that the inflation target has changed in the sample, whereas in the model such thing does not occur, I subtract the prevailing target (quarterly) rate from each of these series. No seasonal adjustment is applied. A level series in log is constructed and detrended with a one-sided HP filter. Inflation rates are then computed by taking differences in the resulting cyclical component.

Selic and TJLP. Raw data comes from the *Sistema Gerenciador de Séries Temporais* (BCB), in daily frequency for Selic and monthly frequency for TJLP. Both are transformed to quarterly by averaging. To deal with the fact that the inflation target has changed in the sample, whereas in the model such thing does not occur, I subtract the prevailing target (quarterly) rate from each of these series. No seasonal adjustment is applied. A level series in log is built and detrended with a one-sided HP filter. The final Selic and TJLP series used for estimation are then computed by differencing the resulting cyclical component.

Visualization. In figure B.1, the dashed blue lines represent demeaned data and the solid red lines represent the data filtered with the one-sided HP filter. The former is presented for illustrative purpose, as only the later is used for estimation.

Figure B.1: Data



B.5
Parameters' marginal distributions: prior vs. posterior

Figure B.2: Prior vs. posterior distributions (1/2)

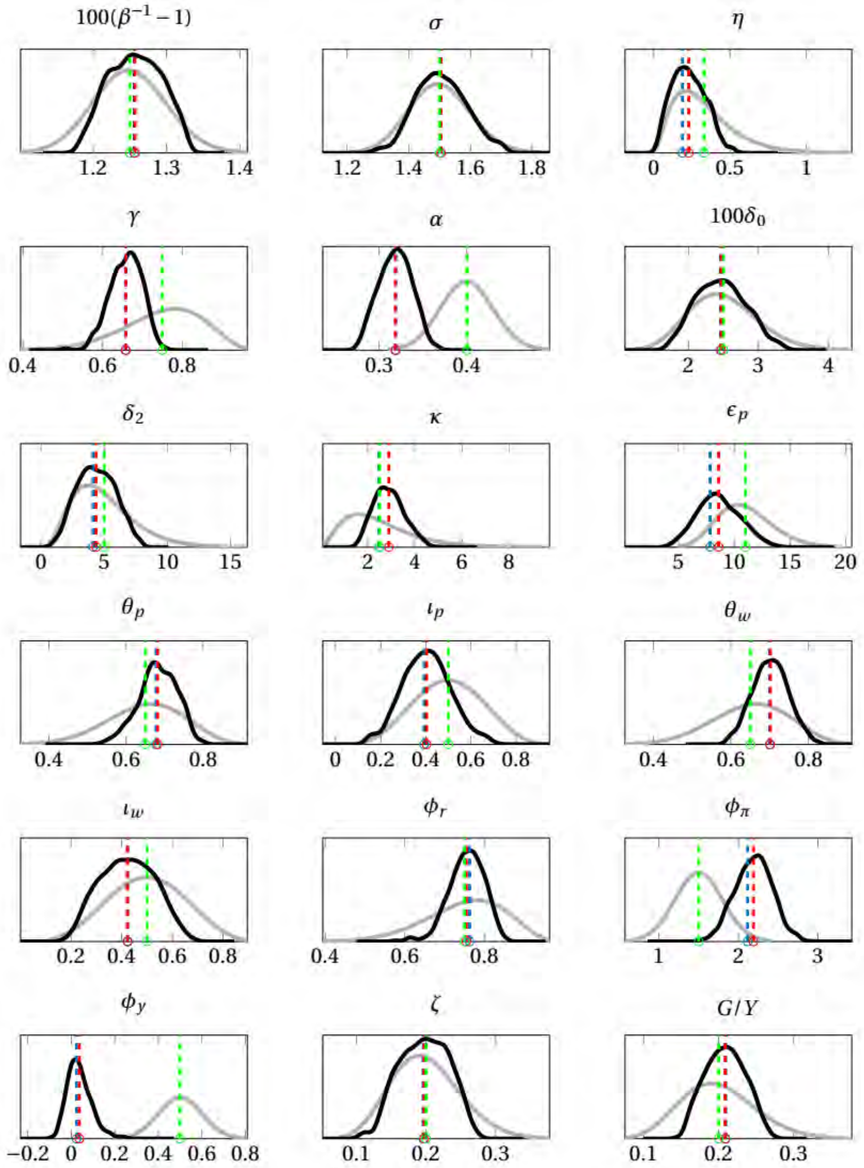
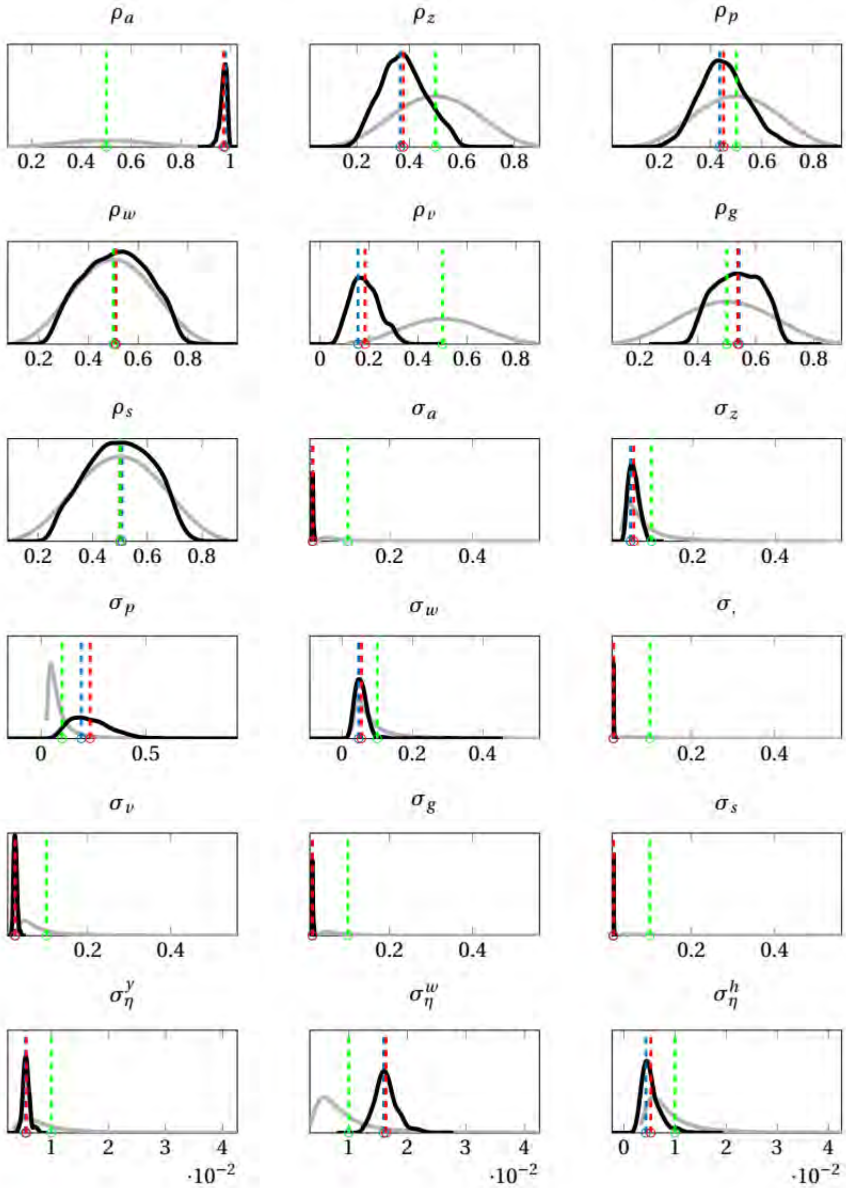


Figure B.3: Prior vs. posterior distributions (2/2)



B.6
IRFs to other shocks

Figure B.4: IRFs to a positive 1 st. dev. TFP shock — posterior distribution

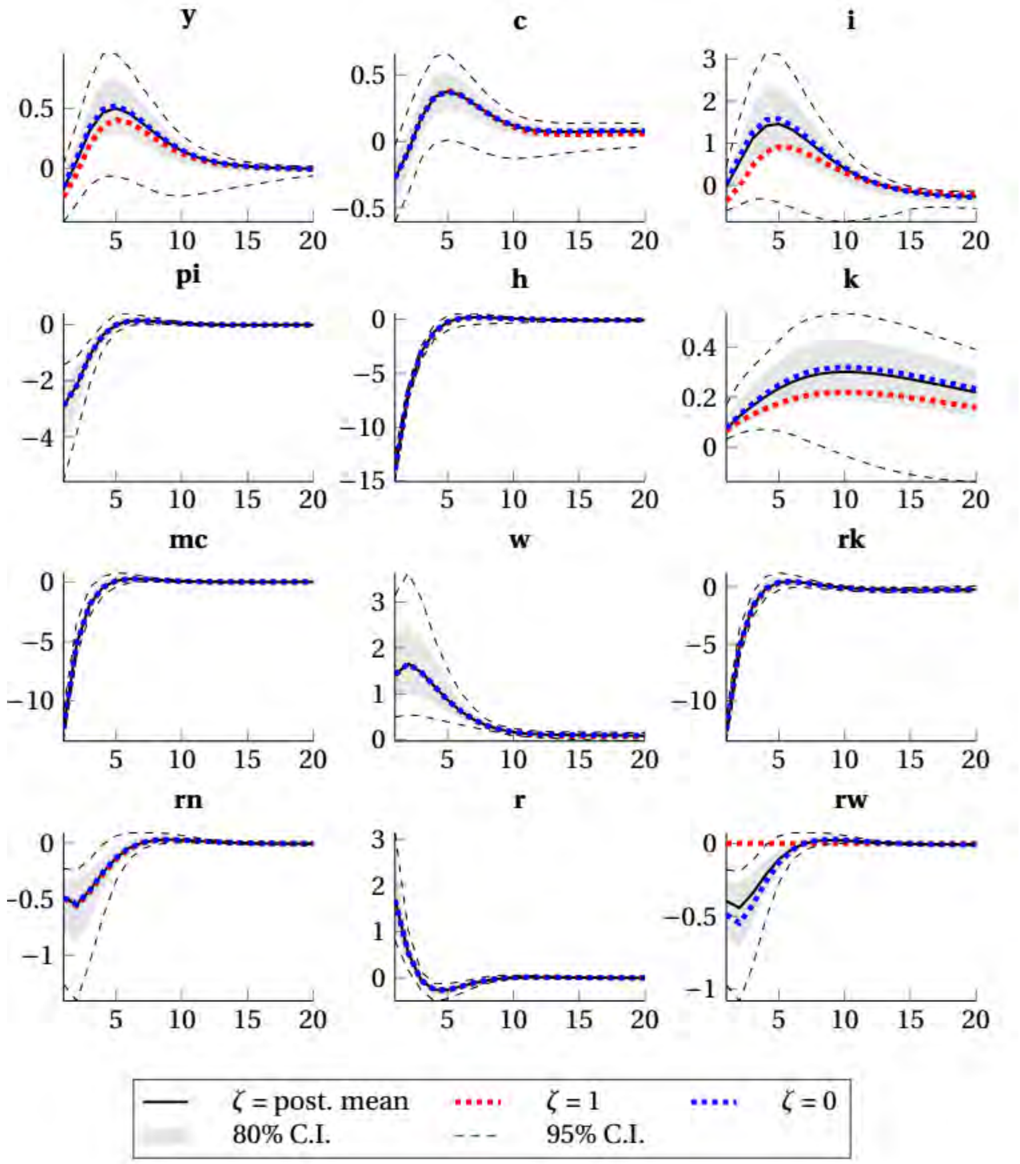


Figure B.5: IRFs to a positive 1 st. dev. investment-specific productivity shock — posterior distribution

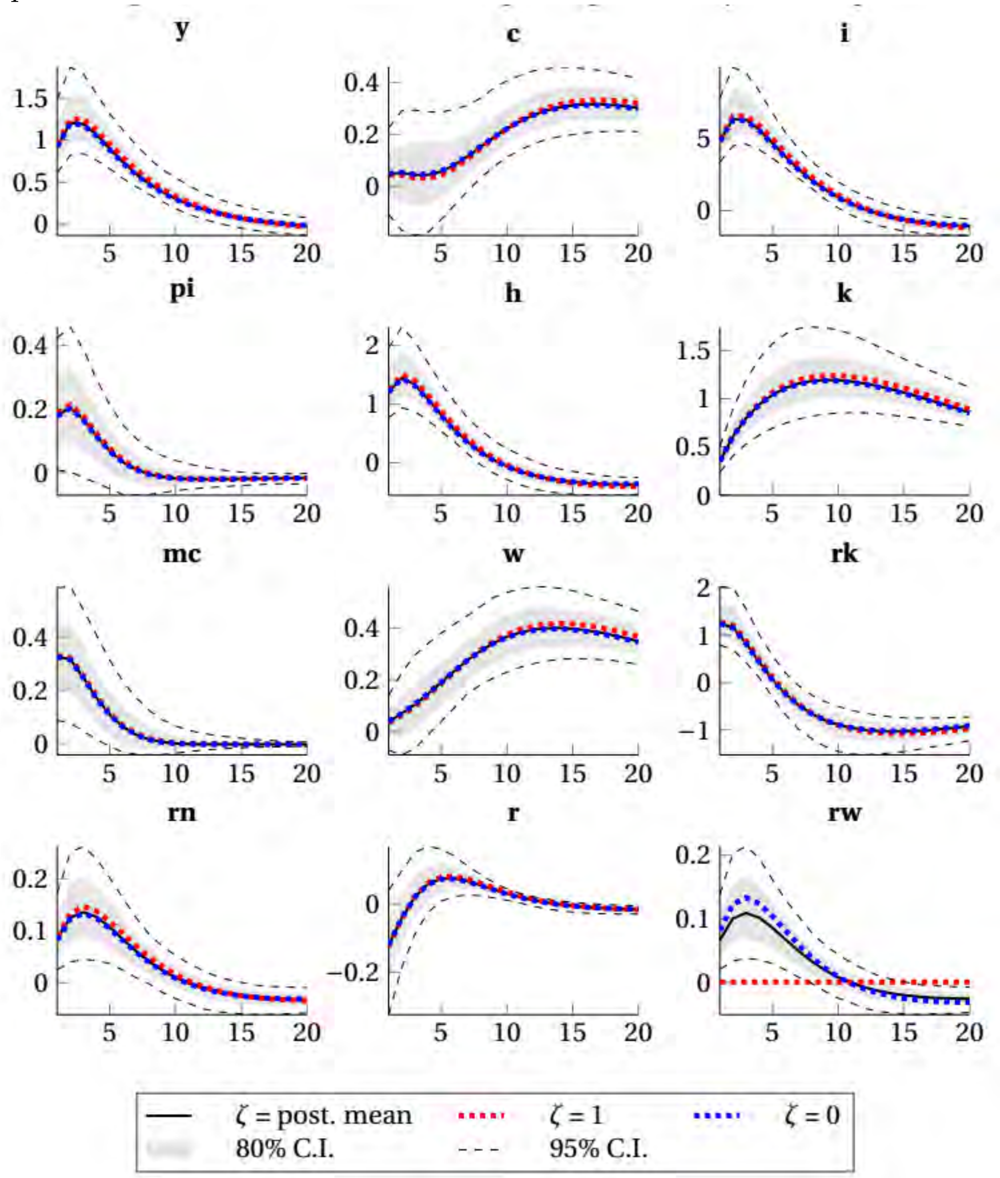


Figure B.6: IRFs to a negative 1 st. dev. price mark-up shock — posterior distribution

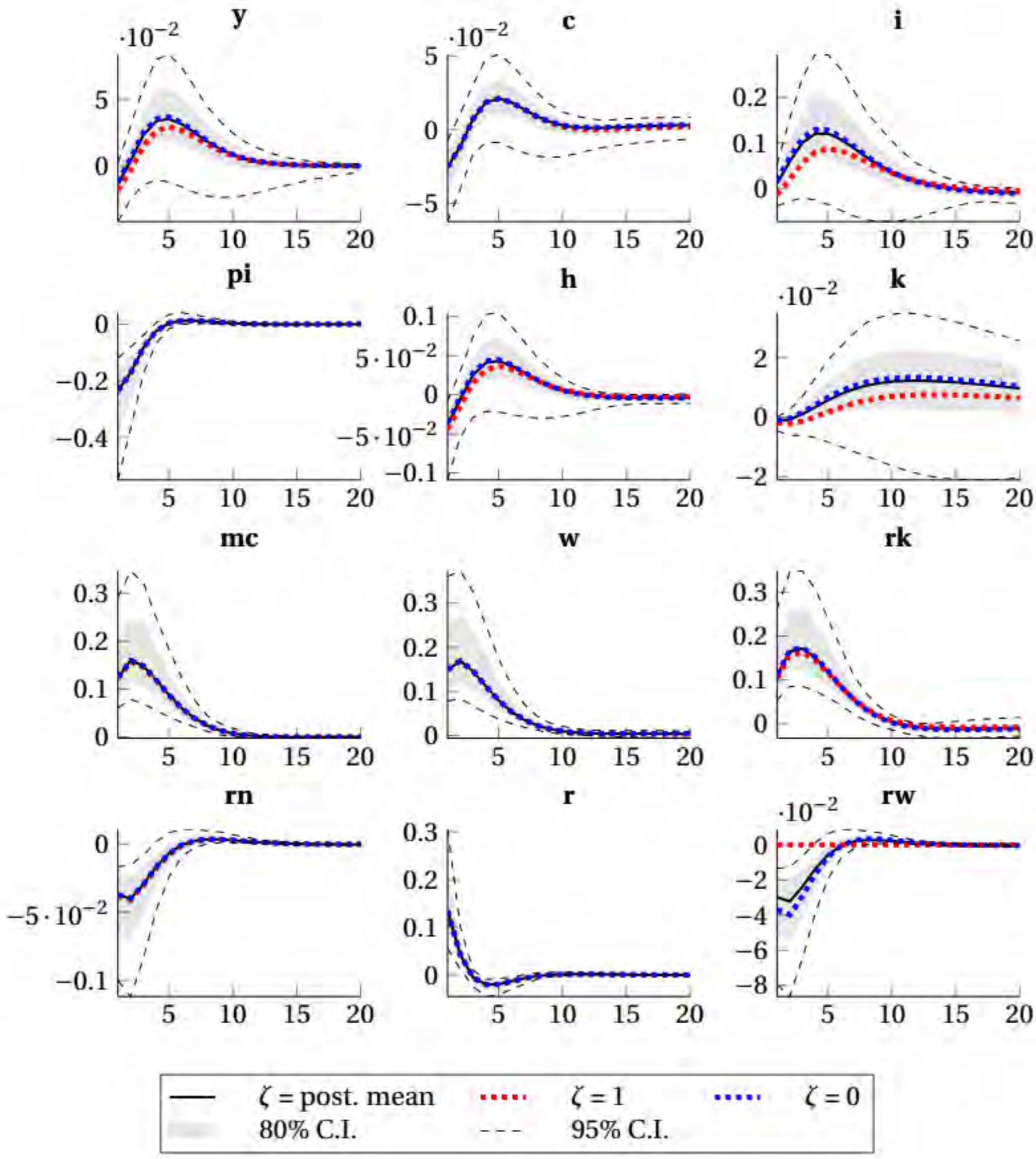


Figure B.7: IRFs to a negative 1 st. dev. wage mark-up shock — posterior distribution

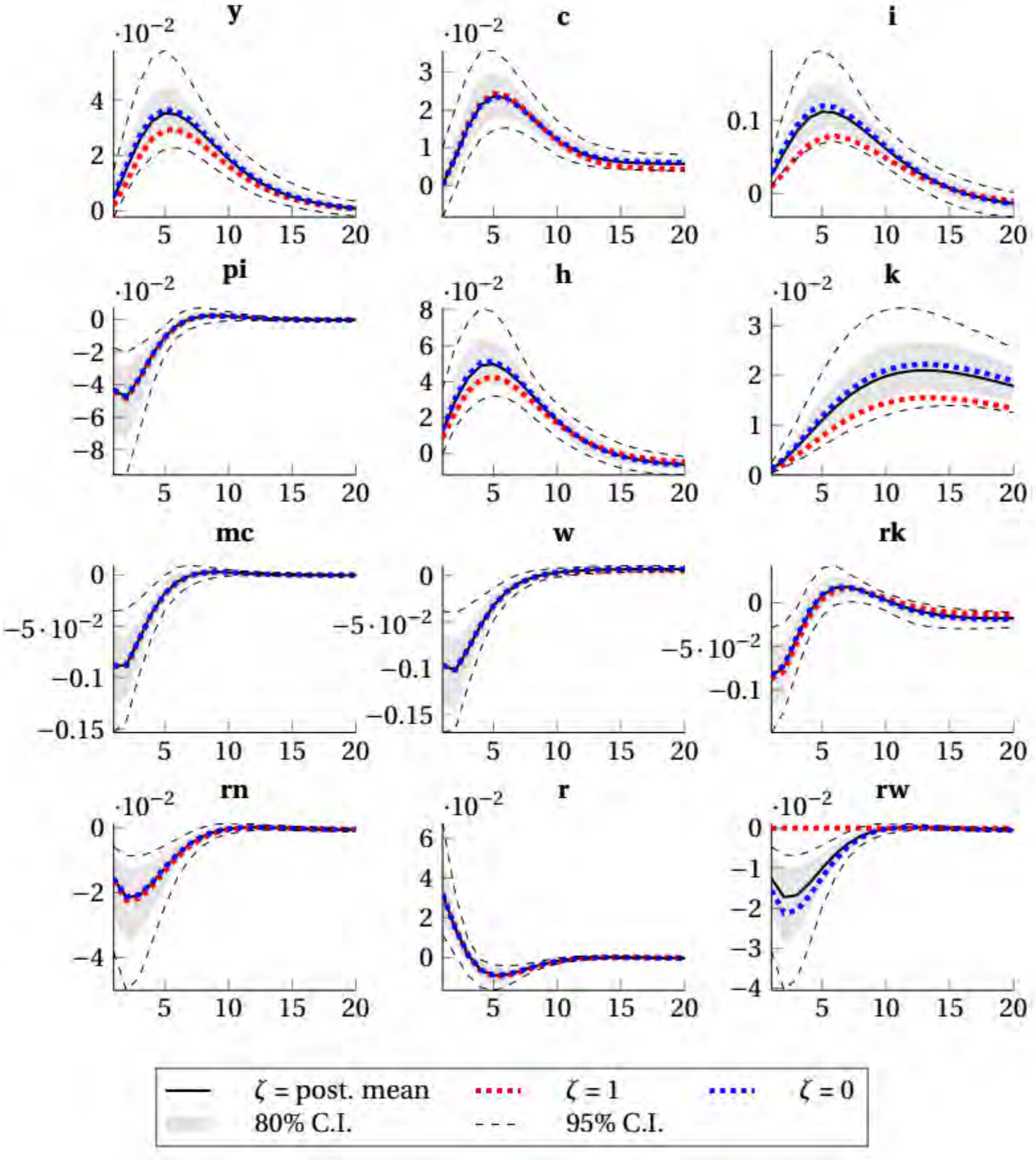


Figure B.8: IRFs to a positive 1 st. dev. government spending shock — posterior distribution

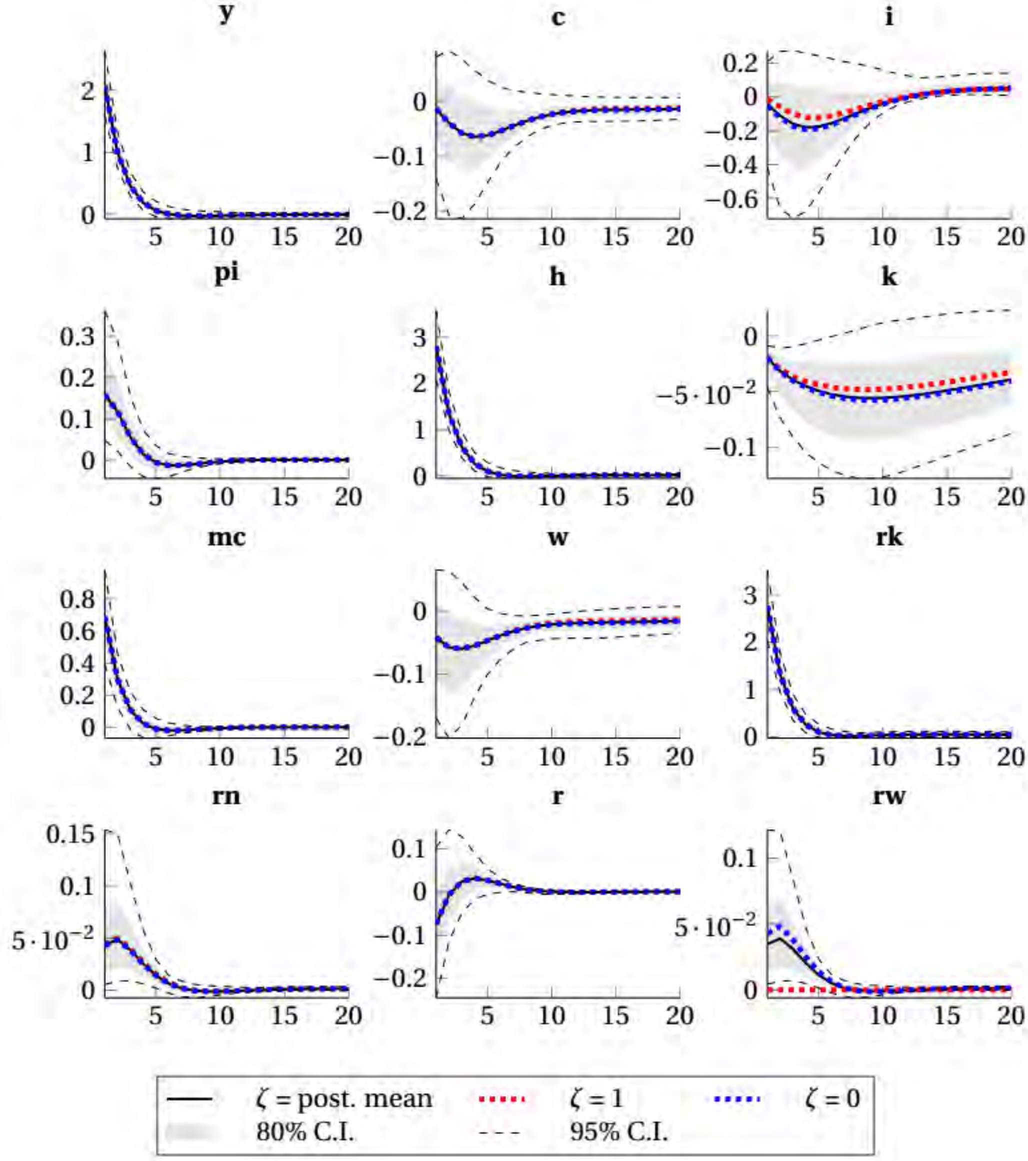


Figure B.9: IRFs to a positive 1 st. dev. risk-premium shock — posterior distribution

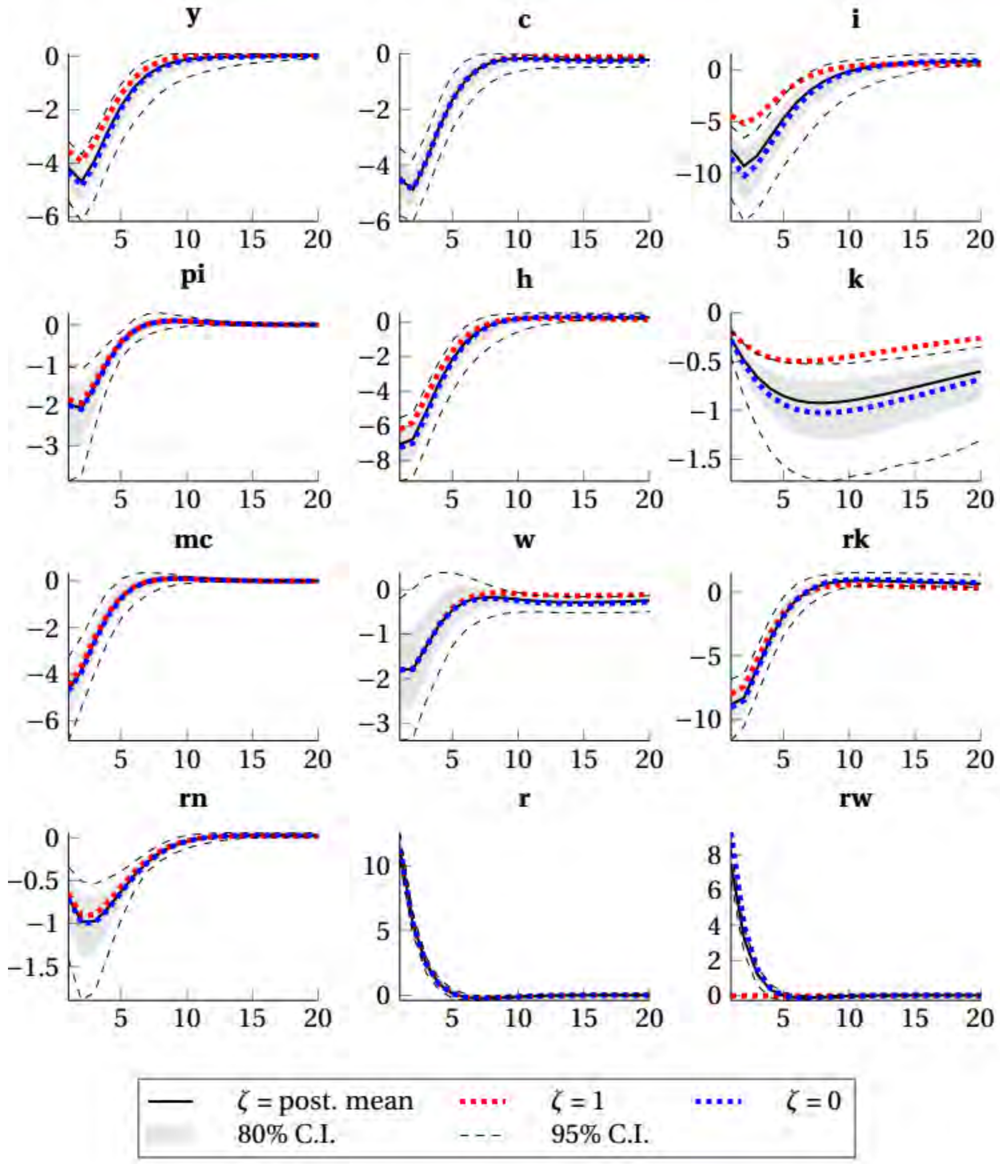
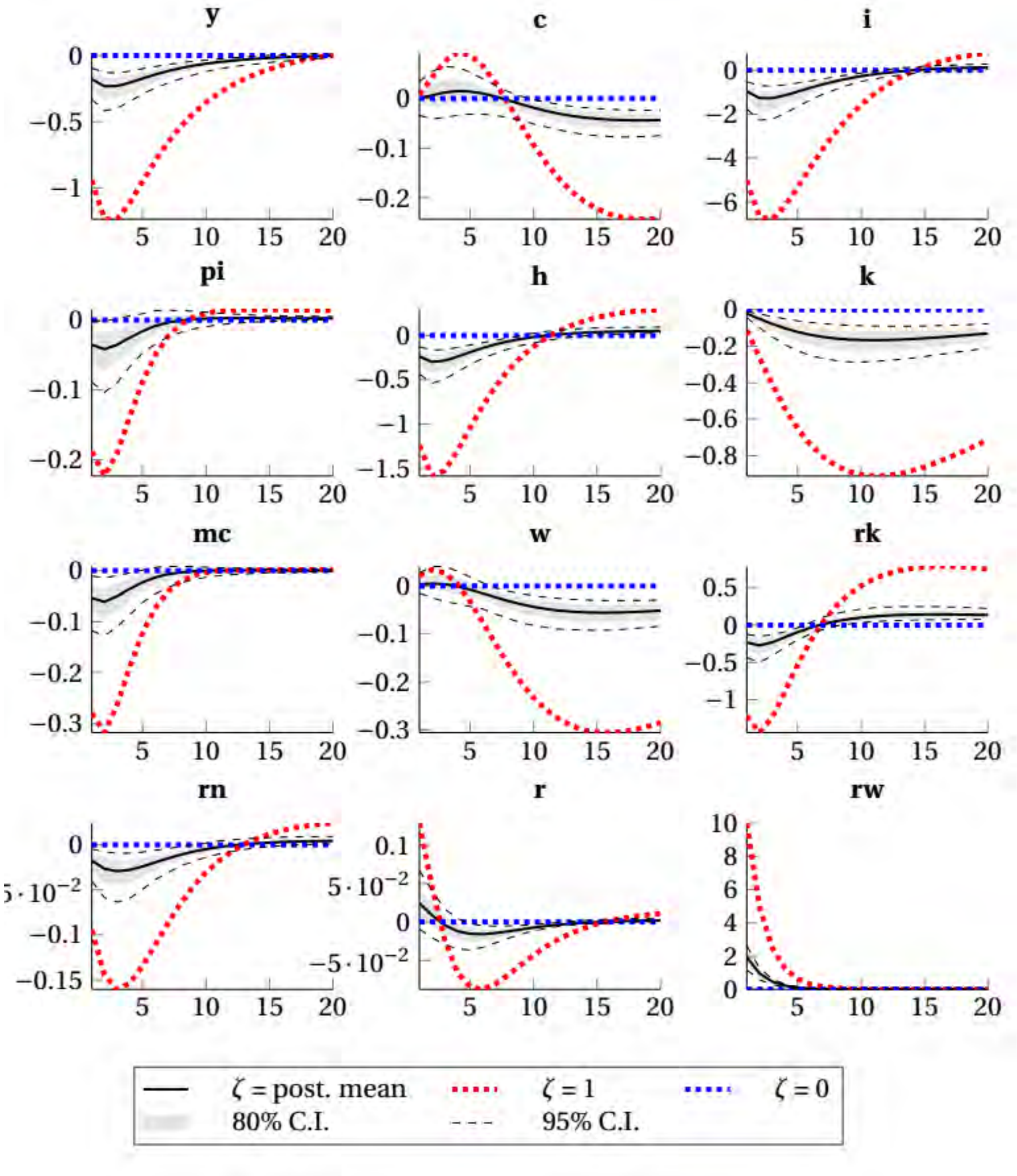


Figure B.10: IRFs to a positive 1 st. dev. subsidized interest rate shock — posterior distribution



B.7
Macro, micro and external effects for other variables

Figure B.11: Macro, micro and external effects — posterior distribution (1/2)

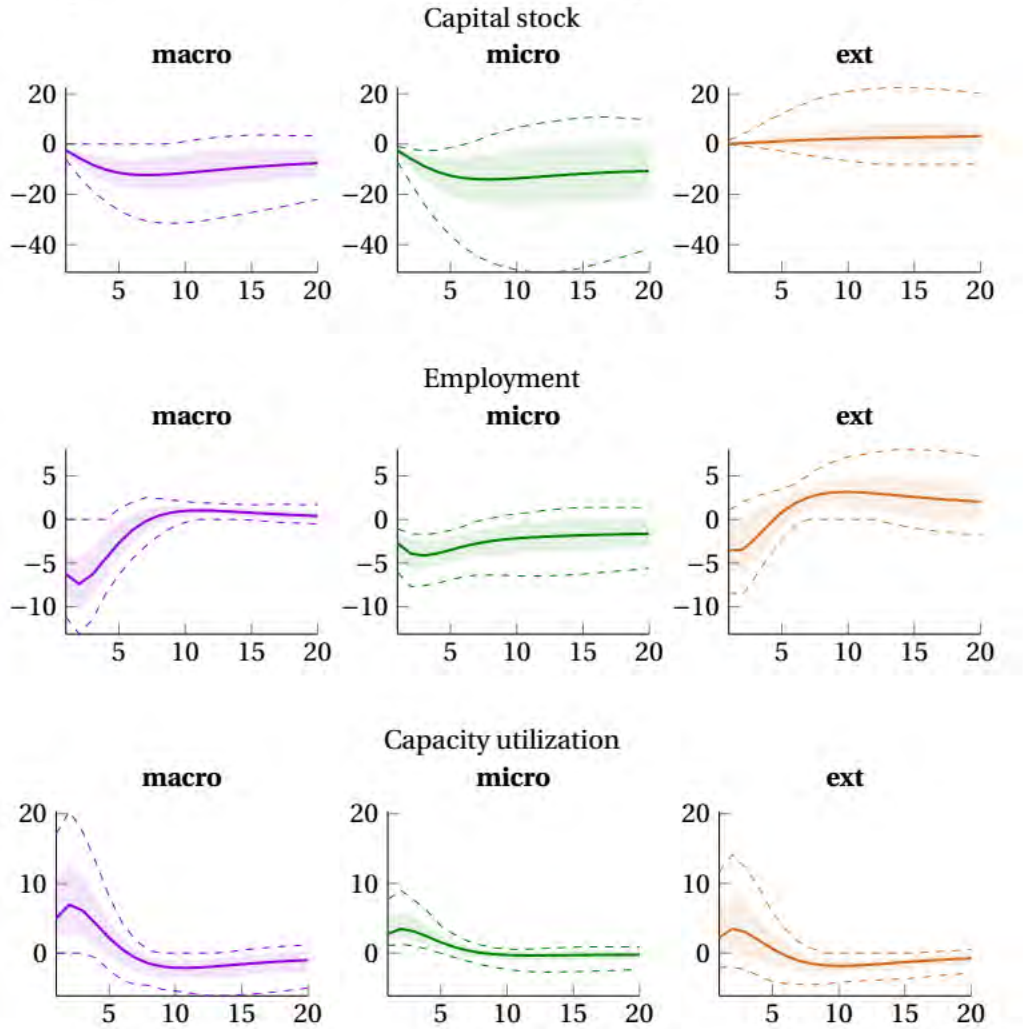
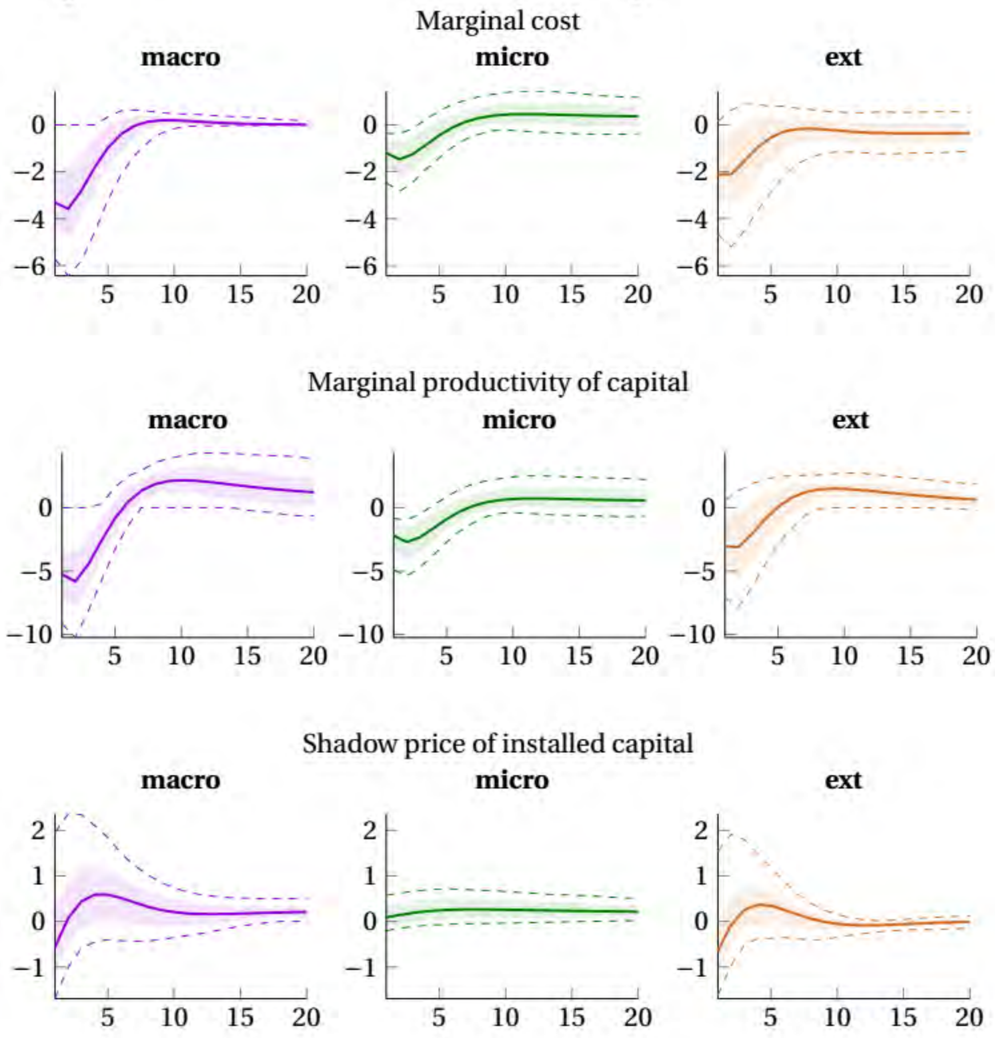


Figure B.12: Macro, micro and external effects — posterior distribution (2/2)



C
Appendix to Chapter 3

C.1
(39)'s model

There are two domestic assets, called *bonds* and *non-bonds*.The central bank controls the rate of return of bonds, by suppling just the right amount of this assets. Hence, the market-clearing condition for bonds is ignored. The market-clearing condition for non-bonds is given by:

$$\hat{N}_D + \hat{N}_F = 0$$

Here, a "hat" means a deviation from the a given steady-state. In order for foreigners to increase their holdings of domestic non-bonds ($\hat{N}_F > 0$) residents have to decrease their holdings by the same amount.

Also important is the balance of payments equilibrium. For simplicity, it is assumed that the current account balance is constant, which implies that changes in the total current account should be equal to zero:

$$\hat{B}_F + \hat{N}_F = E\left(\hat{B}_D^*\right)$$

Note that $\hat{B}_D^*$ denotes residents holdings of foreign bonds, which are denominated in USD. The exchange rate is BRL/USD, so an increase represents depreciation of the domestic currency. For simplicity, it is assumed that domestic residents do not buy foreign non-bonds.

Now, the model includes behavioral equations for assets demand, in the spirit of the portfolio-balance channel. It is assumed that demand for an asset is increase in the return differential to other assets:

$$\begin{aligned}
\hat{N}_D &= \beta\left(R_N - R_B\right) + \beta\left(R_N - \frac{E^e}{E}R^*\right) \\
E\hat{B}_D^* &= \beta\left(\frac{1}{E}R^* - R_B\right) + \beta\left(\frac{E^e}{E}R^* - R_N\right) \\
\hat{B}_F &= \beta\left(R_B - R_N\right) + \beta\left(R_B - \frac{E^e}{E}R^*\right) + \beta s_B \\
\hat{N}_F &= \beta\left(R_N - R_B\right) + \beta\left(R_N - \frac{E^e}{E}R^*\right) + \beta s_N
\end{aligned}$$

Note that in foreigners demand for domestic assets it was included exogenous shifters, s_B for bonds and s_N for non-bonds. That all elasticities are equal to β is to make the math simple. Also to make the math simple, in order to focus on the effect of the exogenous demand shifts, consider the case where $R_B = R_B^* = E^e = 1$. Then:

$$R_N = 1 + \frac{1}{6}s_B - \frac{1}{6}s_N$$
$$\frac{1}{E} = 1 + \frac{1}{3}s_B + \frac{1}{6}s_N$$

We see that the appreciation of the domestic currency following the bond inflow shock s_B is higher (1/3) than the one following the non-bond inflow shock s_N (1/6). It is argued that this makes bond inflows more contractionary than non-bond inflows, because of the impact of the exchange rate appreciation on the net-exports component of aggregate demand (even though the current account is assumed fixed in the analysis). Also, the rate of return on non-bonds increase following a bond inflow shock (+1/6), but decreases following a non-bond inflow shock (-1/6). It is argued that increases in the rate of return on non-bonds are contractionary because it decreases the cost of financial intermediation. Hence, the rate of return channel reinforces the contractionary effect of the exchange rate channel in the case of bond inflows, but mitigates it in the case of non-bond inflows.

C.2 Modeling financial intermediaries

Why did I modeled financial intermediaries in the way I did? Why have different types of intermediaries, each specialized in a different kind arbitrage between asset returns?

C.2.1 Why not only one type of intermediary

Remember the framework of (40), from which (41) builds upon. Intermediaries finance long positions in one asset with short positions in another, maximize one-period expected profits and are subject to a incentive compatibility constraint. Let us consider generalizing their framework to consider the case where the intermediary is able to deal with more then two assets. His problem is given by:

$$\begin{aligned} \max_{X_1 \cdots X_n} \quad & \mathbb{E}\Big\{ R_1 X_1 + R_2 X_2 + \cdots R_n X_n \Big\} \\ \text{s.t.} \quad & X_1 + X_2 + \cdots X_n = 0 \\ & \mathbb{E}\Big\{ R_1 X_1 + R_2 X_2 + \cdots R_n X_n \Big\} \geq \Gamma \left(\sum_{i:X_i>0} X_i \right)^2 \end{aligned}$$

Explaining the incentive compatibility constraint. Remember that intermediary's outside option is to divert a fraction ΓA of total assets A, i.e., to runway with the amount ΓA^2. If $X_i > 0$ it is counted as an asset in intermediary's balance-sheet, and if $X_i < 0$ it is counted as a liability.[1] Total assets are then given by the sum of all values of all assets whose holding by the intermediary is positive: $A = \sum_{i:X_i>0} X_i$.

We now characterize the solution. Let λ and μ be the multipliers of balance sheet and incentive compatibility constraints, respectively. Looking for a critical point different from zero, where there is a discontinuity, we find the following first order condition for X_j:

$$\mathbb{E}\left\{R_j\right\} - \lambda + \mu \left(\mathbb{E}\left\{R_j\right\} - 2\Gamma \left(\sum_{i:X_i>0} X_i \right) \mathbb{1}\Big[X_j > 0 \Big] \right) = 0$$

where $\mathbb{1}[\cdot]$ is the indicator function. The multipliers are both positive. Remember that $A = \sum_{i:X_i>0} X_i$. Hence:

$$\mathbb{E}\left\{R_j\right\} = \frac{\lambda + 2\mu\Gamma A \mathbb{1}\Big[X_j > 0 \Big]}{1+\mu}$$

Now consider the expected return differential between assets j and k. Assume $X_j > X_k$, without loss of generality. We have

$$\mathbb{E}\left\{R_j\right\} - \mathbb{E}\left\{R_k\right\} = \begin{cases} 0 & , \text{ if } \quad (X_i, X_j > 0) \text{ or } (X_i, X_k < 0) \\ \frac{2\mu\Gamma A}{1+\mu} > 0 & , \text{ if } \quad (X_j > 0) \text{ and } (X_k < 0) \end{cases}$$

If the intermediary longs two different assets these assets must have the same expected return. If it shorts two different assets, these liabilities (for the intermediary) must also pay the same expected return. If the intermediary is long one asset and short the other, it must be the case that the first has higher expected return, so that the intermediary is expected to make a profit in this

[1] $X_i = 0$ means it is neither an asset or a liability in the balance-sheet.

long-short position.

So in this generalized model there is only one wedge between expected returns: a wedge between longed and shorted assets. Imperfect substitutability only arises between longed and shorted assets. In equilibrium assets that are longed are perfect substitutes to one another, and the same happens for shorted assets. But this is not a feature we want our model to have. We want to allow for different wedges between all possible pair of assets, such that is true imperfect substitution across all assets.

C.2.2
(41)'s approach

The problem we identified above has been dealt with by (41) in the same way we deal with it here: by segmenting the market. In his model there is a continumm of bonds — one for each small open economy in the world. There is a double continuum of intermediaries — intermediary indexed by i, j arbitrage returns between bonds issued by countries i and j, only, ignoring all other bonds. As we have pointed out, above, his model would not work had he introduced only one intermediary which could invest in any country.

C.3
Balance of payment derivation

Start with households' real budget constraint:

$$C_t + D_t + F_{S,t} + \mathcal{E}_t F_{B^*,t} + \mathcal{E}_t F_{S^*,t} = W_t L_t + R_t D_{t-1} + R_t^s F_{S,t-1} + \mathcal{E}_t R_t^* F_{B^*,t-1} + \mathcal{E}_t R_t^{s*} F_{S^*,t-1} + \Omega_t + T_t$$

Combining equations (3-7), (3-8), (3-46) and (3-47) we get:

$$C_t = p_{F,t} Z_{F,t} + p_{H,t}\Big(Y_{H,t} - Z_{H,t}^*\Big) - I_t$$

Equation (3-48) can be rewritten as $W_t L_t = (1-\alpha)\mu_t \emptyset_t Y_{H,t}$.

Remember that (3-36) reads $Q_t K_t = S_t$. Moreover, combining it to (3-37) and (3-48) yields:

$$R_t^s S_{t-1} = \alpha \mu_t \emptyset_t Y_{H,t} + (1-\delta) Q_t K_{t-1}$$

Profits distributed to households come from input producing firms, from capital producing firms and the local financial intermediary: $\Omega_t = \Omega_{h,t} + \Omega_{f,t} + \Omega_{k,t} + \Omega_{1,t}$. Profits from global intermediaries are equally distributed among countries but domestic economy's share is infinitesimal. Hence:

$$\Omega_t = \Big\{ \big[p_{H,t} - \emptyset_t \mu_t \big] Y_{H,t} \Big\} + \Big\{ \big[p_{F,t} - \emptyset_{F,t} \mu_{F,t} \big] Z_{F,t} \Big\}$$
$$+ \left\{ Q_t I_t \left(1 - f\left(\frac{I_t}{I_{t-1}} \right) \right) - I_t \right\} + \Big\{ R_t^s S_{1,t-1} + R_t B_{1,t-1} \Big\}$$

Now, remember that capital law of motion (3-35) implies $I_t \left(1 - f\left(\frac{I_t}{I_{t-1}} \right) \right) = K_t - (1-\delta) K_{t-1}$. Also, remember that government transfers are given by $T_t = \mathcal{E}_t R_t^* F_{t-1}^{\text{CB}} + R_t B_{G,t} - \tau p_{H,t}$.

Appropriately substituting the above mentioned expressions in the households' budget constraint:

$$\Big(\emptyset_{F,t} \mu_{F,t} Z_{F,t} - \mu_{H,t}^* Z_{H,t}^* \Big) + D_t + R_t S_{t-1} + F_{S,t} + \mathcal{E}_t F_{B^*,t} + \mathcal{E}_t F_{S^*,t} =$$
$$S_t + R_t D_{t-1} + \mathcal{E}_t R_t^* F_{t-1}^{\text{CB}} + R_t B_{G,t-1} + R_t^s S_{1,t-1} + R_t B_{1,t-1} + R_t^s F_{S,t-1}$$
$$+ \mathcal{E}_t R_t^* F_{B^*,t-1} + \mathcal{E}_t R_t^{s*} F_{S^*,t-1}$$

Now, summing to this equation the balance-sheet of the central bank — $\mathcal{E}_t F_t^{\text{CB}} + B_{G,t} = 0$ — and of local financial intermediaries — $S_{1,t} + B_{1,t} = 0$ — we find:

$$\Big(\emptyset_{F,t} \mu_{F,t} Z_{F,t} - \mu_{H,t}^* Z_{H,t}^* \Big) + \Big(D_t + B_{1,t} + B_{G,t} \Big) + \mathcal{E}_t \Big(F_t^{\text{CB}}$$
$$+ F_{B^*,t} + F_{S^*,t} \Big) + R_t^s \Big(S_{t-1} - S_{1,t-1} - F_{S,t-1} \Big) =$$
$$\Big(S_t - S_{1,t} - F_{S,t} \Big) + R_t \Big(D_{t-1} + B_{1,t-1} + B_{G,t-1} \Big)$$
$$+ \mathcal{E}_t R_t^* \Big(F_{t-1}^{\text{CB}} + F_{B^*,t-1} \Big) + \mathcal{E}_t R_t^{s*} F_{S^*,t-1} \tag{C-1}$$

From the bonds market clearing we have that domestic bonds holdings by non-residents satisfy $B_{2,t} + F_{B,t}^* = -(D_t + B_{1,t} + B_{G,t})$. Domestic stock holdings by non-residents satisfy $S_{3,t} + F_{S,t}^* = S_t - S_{1,t} - F_{S,t}$. Finally, $F_t^{\text{CB}} + F_{B^*,t}$ and $F_{S^*,t}$ are residents' holdings of foreign bonds and foreign stock, respectively. Hence, real net-foreign assets of the domestic economy are given by:

$$\mathcal{A}_t = \mathcal{E}_t \Big(F_t^{\text{CB}} + F_{B^*,t} + F_{S^*,t} \Big) - \Big(B_{2,t} + F_{B,t}^* \Big) - \Big(S_{3,t} + F_{S,t}^* \Big)$$

Substituting the definition of net-foreign assets into our previous balance of payments derivation:

$$\begin{aligned}\mathcal{A}_t &= \underbrace{\mu^*_{H,t} Z^*_{H,t} - \emptyset_{F,t}\mu_{F,t} Z_{F,t}}_{\text{Trade balance}} + \\ &+ \underbrace{\mathcal{E}_t R^*_t \left(F^{\text{CB}}_{t-1} + F_{B^*,t-1} \right) + \mathcal{E}_t R^{s*}_t F_{S^*,t-1} - R_t \left(B_{2,t-1} + F^*_{B,t-1} \right) - R^s_t \left(S_{3,t-1} + F^*_{S,t-1} \right)}_{\text{Previous NFA + income/valuation gains}}\end{aligned}$$

C.4
Model summary

Households

$$R_t = \frac{R^n_{t-1}}{\Pi_t} \tag{C-2}$$

$$\Xi_t = \beta\left(\frac{C_t - \psi C_{t-1}}{C_{t-1} - \psi C_{t-2}}\right)^{-1} \tag{C-3}$$

$$W_t = \chi\Big(C_t - \psi C_{t-1}\Big)L_t^{\varphi} \tag{C-4}$$

$$\mathbb{E}_t\left\{\Xi_{t+1}R_{t+1}\right\} = 1 \tag{C-5}$$

Final good and input use

$$Z_{F,t} = \gamma\left(p_{F,t}\right)^{-1}Y_t \tag{C-6}$$

$$Z_{H,t} = (1-\gamma)\left(p_{H,t}\right)^{-1}Y_t \tag{C-7}$$

$$1 = p_{F,t}^{\gamma}p_{H,t}^{1-\gamma} \tag{C-8}$$

Domestic input variety producers

$$\mu_t = \left(\frac{R^r_t}{\alpha}\right)^{\alpha}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha} \tag{C-9}$$

$$X_t = \Big(\Pi_{H,t-1}\Big)^{\iota} \tag{C-10}$$

$$\hat{p}_t = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{u_{1,t}}{u_{2,t}} \tag{C-11}$$

$$u_{1,t} = \Big(p_{H,t}\Big)^{\varepsilon}Y_{H,t}\,\mu_t + \theta\mathbb{E}_t\Big\{\Xi_{t+1}\Big(\Pi_{t+1}\Big/X_{t+1}\Big)^{\varepsilon}u_{t+1}\Big\} \tag{C-12}$$

$$u_{2,t} = \Big(p_{H,t}\Big)^{\varepsilon}Y_{H,t} + \theta\mathbb{E}_t\Big\{\Xi_{t+1}\Big(\Pi_{t+1}\Big/X_{t+1}\Big)^{\varepsilon-1}u_{2,t+1}\Big\} \tag{C-13}$$

$$p_{H,t} = \left((1-\theta)\hat{p}_t^{1-\varepsilon} + \theta\left(p_{H,t-1}\frac{X_t}{\Pi_t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \tag{C-14}$$

$$\emptyset_t = (1-\theta)\left(\frac{\hat{p}_t}{p_{H,t}}\right)^{-\varepsilon} + \theta\left(\frac{\Pi_{H,t}}{X_t}\right)^{\varepsilon}\emptyset_{t-1} \tag{C-15}$$

$$\Pi_{H,t} = \left(\frac{p_{H,t}}{p_{H,t-1}}\right)\Pi_t \tag{C-16}$$

Imports

$$X_{F,t} = \left(\Pi_{F,t}\right)^{\iota_f} \tag{C-17}$$

$$\mu_{F,t} = (1-\tau)\mathcal{E}_t p^*_{F,t} \tag{C-18}$$

$$\hat{p}_{F,t} = \left(\frac{\epsilon_f}{\epsilon_f - 1}\right)\frac{u_{F,1,t}}{u_{F,2,t}} \tag{C-19}$$

$$u_{F,1,t} = \left(p_{F,t}\right)^{\epsilon_f} Z_{F,t}\,\mu_{F,t} + \theta_f \mathbb{E}_t\left\{\Xi_{t+1}\left(\Pi_{t+1}\big/X_{F,t+1}\right)^{\epsilon_f} u_{F,t+1}\right\} \tag{C-20}$$

$$u_{F,2,t} = \left(p_{F,t}\right)^{\epsilon_f} Z_{F,t} + \theta_f \mathbb{E}_t\left\{\Xi_{t+1}\left(\Pi_{t+1}\big/X_{F,t+1}\right)^{\epsilon_f - 1} u_{F,2,t+1}\right\} \tag{C-21}$$

$$p_{F,t} = \left((1-\theta_f)\hat{p}_{F,t}^{1-\epsilon_f} + \theta_f\left(p_{F,t-1}\frac{X_{F,t}}{\Pi_t}\right)^{1-\epsilon_f}\right)^{\frac{1}{1-\epsilon_f}} \tag{C-22}$$

$$\emptyset_{F,t} = (1-\theta_f)\left(\frac{\hat{p}_{F,t}}{p_{F,t}}\right)^{-\epsilon_f} + \theta\left(\frac{\Pi_{F,t}}{X_{F,t}}\right)^{\epsilon_f}\emptyset_{F,t-1} \tag{C-23}$$

$$\Pi_{F,t} = \left(p_{F,t}\big/p_{F,t-1}\right)\Pi_t \tag{C-24}$$

Exports

$$Z^*_H = \left(p^*_{H,t}\right)^{-\epsilon_f}\left(\gamma Y^*\right) \tag{C-25}$$

$$\mu^*_{H,t} = (1-\tau)p_{H,t} \tag{C-26}$$

$$X^*_{H,t} = \left(\Pi^*_{H,t}\right)^{\iota_f} \tag{C-27}$$

$$\hat{p}^*_{H,t} = \left(\frac{\epsilon_f}{\epsilon_f - 1}\right)\frac{u^*_{H,1,t}}{u^*_{H,2,t}} \tag{C-28}$$

$$u^*_{H,1,t} = \left(p^*_{H,t}\right)^{\epsilon_f} Z^*_{H,t}\frac{\mu^*_{H,t}}{\mathcal{E}_t} + \beta\theta_f\mathbb{E}_t\left\{\left(1\big/X^*_{H,t+1}\right)^{\epsilon_f} u^*_{H,1,t+1}\right\} \tag{C-29}$$

$$u^*_{H,2,t} = \left(p^*_{H,t}\right)^{\epsilon_f} Z^*_{H,t} + \beta\theta_f\mathbb{E}_t\left\{\left(1\big/X^*_{H,t+1}\right)^{\epsilon_f - 1} u^*_{H,2,t+1}\right\} \tag{C-30}$$

$$p^*_{H,t} = \left((1-\theta_f)\left(\hat{p}^*_{H,t}\right)^{1-\epsilon_f} + \theta_f\left(p^*_{H,t-1}X^*_{H,t}\right)^{1-\epsilon_f}\right)^{\frac{1}{1-\epsilon_f}} \tag{C-31}$$

$$\Pi^*_{H,t} = \left(\frac{p^*_{H,t}}{p^*_{H,t-1}}\right) \tag{C-32}$$

Capital good producers

$$Q_t\left[1 - f\left(\frac{I_t}{I_{t-1}}\right) - \frac{I_t}{I_{t-1}}f'\left(\frac{I_t}{I_{t-1}}\right)\right] + \mathbb{E}_t\left\{\Xi_{t+1}Q_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'\left(\frac{I_{t+1}}{I_t}\right)\right\} = 1 \tag{C-33}$$

$$K_t = (1-\delta)K_{t-1} + I_t\left[1 - f\left(\frac{I_t}{I_{t-1}}\right)\right] \tag{C-34}$$

Stocks

$$S_t = Q_t K_t \tag{C-35}$$

$$R^s_t = \frac{R^r_t + Q_t(1-\delta)}{Q_{t-1}} \tag{C-36}$$

Financial intermediaries

$$S_{1,t} = \frac{1}{\Gamma_H}\mathbb{E}_t\left\{R^s_{t+1} - R_{t+1}\right\} \tag{C-37}$$

$$B_{2,t} = \frac{\mathcal{E}_t}{\Gamma_F}\mathbb{E}_t\left\{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}R_{t+1} - R^*_{t+1}\right\} \tag{C-38}$$

$$S_{3,t} = \frac{\mathcal{E}_t}{\Gamma_F}\mathbb{E}_t\left\{\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}R^s_{t+1} - R^{s*}_{t+1}\right\} \tag{C-39}$$

Balance of payments

$$\mathcal{A}_t = \mathcal{E}_t\left(F^{\text{CB}}_t + F_{B^*,t} + F_{S^*,t}\right) - \left(B_{2,t} + F^*_{B,t}\right) - \left(S_{3,t} + F^*_{S,t}\right) \tag{C-40}$$

$$\Delta\mathcal{A}_t = \text{TB}_t + \text{IB}_t \tag{C-41}$$

$$\text{TB}_t = \mu^*_{H,t}Z^*_{H,t} - \emptyset_{F,t}\mu_{F,t}Z_{F,t} \tag{C-42}$$

$$\text{IB}_t = \mathcal{E}_t R^*_t\left(F^{\text{CB}}_{t-1} + F_{B^*,t-1}\right) + \mathcal{E}_t R^{s*}_t F_{S^*,t-1} - R_t\left(B_{2,t-1} + F^*_{B,t-1}\right)$$
$$- R^s_t\left(S_{3,t-1} + F^*_{S,t-1}\right) - \mathcal{A}_{t-1} \tag{C-43}$$

Central Bank

$$R^n_t = R^{\rho_m}_{t-1}\left[\frac{1}{\beta}\Pi^{\phi_m}_t\right]^{1-\rho_m} \tag{C-44}$$

Market clearing

$$Y_t = C_t + I_t \tag{C-45}$$

$$Y_{H,t} = Z_{H,t} + Z^*_{H,t} \tag{C-46}$$

$$L_t = (1-\alpha)\frac{\mu_t}{W_t}Y_{H,t}\emptyset_t \tag{C-47}$$

$$K_{t-1} = \alpha\frac{\mu_t}{R^r_t}Y_{H,t}\emptyset_t \tag{C-48}$$

$$S_{1,t} + S_{3,t} + F_{S,t} + F^*_{S,t} = S_t \tag{C-49}$$

Exogenous variables

$$F^*_B \quad F^*_S \quad F_{B^*} \quad F_{S^*} \quad F_S \quad X \quad p^*_F \quad R^* \quad R^{s*} \quad Y^*$$

where $f(x) = \frac{1}{2}\left\{\exp\left[\sqrt{\kappa}(x-1)\right] + \exp\left[-\sqrt{\kappa}(x-1)\right] - 2\right\}$.

C.5 Linear model

I log-linearize the model around the deterministic steady-state. For variables related to asset demands (S_1, B_2, S_3, F_B^*, F_S^*, ${F_B}^*$, ${F_S}^*$, F_S and F^{CB}), for net foreign assets ($\mathcal{A}$) and for the income balance (IB) I just consider a simple linerization.

Households

$$R_t = R_{t-1}^n - \Pi_t \tag{C-50}$$

$$W_t = (1-\psi)^{-1}\Big(C_t - \psi C_{t-1}\Big) + \varphi L_t \tag{C-51}$$

$$C_t = \left(\frac{1}{1+\psi}\right)\mathbb{E}_t\left\{C_{t+1}\right\} + \left(\frac{\psi}{1+\psi}\right)C_{t-1} - \left(\frac{1-\psi}{1+\psi}\right)\mathbb{E}_t\left\{R_{t+1}\right\} \tag{C-52}$$

Final good and input use

$$Z_{F,t} = Y_t - p_{F,t} \tag{C-53}$$

$$Z_{H,t} = Y_t - p_{H,t} \tag{C-54}$$

$$0 = \gamma p_{F,t} + (1-\gamma)p_{H,t} \tag{C-55}$$

Domestic input producers

$$\mu_t = \alpha R_t^r + (1-\alpha)W_t \tag{C-56}$$

$$\Pi_{H,t} = \left(\frac{\iota}{1+\beta\iota}\right)\Pi_{H,t-1} + \left(\frac{\beta}{1+\beta\iota}\right)\mathbb{E}_t\left\{\Pi_{H,t+1}\right\} + \left[\frac{(1-\theta)(1-\beta\theta)}{\theta(1+\beta\iota)}\right](\mu_t - p_{H,t}) \tag{C-57}$$

$$\Pi_{H,t} = (p_{H,t} - p_{H,t-1}) + \Pi_t \tag{C-58}$$

Imports

$$\mu_{F,t} = \mathcal{E}_t + p_{F,t}^* \tag{C-59}$$

$$\Pi_{F,t} = \left(\frac{\iota_f}{1+\beta\iota_f}\right)\Pi_{F,t-1} + \left(\frac{\beta}{1+\beta\iota_f}\right)\mathbb{E}_t\left\{\Pi_{F,t+1}\right\} + \left[\frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f(1+\beta\iota_f)}\right](\mu_{F,t} - p_{F,t}) \tag{C-60}$$

$$\Pi_{F,t} = (p_{F,t}^* - p_{F,t-1}^*) + \Pi_t \tag{C-61}$$

Exports

$$Z_{H,t}^* = Y_t^* - \epsilon_f p_{H,t}^* \tag{C-62}$$

$$\mu_{H,t}^* = p_{H,t}^* \tag{C-63}$$

$$\Pi_{H,t}^* = \left(\frac{\iota_f}{1+\beta\iota_f}\right)\Pi_{H,t-1}^* + \left(\frac{\beta}{1+\beta\iota_f}\right)\mathbb{E}_t\left\{\Pi_{H,t+1}^*\right\} + \left[\frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f(1+\beta\iota_f)}\right](\mu_{F,t}^* - \mathcal{E}_t - p_{F,t}) \tag{C-64}$$

$$\Pi_{H,t}^* = (p_{H,t}^* - p_{H,t-1}^*) \tag{C-65}$$

Capital good producers

$$I_t = \left(\frac{1}{1+\beta}\right) I_{t-1} + \left(\frac{\beta}{1+\beta}\right) \mathbb{E}_t \left\{I_{t+1}\right\} + \left(\frac{1}{\kappa(1+\beta)}\right) Q_t \tag{C-66}$$

$$K_t = (1-\delta)K_{t-1} + \delta I_t \tag{C-67}$$

Stocks

$$\left(\frac{1}{K}\right) S_t = Q_t + K_t \tag{C-68}$$

$$R^s_t = \left(\frac{R^r}{R^s}\right)(R^r_t - Q_{t-1}) + \left(\frac{R^s - R^r}{R^s}\right) \Delta Q_t \tag{C-69}$$

Financial intermediaries

$$S_{1,t} = \left(\frac{R^s}{R^s - R}\right) \mathbb{E}_t \left\{R^s_{t+1}\right\} - \left(\frac{R}{R^s - R}\right) \mathbb{E}_t \left\{R_{t+1}\right\} \tag{C-70}$$

$$B_{2,t} = \left(\frac{R}{\Gamma_F K}\right) \mathbb{E}_t \left\{R_{t+1} - R^*_{t+1} - \Delta\mathcal{E}_{t+1}\right\} \tag{C-71}$$

$$S_{3,t} = \left(\frac{R^s}{\Gamma_F K}\right) \mathbb{E}_t \left\{R^s_{t+1} - R^{s*}_{t+1} - \Delta\mathcal{E}_{t+1}\right\} \tag{C-72}$$

Central Bank

$$R^n_t = \rho_m R^n_{t-1} + (1-\rho_m)\phi_m \Pi_t \tag{C-73}$$

Balance of payments

$$\mathcal{A}_t = \left[F^{\text{CB}} + F_B + F_S\right] \mathcal{E}_t + \left(F^{\text{CB}}_t + F_{B^*,t} + F_{S^*,t}\right) - \left(B_{2,t} + F^*_{B,t}\right) - \left(S_{3,t} + F^*_{S,t}\right) \tag{C-74}$$

$$\Delta\mathcal{A}_t = \text{TB}_t + \text{IB}_t \tag{C-75}$$

$$\text{TB}_t = \mu^*_{H,t} Z^*_{H,t} - \emptyset_{F,t}\mu_{F,t} Z_{F,t} \tag{C-76}$$

$$IB_t = \left[R^*\left(F^{\text{CB}} + F_B\right)\right]\left(\mathcal{E}_t + R^*_t - R_t\right) + \left[R^s F_S\right]\left(\mathcal{E}_t + R^{s*}_t - R^s_t\right) - \mathcal{A}_{t-1} \tag{C-77}$$

$$+ R\left[\left(F^{\text{CB}}_{t-1} + F_{B^*,t-1}\right) - \left(B_{2,t-1} + F^*_{B,t-1}\right)\right] + R^s\left[F_{S^*,t-1} - \left(S_{3,t-1} + F^*_{S,t-1}\right)\right]$$

Market clearing

$$Y_t = \left(\frac{C}{Y}\right) C_t + \left(\frac{\delta K}{Y}\right) I_t \tag{C-78}$$

$$Y_{H,t} = (1-\gamma) Z_{H,t} + \gamma Z^*_{H,t} \tag{C-79}$$

$$L_t = (\mu_t - W_t) + Y_{H,t} \tag{C-80}$$

$$K_{t-1} = (\mu_t - R^r_t) + Y_{H,t} \tag{C-81}$$

$$S_{1,t} + S_{3,t} + F_{S,t} + F^*_{S,t} = S_t \tag{C-82}$$

(C-83)

C.6 Steady-state

We consider a symmetric steady-state across countries. To compute it will write as functions of Y e K the other variables' values. We will then arrive at a two-equations non-linear system in (Y, K). Once this system is solved we can then recover the steady-state value of the other variables. Importantly, in the steady-state computation we treat Γ_H as a variable and equity premium $\mathcal{R} = R^s - R$ as a parameter, because we want to calibrate the value of Γ_H to achieve a target for the equity premium.

We consider a symmetric steady state with zero trend inflation. Hence, $\Pi = \Pi^* = 1$. By symmetry we have that $Y = Y^*$ and $Z_H^* = Z_F$, i.e., production and import levels are equal across countries. Comparing (3-7) for both economies we find $p_F = p_H^*$, and $P_p = p_F^*$ by doing the same with (3-8). Using these relations together with (**??**) and (3-9) we conclude that all prices are equal and that $\mathcal{E} = 1$. Again, using the assumption of symmetry together with (3-7) and (3-8) we find $Y_H = Y$. Finally, from (3-4) and (3-6) we have $R = 1/\beta$, from (3-12), (3-13) and (3-14) we have $\mu = \frac{\varepsilon - 1}{\varepsilon}$, and from (3-49), $R^r = \alpha\mu\frac{Y_H}{K}$.

Now we have two different equations to compute the value of R^s as a function of (Y, K). From the definition of the equity premium, that we take as given for calibration purpose, we have $R^s = \mathcal{R} + R$. From (3-37) we have $R^s = R^r + (1 - \delta)$. Hence, in equilibrium:

$$\mathcal{R} + R = R^r + (1 - \delta) \tag{C-84}$$

From (3-35) and (3-46) we have $C = Y - \delta K$, and (3-10), $W = (1-\alpha)\mu^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{R^r}\right)^{\frac{\alpha}{1-\alpha}}$. We now have two equations determining L as a function of (Y_H, K). First, (3-5): $L = \left(\frac{W}{C}\right)^{\frac{1}{\varphi}}$. Also, (3-48): $L = (1-\alpha)\mu\frac{Y_H}{W}$. Hence:

$$\left(\frac{W}{C}\right)^{\frac{1}{\varphi}} = (1-\alpha)\mu\frac{Y_H}{W} \tag{C-85}$$

Equations (C-84) and (C-85) depend only on (Y, K) once we substitute the values of Y_H, R, R^r, W e C. This system admits a closed form solution:

$$Y = (1-\alpha)^{\frac{1}{1+\varphi}}\left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1+\alpha\varphi}{(1-\alpha)(1+\varphi)}}\left(\frac{\alpha}{\beta^{-1} + \mathcal{R} + \delta - 1}\right)^{\frac{\alpha}{1-\alpha}}\left(1 - \frac{\delta\alpha}{\beta^{-1} + \mathcal{R} + \delta - 1}\frac{\epsilon - 1}{\epsilon}\right)^{\frac{-1}{1+\varphi}}$$

$$K = \left(\frac{\alpha}{\beta^{-1} + \mathcal{R} + \delta - 1}\frac{\epsilon - 1}{\epsilon}\right)Y$$

With the values of steady-state output and capital we can find the values for the other variables, but show all this computation here. Of particular interest is the implied value of the parameter Γ_H, calibrated to target a given equity premium. From the market-clearing condition for stocks we have that:

$$S_1 = K - \left(F_S^* - F_S\right) - S_3$$

Remember that F_S^* and F_S are the steady-state holdings of domestic stocks by domestic households and foreign households, respectively. We treat these as parameters for steady-state computation. Also, we have that $S_3 = 0$ because there is no return differential to global intermediaries to arbitrage. Substituting (3-38) we then find that

$$\Gamma_H = \frac{\mathcal{R}}{K - \left(F_S^* + F_S\right)}$$