TEXTO PARA DISCUSSÃO

Nº 14

Rational Expectations in Keynesian Macro-Models

Francisco L. Lopes

PUC-Rio – Departamento de Economia
www.econ.puc-rio.br

June 1981

Rational expectations are the most important recent development in macroeconomic analysis, and it stands now undoubtedly as the dominant hypothesis on expectations formation. As is well known, it was brought forward, together with continuous market clearing and the Lucas supply function, as one of the three key elements in the new equilibrium approach to business cycle analysis, developed by Lucas (1972a, 1972b, 1973), Sargent (1973) and others[1]. It is obvious, however, that these three ideas are logically independent, notwithstanding the fact that they have been frequently associated in the literature. As a matter of fact, the models in Phelps and Taylor (1977), Fischer (1977) and Taylor (1980) are good examples of how it is possible to put together rational expectations and some form of wage or price stickiness which precludes continuous market clearing.

It seems to me that continuous market clearing is the weak point in the equilibrium approach to business cycles. It implies, for example, that wages and prices are instantaneously reset every time there is a change in money supply, which means that they are continuously flexible over time. Casual observation of wage and price behaviour in the real economy seems to give no support to his notion. It is not obvious, either, that continuous market clearing is a necessary consequence of the choice-theoretic approach to microeconomic theory, as it is often argued by its proponents. Arthur Okun (1980) has forcefully suggested that wage-price stickiness may itself be a result of efficient transactional mechanisms which exhaust all perceived mutually advantageous possibilities for trading. On this matter, I think Solow (1979) still has the last word: "if wages and prices *are* sticky, that fact and its consequences do not go away merely because we have not yet settled on a universally satisfactory theory of *why* they are sticky" (p. 346).

The purpose of this paper is to discuss the role of monetary policy in a class of models that follow the old keynesian tradition of taking it for granted that wages and prices set at the beginning of each period and remain unchanged until the start of next period, while money supply changes continuously over time. It will be assumed that expectations are rational but information on aggregate economy-wide variables is received by economic agents, including the monetary authorities, with a one period lag[2]. It follows that an unanticipated change in the money supply will throw the economy out of equilibrium in the period it occurs. Disequilibrium may also result from velocity or supply shocks, and once established in a given period it may either tend to disappear in the immediately consecutive period or to persist over several periods. In the former case, we have a discrete time analogue of continuous market clearing, which we will call here *discrete market clearing*. It will occur if wages and prices jump at the beginning of each period to the levels that can be expected to

[1] See Barro (1979) for a recent survey of the literature on the equilibrium approach to business cycles.

[2] This means that the monetary policy lag has the same length as the (minimum) wage and price-setting interval, with both equal to the period of analysis. Latter in the paper, the wage setting interval will be made larger than the monetary policy lag when we come to discuss overlapping long-term wage contracts.

produce equilibrium in the period if the economy is not disturbed by some unforeseen event: hence, all markets are expected to clear each in period[3].

The paper will show that in any rational expectations model in which there is discrete market clearing, anticipated monetary policy has no effect on real output, which is the same neutrality proposition derived by Lucas (1972), Sargent (1973), Sargent-Wallace (1975), Barro (1976) and others, from continuous market clearing models. In that kind of model, however, the fact that anticipated money is neutral does not imply that constant money growth is the optimal monetary policy. We will see that, while the neutrality proposition holds, the monetary policy rule that minimizes the variance of the price level may be a feedback rule.

There is an interesting asymmetry in the class of models we will be studying here: while discrete market clearing is sufficient to establish the neutrality proposition, persistence of disequilibria is not by itself sufficient to invalidate it. We will show that, in order to make anticipated money non neutral, we need a combination of staggered long term wage contracts and persistence of disequilibria. In this case, Government can make the economy behave as if there was discrete market clearing, by using an optimal feedback monetary policy rule.

The paper is organized as follows. Section I sets out our basic model, which provides a simple framework for the ensuing discussion. The model can be fixed to generate either discrete market clearing or persistent disequilibria. Section II provides its discrete market clearing solution and shows that anticipated money is neutral in this case. Section III follows by establishing the general point that discrete market clearing makes anticipated money neutral in *any* rational expectations model, and then gives another example of it in an extended market clearing version of the basic model with staggered wage contracts. In section IV it is shown that monetary policy activism is desirable from the point of view of price stability, in spite of the neutrality proposition. Finally, section V deals with models which produce persistent disequilibria.

I. The Basic Model

There are for equations in our basic model. One is a simple quantity theory of aggregate demand:

$$dm_t + \delta_t = dy_t + dp_t \qquad (1)$$

where y_t, p_t and m_t indicate the logarithms of real output, price level and *average* money supply in

[3] If there is continuous market clearing, wages and prices are continuously flexible over time and the economy will be permanently in equilibrium. With discrete market clearing, however, since wages and prices are fixed within each period, the best the market system can do is to try to make the economy return immediately to equilibrium whenever it is thrown out of it by some unpredictable disturbance.

period t, and the letter "d" stands for the difference operator: $dm_t = m_t = m_{t-1}$. Velocity changes are assumed representable by a random white noise disturbance term δ_t. Another equation decomposes the rate of growth of money supply into a systematic fully anticipated term x_t and a white noisc monetary policy shock μ_t:

$$dm_t = x_t + \mu_t \qquad (2)$$

This means that the expected rate of growth of money supply in period t as calculated at the end of period $t-1$ is $x_t = E_{t-1}(dm_t)$.

The price level is related to the nominal wage w_t by a mark-up factor that does not depend on the level of activity but can be affected by supply shocks:

$$dp_t = dw_t + \phi_t \qquad (3)$$

where ϕ_t is a white noise disturbance term representing (external) supply shocks. Note that, since it is assumed that information on aggregate variables is received by all economic agents, including the Government, with a one period lag, the disturbance terms in the model must be independent stochastic variables[4].

Our last equation is a sort of Phillips Curve equation, which decomposes wage inflation into a cyclical component and a trend term[5].

$$dw_t = -a_t(\bar{y} - y_{t-1}) + z_t^e = -a_t h_{t-1} + z_t^e \qquad (4)$$

where $\bar{y}$ indicates full employment output (which is taken as fixed here), $h_t = \bar{y} - y_{t-1}$ is the output gap measured by the logarithmic deviation of real output from its full employment level, a_t is the Phillips Curve slope coeffient and z_t^e stands for expected trend inflation. We define trend inflation as the rate of inflation which would occur this period if the economy were in equilibrium last period: therefore $z_t^e = E_{t-1}(dp_t/h_{t-1} = 0)$. Let me emphasize that neither the Phillips Curve slope coefficient, a_t, nor the expected trend inflation term, z_t^e, can be given arbitrary values: they have to be determined as part of the solutíon of the model, and will depend crucially on the market behaviour hypothesis which is adopted.

Note that equation (4) is not the traditional keynesian specification for the Phillips Curve, which would be:

$$dw_t = -a_t h_{t-1} + dp_t^e \qquad (4')$$

where $dp_t^e = E_{t-1}(dp_t)$ is the expected rate of inflation as calculated at the end of period $t-1$. This,

[4] Hence, we cannot follow Taylor (1980) is assuming that the monetary policy rule is $dm_t = gdp_t$, where g is a positive constant smaller than one. This amounts to $dm_t = gdw_t + g\phi_t$, which implies, from (2) – and assuming that dw_t is a fully anticipated variable – that $\mu_t = g\phi_t$. In this case, the stochastic disturbances are not independent, and government could try to compensate supply shocks by means of monetary policy shocks. Such possibility is ruled out in our models because if we give an informational advantage to the monetary authority the case for policy activism becomes a trivial one.

[5] See Dornbusch (1980) for a similar formulation of the Phillips Curve equation.

however, cannot be used in a discrete price-setting-cum-rational expectations context, as shown by McCallum (1980): if we put together (3) and (4'), we have $dp_t = -a_t h_{t-1} + dp_t^e + \phi_t$ and, by taking expectations on both sides of this equation, we conclude that h_{t-1} is *identical* to zero, which cannot be true in discrete price-setting models[6].

To understand the relationship between expected inflation and expected trend inflation, let us put together (3) and (4) into

$$dp_t = -a_t h_{t-1} + z_t^e + \phi_t \qquad (5)$$

and, by taking expectations on both sides of the equation, derive

$$dp_t^e = -a_t h_{t-1} + z_t^e \qquad (6)$$

This shows that a) if the economy is in equilibrium in period $t-1$, expected inflation and expected trend inflation for period t (as calculated at the end of $t-1$) will be same: $dp_t^e = z_t^e = E_{t-1}(dp_t/h_{t-1} = 0)$; and b) if the economy is out of equilibrium in period $t-1$, inflation will be expected to differ from trend inflation (which, as will be seen bellow, is usually the same as expected money growth), but this is perfectly rational from the point of view of economic agents because this difference between inflation and trend inflation is a crucial part of the adjustment process by which the economy moves toward equilibrium in period t.

From (5) and (6) we have

$$dp_t = dp_t^e + \phi_t \qquad (7)$$

which confirms the rationality of expectations in our model: actual and expected prices may differ only by a random forecast error resulting from supply shocks. Note that we are dealing in (7) with the expectation of the price level for period t, as calculated at the end of period $t-1$ with full knowledge of the output gap for that period. If we consider instead the expectation for the same variable as calculated at the end of period $t-2$, without knowledge of real output in period $t-1$, we have[7]:

$$dp_t^* = -a_t h_{t-1}^e + z_t^* \qquad (8)$$

[6] We are assuming that economic agents have perfect knowledge of the true model of the economy, so that the Phillips Curve slope coefficient a_t is fully anticipated at the end of period $t-1$. Therefore, $E_{t-1}(a_t h_{t-1}) = a_t h_{t-1}$, and since a_t is always different from zero – as will be shown later – it follows that h_{t-1} is identical to zero. Note that we cannot avoid this problem by writting h_t instead of h_{t-1} in (4'), because in a discrete price-setting economy the value of h_t cannot be known at the beginning of period t when $dw_t = w_t - w_{t-1}$ determined. For the same reason, we cannot follow Fischer (1977) by writing the mark-up equation as $dp_t = dw + f dy_t + \phi_t$, as $dy_t = y_t - y_{t-1}$ cannot be known at the beginning of period t, when d_p is determined. Fischer's model does not belong to the class of models we are studying here, since it has discrete wage setting but continuously flexible prices. Phelps (1978) wrote $dw_t = -a_t h_t^e + dp_t^e$, where $h_t^e = E_{t-1}(h_t)$. This, in conjunction with (3), implies that h_t^e is identical to zero, which – as will be seen later – amounts to discrete market clearing. Hence, this form of the Phillips Curve equation is a valid alternative to (4) in the case of discrete market clearing, but cannot be used when we want to make the model generate persistent disequilibria.

[7] Throughout this paper the notation will be p_t^e for the expectation of p_t as calculated at the end of period $t-1$, and p_t^* for the expectation of the same variable at the end of period $t-2$. It follows that $dp_t^e = p_t^e - p_{t-1}$ and $dp_t^* = p_t^* - p_{t-1}^e$.

where $h_{t-1}^e = E_{t-2}(h_{t-1})$ and $z_t^* = E_{t-2}(z_t^e)$. Assuming $z_t^* = z_t^e$[8], from (5) and (8) comes

$$dp_t = dp_t^* + \phi_t + a_t(h_{t-1}^e - h_{t-1}) \qquad (9)$$

showing that actual and expected prices may differ in this case as a result of either supply shocks or lagged demand shocks.

II. Discrete Market Clearing and the Neutrality Proposition

Under discrete market clearing, at the end of each period all markets are expected to clear in the following period, regardless of whether the economy is currently in equilibrium or not. If there is equilibrium in period $t-1$, the Economy is expected to remain in equilibrium in period t; if there is disequilibrium in period $t-1$, the economy is expected to return to equilibrium in period t. In terms of our basic model, it means that $E_{t-1}(h_t) = 0$ must hold as an identity. From this condition, we may find out the values for the expected trend inflation term, z_t^e, and the slope coefficient, a_t, of the Phillips Curve equation (4) which are consistent with discrete market clearing.

By substituting (2) and (5) into (1), we have

$$dy_t = a_t h_{t-1} + x_t - z_t^e + \varepsilon_t$$

where $\varepsilon_t = \mu_t + \delta_t - \phi_t$ is the net (expansionary) effect of random shocks on real output. Since $dy_t = y_t - y_{t-1} = h_t + h_{t-1}$, if follows that

$$h_t = (1 - a_t)h_{t-1} + z_t^e - x_t - \varepsilon_t \qquad (10)$$

Suppose first that $h_{t-1} = 0$; then $E_{t-1}(h_t) = 0$ only if $z_t^e = x_t$, which means that expected trend inflation must be equal to the anticipated rate of growth of money supply. This reduces (10) to

$$h_t = (1 - a_t)h_{t-1} - \varepsilon_t \qquad (11)$$

which shows that if $h_{t-1} \neq 0$, $E_{t-1}(h_t) = 0$ implies $a_t = 1$. Hence, with discrete market clearing the Phillips Curve equation (4) must be written as

$$dw_t = -h_{t-1} + x_t \qquad (12)$$

and the rate of inflation is given by

$$dp_t = -h_{t-1} + x_t + \phi_t \qquad (13)$$

If $z_t^e = x_t$ and $a_t = 1$, (10) is reduced to

$$h_t = -\varepsilon_t \qquad (14)$$

which is the same as

$$y_t = \bar{y} + \varepsilon_t \qquad (15)$$

showing that, under discrete market clearing, real output will depart from its full employment level

[8] It will be seen in section V that this may not be true, if the monetary policy rule determines x_t as a function of some period $t-1$ variable, such h_{t-1}. Neverthless, with rational expectations, the diference $(z_t^* - z_t^e)$ will always be a rondom white-noise term.

only as a result of unanticipated disturbances. The neutrality proposition holds in this case: real output responds only to monetary policy shocks (μ_t) and the systematic anticipated part of money supply growth (x_t) has no effect whatsoever on its behaviour.

Equation (15) may be read as saying that real output is a random deviation from its constant full employment level, and therefore must be serially uncorrelated[9]. This is of course inconsistent with the well-known evidence on real output persistence, but the problem can be circumvented – as it is done in the equilibrium approach literature – if it is assumed that full employment output is determined by a Lucas supply function such as

$$\bar{y}_t = s_0 + s_1(p_t - p_t^*) + s_2\bar{y}_{t-1} \qquad (16)$$

were p_t^* is the expectation of the price level for period t as calculated at the end of period $t-2$[10], and the lagged value of full employment output appears on the right-hand side of the equation on account of capital stock inertia (Lucas-1975), inventory inertia (Blinder-Fischer-1981) or labour force adjustment costs (Sargent-1979). With this additional equation, the model will generate real output persistence from its supply side, as a result of the first-order autocorrelation in full employment output, even though discrete market clearing forces the output gap to be random, as shown by (14)[11]. The neutrality proposition obviously still holds in this case[12]. The reader should be aware that although we will cling to the basic model assumption of a constant full employment output level for the remainder of this paper, nothing in our results would be changed by the introduction of the Lucas supply mechanism of (16).

III. Discrete Market Clearing with Staggered Wage Contracts

It should be really no surprise that discrete Market clearing has produced the neutrality proposition in our rational expectations discrete price-setting basic model. With discrete market clearing, all markets are expected to clear in each period, and consequently, on account of rational

[9] If $\bar{y}$ follows a fixed time trend, instead of remaining constant over time as in our basic model, (15) implies that the log of detrended real output is serially uncorrelated, which is also inconsistent with the empirical evidence.

[10] If we had used here the expectation of the price level for period t as calculated at the end of period $t-1$, only supply shocks would affect full employment output, as shown by (7). A rationale for the use of end of period $t-2$ expectations in the Lucas supply function when there is discrete price-setting is this: for technological reasons, producers in each local market must decide their supply for period t at the beginning of period $t-1$, without knowledge of real output for this period; consequently they can only use (8) in forming their expectations for the price level in period t.

[11] Note that this is not inconsistent with the empirical evidence on serial correlation in the logarithm of detrended output, because in the present case this variable is not a measure of the output gap $(\bar{y}_t - y_t)$; it is a measure of the variable $(\bar{y} - y_t)$, where $\bar{y} = s_0/(1-s_2)$ indicates the perfect information equilibrium value of full employment output, which we may assume to follow a fixed trend over time.

[12] To check this, note that $p_t - p_t^* = (p_t - p_{t-1}) - (p_t^* - p_{t-1}^e + (p_{t-1}^e - p_{t-2}) - (p_{t-1} - p_{t-2}) = dp_t - dp_t^* - (dp_{t-1}^e - dp_{t-1})$, so from (7) and (9), we have $p_t - p_t^* = \phi_t + a_t(h_{t-1}^e - h_{t-1}) + \phi_{t-1}$. We know that with discrete market clearing, $h_{t-1}^e = 0$ and, from (14), $h_{t-1} = -\varepsilon_{t-1}$. Therefore, $p_t - p_t^* = \phi_t + a_t\varepsilon_{t-1} + \phi_{t-1}$, showing that full employment output can be affected only by unanticipated disturbances, including monetary policy shocks.

expectations, the economy will be out of equilibrium only in those periods in which it is hit by random unpredictable shocks. If equilibrium is defined by a constant full employment output level (or by a Lucas supply function), this means that real output may be affected by monetary policy shocks but not by anticipated monetary policy[13]. Hence, anticipated money must be neutral in *any* rational expectations model with discrete Market clearing,

This section gives another example of this general statement by considering a more complex discrete market clearing version of the basic model, which results from adding to it the assumption of staggered wage contracts, as in Fischer (1977), Phelps (1978) or Taylor (1980). This will provide additional insights on the mechanics of solving discrete market clearing models, while at the same time laying in the background for the discussion, in section V, of persistent disequilibria in models with staggered wage contracts.

Suppose labour contracts fix the nominal wage for two periods ahead[14]. At the start of any given period, half of the labour force renegotiate its nominal wage for the next two periods, while the other half must still work under the terms set at the beginning of the previous period. Let c_t be the contract wage set in period t. It follows that the average nominal wage in this same period is:

$$w_t = 0.5c_t + 0.5c_{t-1} \qquad (17)$$

and, consequently,

$$dw_t = w_t - w_{t-1} = 0.5(dc_t + dc_{t-1}) \qquad (18)$$

The contract wage is determined by a Phillips Curve equation analogous to (4):

$$dc_t = -a_t h_{t-1} + j_t^e \qquad (19)$$

where $j_t^e = E_{t-1}\ (dc_t/h_{t-1} = 0)$ is the expected trend rate of contract wage inflation, which is not, however, the same thing as the expected trend rate of price inflation, z_t^e. To uncover the relationship between these two variables, consider the equation for the rate of inflation that comes from (19), (18) and (3):

$$dp_t = -0.5a_t h_{t-1} - 0.5a_{t-1}h_{t-2} + 0.5(j_t^e + j_{t-1}^e) + \phi_t \qquad (20)$$

With rational expectations,

$$z_t^e = E_{t-1}(dp_t/h_{t-1} = 0) = -0.5a_{t-1}h_{t-2} + 0.5(j_t^e + j_{t-1}^e)$$

or,

$$j_t^e = 2z_t^e + a_{t-1}h_{t-2} - j_{t-1}^e \qquad (21)$$

Thus $j_t^e = z_t^e$ only if $h_{t-2} = 0$ and $j_{t-1}^e = z_t^e$; the trend rates of inflation in contract wages and

[13] If $E_{t-1}(h_t) = 0$, where $E_{t-1}(.)$ is the expectation conditional on all information available at the end of period $t-1$, including the true model of the economy and the autoregressive structute of all variables, it follows that $h_t = \xi_t$ where ξ_t is a white noise error term. This last equation implies the neutrality proposition.

[14] The model discussed here is similar to that used by Phelps and Taylor. Fischer assumes that the nominal wage is indexed to future price levels as expected at the time contracts are written. Therefore, the nominal wage will not, in general, be constant within the contract span.

prices will be the same only in very particular circumstances.

Substitution of (21) into (20) shows that

$$dp_t = -0.5a_t h_{t-1} + z_t^e + \phi_t \qquad (22)$$

which, together with (1) and (2) leads to

$$dy_t = 0.5a_t h_{t-1} + x_t - z_t^p + \varepsilon_t$$

or, since $dy_t = -h_t + h_{t-1}$,

$$h_t = (1 - 0.5a_t)h_{t-1} + z_t^e - x_t - \varepsilon_t \qquad (23)$$

Discrete market clearing requires that $h_t^e = 0$ holds as an identity; hence $a_t = 2$ and $z_t^e = x_t$, and (23) is reduced to

$$h_t = -\varepsilon_t \qquad (24)$$

which is the same as (14), and shows that anticipated money is neutral, as expected.

Note that equation (22) for the rate of inflation can be rewritten as

$$dp_t = h_{t-1} + x_t + \phi_t \qquad (25)$$

which is the same as equation (13) for the rate of inflation in the basic model under discrete market clearing. Obviously, this same equation for the rate of inflation will be derived in any rational expectations discrete price-setting model with discrete market clearing[15].

IV. Optimal Monetary Policy

The neutrality proposition is usually assumed to provide strong intellectual support for Milton Friedman's (1959) proposal that monetary policy should follow a constant money growth rate full. If anticipated money can have no effect on real output, monetary policy should be concerned only with price stability, and the optimal policy would be the one that minimizes the variance of the rate of inflation around a desired target. This, so the argument goes, is what results from a constant rate of growth of the money supply. We know that this argument is correct in the case of continuous market clearing models, but here we want to show that, in the case of discrete market clearing models, monetary policy activism may be desirable from the point of view of price stability.

Suppose the monetary policy rule is given by

$$x_t = \bar{x} + bh_{t-1} \qquad (26)$$

where $\bar{x}$ is the target rate of inflation; if $b = 0$ we have a constant money growth rate rule, and if b is positive we have a feedback rule. We have seen that, in any discrete market clearing model, the neutrality proposition holds and the rate of inflation is given by

$$dp_t = -h_{t-1} + x_t + \phi_t = \varepsilon_{t-1} + x_t + \phi_t \qquad (27)$$

[15] Note that from (1), (2) and (24) we derive (25).

since $h_{t-1} = -\varepsilon_{t-1}$.

From (26) and (27) it follows that

$$dp_t = \bar{x} + (1-b)\varepsilon_{t-1} + \phi_t \tag{28}$$

and, by computing variances in this equation:

$$var(dp_t) = (1-b)^2 var(\varepsilon_{t-1}) + var(\phi_t) \tag{29}$$

which shows that the variance of the rate of inflation is minimized when $b = 1$. The optimal monetary policy rule is the feedback rule $x_t = \bar{x} + h_{t-1}$, in which the rate of growth of the money supply must deviate from trend in period t by exactly the output gap in period $t-1$[16].

A simple example may enhance our understanding of this result. Suppose the economy is in equilibrium until period t when, *ceteris paribus*, it is hit by an expansionary velocity shock $\delta_t = -\bar{\delta}$. The consequence is a negative output gap $h_t = -\bar{\delta}$, from (14), while the rate of inflation remains constant, $dp_t = \bar{x}$, as shown by (28). Assume there are no random, shocks in period $t+1$. In that case, discrete market clearing will make the economy return to equilibrium in this same period, hence $h_{t+1} = 0$ and $dy_{t-1} = -h_{t+1} + h_t = -\bar{\delta}$.

From (1), it follows that $dp_{t+1} - dp_{t+1} = -\bar{\delta}$, showing thal the return to equilibrium must be accomplished through a reduction in the real quantity of money. This, however, can be done in many different ways. Consider first the case of a constant money growth rule, where $b = 0$; from (28) we see that $dp_{t+1} = \bar{x} + \bar{\delta}$, therefore $dm_{t+1} = \bar{x}$. In this case the reduction in real money results from a rise in the rate of inflation while money supply grows at a constant rate. Consider now the optimal feedback monetary policy rule, in which $b = 1$; from (28) we have $dp_{t+1} = \bar{x}$, and consequently $dp_{t+1} = \bar{x} - \bar{\delta}$. In this case the real quantity of money falls because the rate of growth of money supply is reduced while the rate of inflation remains constant. Obviously the second policy is the best one.

An interesting corollary from this discussion is that if monetary policy is optimal, with $b = 1$, the model will not generate a statistical Phillips Curve, in the sense of a negative covariance between h_{t-1} and dp_t. Note that, from (14) and (28), $cov(dp_t, h_{t-1}) = E\{[(1-b)\varepsilon_{t-1} + \phi_t](-\varepsilon_{t-1})\} =$

[16] Constant money growth would be optimal, however, it the monetary authority only had access to information on aggregate economy-wide variable with a two period lag, in which case (26) would not be a feasible policy rule. It is easy to see that if monetary policy sets $x_t = \bar{x} + bh_{t-2}$, we have $dp_t = \bar{x} + \varepsilon_{t-1} - b\varepsilon_{t-2} + \phi$, instead of (28), and the variance of the rate of inflation is minimized with $b = 0$. Hence, if there is discrete market-clearing and private agents have an informational advantage over the monetary authority, a constant money growth rule is indeed desirable from the point of view of price stability. Lucas (1980, p. 207) seems to agree that is the only possible basis for a full indictment of policy activism: "Friedman's case", he writes, "was built largely on the presumption of *ignorance* of the nature of business cycles. Many of us confused the methodological advances in economic dynamics that took place in the 1950s and 1960s with the substantive narrowing of this ignorance and consequently with the increasing feasibility of sophisticated reactive countercyclical policy. We have learned, I believe, that the list of economic propositions sufficiently well-grounded in theory and evidence to be useful in formulating aggregative policy is no longer now than it was in 1948".

$-(1-b)var(\varepsilon_{t-1})$ which will be zero if $b = 1$. Hence, if we do observe a statistical Phillips Curve in the real world, and want to believe there is discrete market clearing in it, we must also believe that either monetary policy is typically sub-optimal or that full employment output is determined by a Lucas supply function, such as (16), with the observed output gap being a measure of $(\tilde{y} - y_t)$, where $\tilde{y} = s_0/(1-s_2) \neq \bar{y}_t$ is the perfect information equilibrium value for full employment output.

V. Persistent Disequilibria

Let us turn now our attention to rational expectations discrete price-setting models which are not constrained by the discrete market clearing assumption. In these models, unanticipated disturbances may, *ceteris paribus*, generate persistent disequilibria that will last for at least two consecutive periods. We will first consider a simple non-market clearing version of our basic model, and then move on to discuss two models with staggered wage contracts.

A. Cyclical Stickiness of Wages and Prices

The simplest way to obtain a rational expectations discrete price-setting model that generates persistent disequilibria is by adding to our basic model the particular assumption on wage and price stickiness, suggested by McCallum (1978), which makes them sluggish when reacting to disequilibrium but fully flexible when adjusting to changes in anticipated money. One rationale for such "cyclical" stickiness of wages and prices is that when the economy departs from equilibrium, as a result of wrong expectations, allocation errors inevitably occur that are costly to correct rapidly and an immediate return to equilibrium would not be optimal from the point of view of utility maximizing economic agents; on the other hand, while the economy is in equilibrium there is no such restriction on wage and price flexibility in response to shifts in trend inflation.

Suppose then that in our basic model, $z_t^e = x_t$, as in the discrete market clearing case, while the Phillips Curve slope coefficient is smaller than its discrete market clearing value, say $a_t = a_1$. The rate of inflation will be given by

$$dp_t = -ah_{t-1} + x_t + \phi_t \qquad (3)$$

which, in conjunction with (1) and (2), leads to:

$$h_t = (1-a)h_{t-1} - \varepsilon_t \qquad (31)$$

showing that in this case, as in any discrete market clearing model, the economy can be thrown out of equilibrium only by a random unpredictable shock, but also that, once established, a State of disequilibrium will persist over time. Hence, cyclical stickiness of wages and prices is sufficient to make our basic model generates persistent disequilibria but is not enough to invalidate the neutrality proposition.

We also have a case here for monetary policy activism aiming at minimizing the variance of the rate of inflation. From (26) and (30) we have $dp_t = (b-a)h_{t-1} + \bar{x} + \phi_t$ which shows that the optimal monetary policy rule is

$$x_t = \bar{x} + ah_{t-1} \qquad (32)$$

in which the rate of growth of money supply deviates from trend in period t by a fraction of the output gap in period $t-1$, the fraction being given by the slope coefficient of the Phillips Curve.

Note also that, as in the discrete market clearing case of section IV, a statistical Phillips Curve will be generated here only when monetary policy is not optimal, that is to say, when $(b-a)$ is a negative number.

B. Real Wage Stickiness

Persistent disequilibria may also occur in models that put together staggered wage contracts and some form of real wage stickiness, as in Fischer (1977). Real wage stickiness has been rationalized by the implicit contract theories of Baily (1974) and Azariadis (1975), but it should be understood that it does not by itself imply the possibility of persistent disequilibria. In a model with synchronous wage and price setting, such as our basic model, the real wage may be expected to be constant over time – as shown by mark-up equation (3) – while the nominal wage and the price level jump around to perform the market clearing job. It is only when there are staggered wage contracts, that real wage stickiness will translate itself into nominal wage and price stickiness, and thereby preclude discrete market clearing. In this case, a change in the average nominal wage must be a change in the nominal wage on new contracts, while the nominal wage on old contracts remains fixed. As this cannot occur without differentiated real wage changes for workers in each contract group, any form of real wage stickiness will necessarily reduce the degree of price flexibility in the economy.

The real wage per period received from a given contract may change over its two period duration; therefore, it seems reasonable to assume that real wage stickiness means that the *average* real wage paid within each contract must be expected to remain constant over time[17]. The simplest way to model this is to assume that the contract wage set in period t must be consistent with

$$(c_t - p_t^e) + (c_t - p_{t+1}^e) = (c_{t-1}) + (c_{t-1} - p_t^e) \qquad (33)$$

where, as before, p_{t+1}^* and p_t^e are the expected price levels for periods $t+1$ and t, respectively, as calculated at the end of period $t-1$. This equation says that the new contracts being written at the

[17] In my paper John Williamson (1981), this wage setting rule has been called "consistent indexation". It was the basic idea behind the mandatory wage indexation schemes applied in Brazil from 1965 to 1979.

end of period $t-1$ must be expected to produce a geometric average real wage over their duration, encompassing periods t and $t+1$, equal to the average real wage that can be expected to result from contracts written in the previous period[18].

An alternative way to write (33) is

$$c_t - c_{t-1} = dc_t = 0.5(dp_{t+1}^* + dp_t^e)$$

since $dp_{t+1}^* = p_{t+1}^* - p_t^e$ and $dp_t^e = p_t^e - p_{t-1}$. Hence,

$$dc_t + dc_{t-1} = 0.5(dp_{t+1}^* + dp_t^e + dp_t^* + dp_{t-1}^e) \quad (34)$$

From (18), (3) and (22) we find that

$$dc_t + dc_{t-1} = 2dw_t = 2(dp_t - \phi_t) = -a_t h_{t-1} + z_t^e$$

and from (22) we may also find that similarly to (6) and (8):

$$dp_{t-1}^* = -0.5a_{t+1}h_t^e + z_{t+1}^*$$
$$dp_t^e = -0.5a_t h_{t-1}^e + z_t^e$$
$$dp_t^* = -0.5a_t h_{t-1}^e + z_t^*$$
$$dp_{t-1}^* = -0.5a_{t-1}h_{t-2} + z_{t-1}^e$$

It follows that (34) is equivalent to

$$0.5a_{t+1}h_{t-1}^e + 0.5a_t h_{t-1} + 0.5a_t h_{t-1}^e + 0.5a_{t-1}h_{t-2} = z_{t-1}^* - 3z_t^e + z_t^* + z_{t-1}^e \quad (35)$$

which defines the multi period constraint on the slope coefficients and expected trend inflation terms of successive Phillips Curve equations that results from real wage stickiness as defined by (3).

A simple case of inconsistency between discrete market clearing and real wage stickiness happens when the rate of growth of the money supply is expected to be constant over time, say $x_t = \bar{x}$ for all t. We have seen, in section III, that discrete market clearing occurs, in a model with two-period staggered wage contracts, when $a_t = 2$ and $z_t^e = x_t$. From the latter condition if follows, in the present case, that $z_{t+1}^* = z_t^e = z_t^* = z_{t-1}^e = \bar{x}$, and hence $z_{t+1}^* - 3z_t^e + z_t^* + z_{t-1}^e = 0$. We also know that with discrete market clearing $h_t^e = 0$ and $h_t = -\varepsilon_t$, for all t. Consequently, (35) is reduced to $3\varepsilon_{t-1} - \varepsilon_{t-2} = 0$, which in general will not be true.

With real wage stickiness, the degree of persistence of disequilibria will be influenced by the monetary policy rule. Let the monetary policy rule be $x_t = \bar{x} + bh_{t-1}$, as in section IV. Suppose the market system makes $z_t^e = x_t = \bar{x} + bh_{t-1}$ for all t, so that – as shown by (23) – if the economy is in equilibrium in period $t-1$ it will be expected to remain in equilibrium in period t. It follows that $z_t^* = \bar{x} + bh_{t-1}^e$, for all t, and (35) is reduced to:

$$(0.5a_{t-1} - b)h_t^e - 3(0.5a_t - b)h_{t-1} + (0.5a_t - b)h_{t-1}^e + (0.5a_{t-1} - b)h_{t-2} = 2 \quad (36)$$

[18] It should be noted that (33) assumes that new contracts do not attempt to eliminate changes in the average real wage paid by old contracts which have not been forecasted when these contracts were written. Otherwise, we should have written $(c_t - p_t^e) + (c_t - p_{t-1}^*) = (c_{t-1} - p_{t-1}^e) + (c_{t-1} - p_t^*)$, but this would make the analysis substantially more complex, though with basically the same results.

which holds as an identity $a_t = 2b$ for all t[19]. Hence, from (23), we derive:

$$h_t = (1-b)h_{t-1} - \varepsilon_t \qquad (37)$$

showing that the degree of persistence of disequilibria is determined by the monetary policy feedback parameter b. Obviously, an optimal monetary policy will set $b = 1$, making the economy behave as if there was discrete market clearing, that is, $x_t = \bar{x} + h_{t-1}$ implying $h_t = \varepsilon_t$.

A noteworthy consequence of real wage stickiness, as defined by (3), is that in this case the model will not generate a statistical Phillips Curve even if monetary policy is suboptimal. If $a_t = 2b$ and $x_t = \bar{x} + bh_{t-1}$, it follows from (22) that:

$$dp_t = -bh_{t-1} + \bar{x} + bh_{t-1} + \phi_t = \bar{x} + \phi_t \qquad (38)$$

and hence, from (37) and (38), we find that, as ϕ_t and ε_{t-1}^ are stochastically independent, $cov(dv_t, h_{t-1}) = 0$ for any value of b. This means that real wage stickness, as defined by (33), will be consistent with the empirical evidence supporting a statistical Phillips Curve only if there is a Lucas supply function, such as (16), in the economy.

C. Relative Wage Stickiness

Models that put together staggered wage contracts and relative wage stickiness have been studied by Phelps (1978) and Taylor (1979, 1980). In the simplest, two period contracts case, it is assumed that the contract wage set in period t must be such that, over periods t and $t+1$, all workers can be expected to receive the same average real wage, irrespectively of whether they have entered a new contract at period t or not. This means that contract c_t must be consistent with:

$$(c_t - p_t^e) + (c_t - p_{t+1}^*) = (c_{t-1} - p_t^e) + (c_{t+1}^* - p_{t+1}^*) \qquad (39)$$

which can be simplified into

$$c_{t+1}^* - c_t = c_t - c_{t-1} \qquad (40)$$

with $c_{t+1}^* = E_{t-1}(c_{t+1})$

From (19) we have

$$c_{t+1}^* - c_t = dc_{t+1}^* = -a_{t+1}h_t^e + j_{t+1}^*$$

$$c_t - c_{t-1} = dc_t = -a_t h_{t-1} + j_t^e$$

with $j_{t+1}^* = E_{t-1}(dc_{t+1}/h_t^e = 0)$, showing that (40) may be rewritten as

$$-a_t h_t^e + j_{t+1}^* = -a_t h_{t-1} + j_t^e \qquad (41)$$

[19] Note that (36) is a non-linear stochastic second order difference equation on a_t. If we assume that, after an initial random shock, the economy returns to equilibrium along a perfect foresight disequilibrium path, we have $h_t^e = h_t = (1 - 0.5a_t)h_{t-1}$, which reduces (36) to $(0.5a_{t+1} - b)(1 - 0.5a_t)(1 - 0.5a_{t-1}) - 2(0.5a_t - b)(1 - 0.5a_{t-1}) + (0.5a_{t-1} - b) = 0$. I suspect (though I cannot prove it) that $a_t = 2b$ is the only solution for this equation that makes $h_t = (1 - 0.5a_t)(1 - 0.5a_{t-1}) \ldots (1 - 0.5a_1)h_0$ tend to zero as t increases.

Note also that, by using (21), we may calculate

$$j_{t+1}^{*} = 2z_{t+1}^{*} + a_t h_{t-1} - j_t^{e}$$

and thereby restate (41) as

$$-a_{t+1}h_t^{e} + 2z_{t+1}^{*} = -2a_t h_{t-1} + 2j_t^{e} \tag{42}$$

Consider now the same equation, written with a one period lag

$$-a_t h_{t-1} + 2z_t^{*} = -2a_t h_{t-1} + 2j_{t-1}^{e} \tag{43}$$

If we add (42) to (43), and note that from (21), $j_t^{e} + j_{t+1}^{e} = 2z_t^{e} + a_{t-1}h_{t-2}$, we get

$$a_{t+1}h_t^{e} + a_t h_{t-1}^{e} - 2a_t h_{t-1} = 2z_{t+1}^{*} + 2z_t^{*} - 4z_t^{e} \tag{44}$$

This is the constraint on slope coefficients and expected trend inflation terms of successive Phillips Curve equations that results from relative wage stickiness, as defined by (39).

If we assume, as before, that the monetary policy rule is $x_t = \bar{x} + bh_{t-1}$, and that the market system makes $z_t^{e} = x_t$ for all t, we have $z_{t+1}^{*} = \bar{x} + bh_t^{e}$ and (44) can be reduced to

$$(a_{t+1} - 2b)h_t^{e} + (a_t - 2b)h_{t-1}^{e} - 2(a_t - 2b)h_{t-1} = 0 \tag{45}$$

which holds as an identity if $a_t = 2b$ for all t. The results here are the same as in the case of real wage stickiness: from (23) we get stickiness, but these models seem to be embarrassingly hard to reconcile with the empirical evidence supporting the notion of a statistical Phillips Curve.

Unexpectedly, however, we have found that the neutrality proposition does not necessarily imply that the optimal monetary policy rule is constant money growth. It was shown that, in discrete price-setting models where anticipated money is neutral but private agents have no international advantage over the monetary authority, optimal monetary policy is given by a feedback rule that links the rate of growth of money supply in the current period with the output gap in the previous period.

Overall the paper has developed a simple technique of analysis for rational expectations macroeconomic models, which does not assume continuous market clearing, brings back to the foreground the familiar concept of the Phillips Curve, and can also incorporate a Lucas supply mechanism for the determination of full employment output.

References

Azariadis, C. (1975) "Implicit Contracts and Underemployment Equilibrium". *Journal of Political Economy*, 83, 1183-1202, December.

Baily, K. N. (1974) "Wages and Employment under Uncertain Demand", *Review of Economic Studies*, January, 41, 37-50.

Barro, Robert J. (1976) "Rational Expectations and the Role of Monetary Policy", *Journal of Monetary Economics*, January, 2, 1-32.

Barro, Robert J. (1979) "The Equilibrium Approach to Business Cycles", Unpublished manuscript.

Blinder, Alan S. and Stanley Fischer (1979) "Inventories, Rational Expectations, and the Business Cycle", National Bureau of Economic Research Working Paper. Nº 381, August.

Dornbusch, Rudiger (1980), Inflation Stabilization and Capital Mobility, Unpublished manuscript.

Fischer, Stanley (1977) "Long Term Contracts, Rational Expectations and the Optimal Monetary Policy Rule", *Journal of Political Economy*, February, 85, 191-205.

Friedman, Milton (1959) *A Program for Monetary Stability*, New York: Fordham University Press.

Lopes, Francisco L. and J. Williamson (1981) "Wage Indexation and Anti-Inflation Policy", Unpublished manuscript.

Lucas, Robert E. Jr. (1972) "Expectations and the Neutrality of Money". *Journal of Economic Theory*, April, 4, 103-124.

Lucas, Robert E. Jr. (1973) "Some International Evidence on Output-Inflation Tradeoffs", *American Economic Review*; June, 63, 326-334.

Lucas, Robert E. Jr. (1975) "An Equilibrium Model of the Business Cycle" *Journal of Political Economy*, December, 83, 1113-1114.

Lucas, Robert E. Jr. (1980) "Rules, Discretion and the Role of the Economic Advisor" in *Rational Expectations and Economic Policy*, edited by Stanley Fischer, Chicago, The University of Chicago Press.

McCallum, Bennett T. (1978) "Price Level Adjustments and the Rational Expectations Approach to Macroeconomic Stabilization Policy", *Journal of Money, Credit, and Banking*, November, 10, 418-36.

McCallum, Bennett T. (1980) "Rational Expectations and Macroeconomic Stabilization Policy", *Journal of Money, Credit and Banking*, November, 12, 716-746.

Okun, Arthur M. "Rational-Expectations-with-Misperceptions as a Theory of the Business Cycle", *Journal of Money Credit and Banking*, November, 12, 817-825.

Phelps, Edmund S. and John B. Taylor (1977) "Stabilizing Properties of Monetary Policy Under Rational Expectations", *Journal of Political Economy*, February, 84, 163-190.

Sargent, Thomas J. (1973) "Rational Expectations, the Real Rate of Interest and the Natural Rate of Unemployment", *Brookings Papers on Economic Activity*, 2, 429-472.

Sargent, Thomas J. (1979) *Macroeconomic Theory*, New York, Academic Press.

Sargent, Thomas J. and Neil Wallace (1975) "Rational Expectations, the Optimal Monetary

Instrument and the Money Supply Rule" *Journal of Political Economy*, April, 83,241-254.

Solow, Robert M. (1979) "Alternative Approaches to Macroeconomic Theory: A Partial View", *Canadian Journal of Economics*, 12, 339-354.

Taylor, John B. (1979) "Staggered Wage Setting in a Macro Model", *American Economic Review*, May, 59, 108-113.

Taylor, John B. (1980) "Aggregate Dynamics and Staggered Contracts" *Journal of Political Economy*, 88, 1-23.