TEXTO PARA DISCUSSÃO

Nº 17

Energy Prices, Inflation and Growth

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I. Introduction

Ever since the oil embargo of 1973, that culminated with the quadrupling of the international prices, numerous articles, both empirical and theoretical, have been written on the subject of the energy-economy relations (See Manne Sweeney [14], Hudson & Jorgenson [7], Nordhaus [12,13] to cite a few). Most of the macro-energy models developed and analysed had the framework of general equilibrium in a real system with a long-run perspective. Imported energy independence being the ultimate target, would be properly stimulated by an increase in relative prices that would trigger the process of substitution and conservation. Structural rigidities that characterize most economies, at least in the short-run, have been by large neglected in these studies. As a result, their energy policy implications could always be recorded on the basis of short-term arguments and their implementation surprised in the short-run. It is widely recognized by now that these models were overly optimistic in their projections (see Deagle et. all [3]) which suggests that there might have been some “missing-links” in their representation of the energy-economy interactions.

The problems faced by most oi1-importing economies, developed and less-developed, in the process of imported energy substitution very much resemble those faced by industrializing economies during the stage of imports substitution (see Hirschman [6] & Furtado [4]) in the 50’s and 60’s. The impact of the energy crisis upon the economic system is similar to that of other supply shocks as, for instance, a crop failure. The dilemma between inflation, foreign borrowing and recession characterizes all sorts of supply shocks.

The peculiarity of the energy problem that distinguishes it from other supply shocks is that the increases in oil prices shall continue into the future. Hence, an adequate response of the economic system must have as its main objective the reduction of energy dependence, which is not the case for agricultural sector after a crop failure. For the latter the choice between an accommodating policy (inf1ationary) and a non-accommodating policy (contractionary) depends only upon how much society is willing to pay in terms of output recession to avoid strengthening the inflationary process.

In this paper imported energy substitution is analysed from the point of view of an economy restrained by the balance-of-payments, that follows an indexation rule for domestic factor prices and has chronical unemployment. While not many economies have all three characteristics, the issues of unemployment, indexation or the inflationary spiral and the foreign exchange bottleneck are, to a certain extent, common to all less developed oil importing economies.

In section two we establish our basic framework of analysis. The distribution of losses when imported energy prices increase is discussed under different hypothesis on domestic factors price rigidity. In section three, the equilibrium analysis of section two is extended to an intertemporal context. An approximate representation of the process of indexation allows us to relate the acceleration of inflation with the increase in the domestic real price of energy imports. Via substitution output growth is also related to the price increase. Section four establishes a structural trade-off between output growth and the of inflation for indexed economies restrained by the balance-of-payments. Alternative energy price is evaluated, through the minimization of the integral of a loss function relating the rate and the acceleration of inflation, in section five. In section six the growth implications of alternative energy price trajectories are examined. The growth/inflation trade-off is further explored in section seven when we assume for the determination of energy price paths that society is willing to accept further acceleration of inflation in exchange for higher output growth rates. Lastly, section eight concludes this work summarizing its main results.

II. Income Distribution and Energy Prices

Following Manne [9, 10] and Sweeney [14] we characterize the economy by a single output aggregate production function F with two inputs: energy E and non-energy R, an aggregate of the domestic factors of production capital and labour[1]. Production possibilities are then represented by

$$Y = F(R, E) \qquad F_R > 0, F_E > 0 \qquad (1)$$

where Y denotes the output level. We further assume that F displays constant returns to scale and marginal productivities are positive. Under these conditions we can derive the economy's input price frontier

$$f(v, q) = 0 \qquad f_v > 0, f_q > 0 \qquad (2)$$

which establishes an inverse relationship between real aggregate value per unit produced v and the real energy price q. Observe that unit real aggregate value, the aggregate price of domestic production factors, is defined by the real wage divided by average labour productivity plus unit profit.

Our economy is subject to a balance-of-payments restriction on payments for energy imports[2] of the form

$$B = q^* \frac{E}{Y} \qquad (3)$$

where q^* denotes the international price of energy and B is the current account deficit plus liquid exports of goods and services. We assume that B is given exogenously by international liquidity conditions, world trade and the levei of the accumulated foreign debt. Domestic real prices and international prices are related by

$$q = (1 - z)eq^* \qquad (4)$$

[1] Empirical evidence on the possibilities of such aggregation is contradictory see Berndt & Wood [1, 2], Griffin & Gregory [5] and Hudson & Jorgenson [7].

[2] We assume from here on that all energy is imported. The domestic energy sector has been merged with the rest of the economy. For a disaggregated analysis along these lines see Lopes/Modiano [11].

where e denotes the real exchange rate assumed constant throughout by a purchasing power parity rule and z the subsidy rate.

Without loss of generality we specialize our results to a constant elasticity of substitution production function in (1) which provides us with the minimal representation required to study the process of imported energy substitution. In this case

$$E = aYq^{-\sigma} \qquad 0 \leq \sigma \leq 1 \qquad (5)$$

where a is a constant and σ denotes the elasticity of substitution.

The simple model of the economy described by (1) to (5) permits us to derive the following conclusions concerning the distribution of real income losses generated by imported energy prices increases:

i) In an economy where real wages and profit margins are fixed, through for instance perfect indexation, energy substitution will not be promoted via domestic energy price increases. As v is constant in (2) if the increase in international prices is passed along to the nominal domestic price it will on1y provoke a corresponding increase in the output price so as to keep the real price q unchanged. From (5) the energy/output ratio remains therefore constant. The acceptance of higher rates of inflation does not relieve the economy of the foreign exchange bottleneck upon growth implied by (3). From the viewpoint of controlling inflation, the optical solution consists of increasing subsidies z in (4) to stabilize the domestic nominal energy price. Clearly, this policy cannot be supported for a long period of time.

ii) When profit margins are fixed but the real wage adjusts, the increase in the domestic real energy price requires a decline in real wages according to the input price frontier in (2). Substitution is induced with the retraction in energy demand for $\sigma > 0$ as given by (5). In developed economies we could expect the decline in real wages, at least in the short-run, to follow the retraction in labour demand associated to the existing capital equipment. However, this natural adjustment process vii the labour market is most unlikely in less developed economies for which labour supply is, for all purposes, perfectly elastic and there are well-defined wage policy rules such as explicit indexation. In this case the decline in employment will not have a significant impact upon real wages. Even if the government is willing to accept the onus of a wage control policy, the substitution of energy imports in less developed economies will not be without major social costs when profit margins are kept unchanged.

iii) Lastly, there is the possibility of adjustment to higher world prices through a decline of the profit margins which also reduces v in (2). As the energy/output coefficient falls, in response to the increase the domestic price, a slack in the ba1ance-of-payments restriction (3) permits a rise in aggregate output. Hence, it is not possible to evaluate a priori the effect of the reduction of profit margins on aggregate profit. However, if as a result of this policy, profit margins in the economy fall significantly the investment financing required for actual energy substitution may be impaired.

We conclude that the adjustment of the economy to an external shock, such as an increase in imported energy prices, requires a change in the relative price of the factors of production with considerable impact upon the distribution of income. The resolution of distributive conflicts is essential to the effective promotion of imported energy substitution. In the following sections the analysis is, for the sake of simplicity, carried only in terms of a real wage adjustment. The main results are still valid for an adjustment through profit margins or a combination of both under a social pact.

III. The Dynamics of Growth and Inflation

Under the assumption of constant returns to scale for the aggregate production function F and efficiency in production input payments exhaust output or, in other words

$$P = Q\frac{E}{Y} + W\frac{R}{Y}$$

where P denotes the output price (general price index), Q denotes the nominal energy domestic price and W denotes the nominal price of the aggregate non-energy factors of production. Taking natural logarithms and differentiating with respect to time we obtain that[3]

$$\hat{P} = s\hat{Q} + (1-s)\widehat{W} \qquad (6)$$

where s denotes the energy share QE/PY. Furthermore, since $w = W/P$ and $q = Q/P$,

$$\widehat{W} = \widehat{w} + \hat{P} \qquad (7)$$

and

$$\hat{Q} = \hat{q} + \hat{P} \qquad (8)$$

Substitution of (7) and (8) into (6) yields the dynamic version of the input-price frontier

$$s\hat{q} + (1-s)\widehat{w} = 0 \qquad (9)$$

The economy is indexed by assumption. The process of indexation is represented in continuous time by

$$\widehat{W}(t) = \hat{P}(t-\tau), \tau > 0 \qquad (10)$$

where τ denotes the average lag between wage adjustments and the general price index increases. For

[3] The hat "$\widehat{\ }$" denotes the derivative of the log of the variable with respect to time.

a perfectly indexed economy $\tau = 0$ and increases in output prices are instantly passed along to the domestic factors price. For a highly indexed economy we expect τ to be small and (10) can be approximated by[4]

$$\widehat{W} = \hat{P} - \tau\dot{\hat{P}} \qquad (11)$$

Which is the continuous-time analogue of Lopes-Bacha [8] discrete-time representation of indexation. In the discrete analysis τ is inversely related to the number of wage adjustments per period. Equation (11) can be combined with expression (7) to yield

$$\widehat{w} = -\tau\dot{\hat{P}} \qquad (12)$$

which states that the real wage falls when inflation accelerates ($\dot{\hat{P}} > 0$) and rises when inflation decelerates ($\dot{\hat{P}} < 0$). The real wage only remains unaltered for a constant rate of inflation ($\dot{\hat{P}} = 0$) or under perfect indexation ($\tau = 0$).

Using expressions (9) and (12) we can solve for the acceleration of inflation consistent with the change of the energy domestic real price

$$\dot{\hat{P}} = \frac{s}{(1-s)\tau}\hat{q} \qquad (13)$$

The coefficient $s/(1-s)$ measures the vulnerability of the economy to changes in the energy price as, for given τ and $\hat{q}$, the larger is this coefficient the larger is the inflationary impact of an energy price increase. For small energy shares the vulnerability of the economy can be approximated by s. Also the higher is the degree of indexation of the economy, the lower is τ and the larger is the inflationary impact of a real energy price increase. Equation (13) expresses the distributive conflict of section two in terms of an indexed economy. If real domestic factors' prices are kept constant by perfect indexation ($\tau = 0$) any attempt to increase the energy real price q will only generate a step increase ($\dot{\hat{P}} = +\infty$) of the same magnitude as $\hat{Q}$ in the rate of inflation $\hat{P}$.

Graphically relation (13) is represented by the inflation- line in Figure 1. When energy demands have the format (5) the share is given by

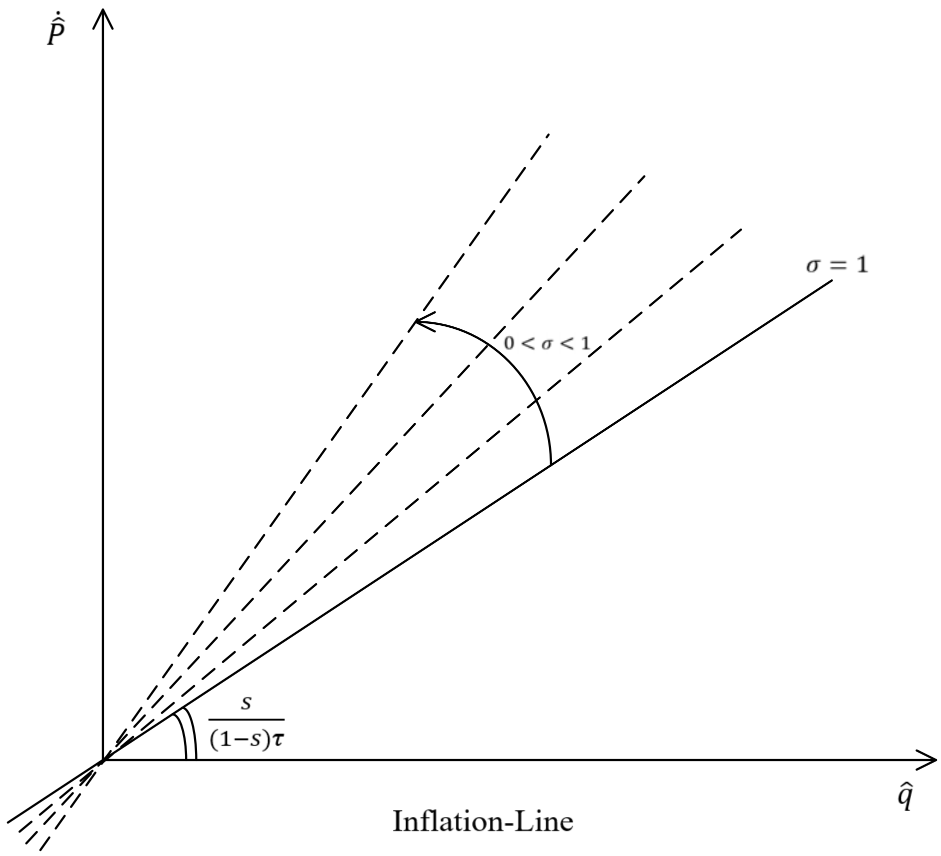
$$s = aq^{1-\sigma} \qquad (14)$$

Notice that for $\sigma < 1$, the energy share increases as its real domestic price rises according to

$$\hat{s} = (1-\sigma)\hat{q}$$

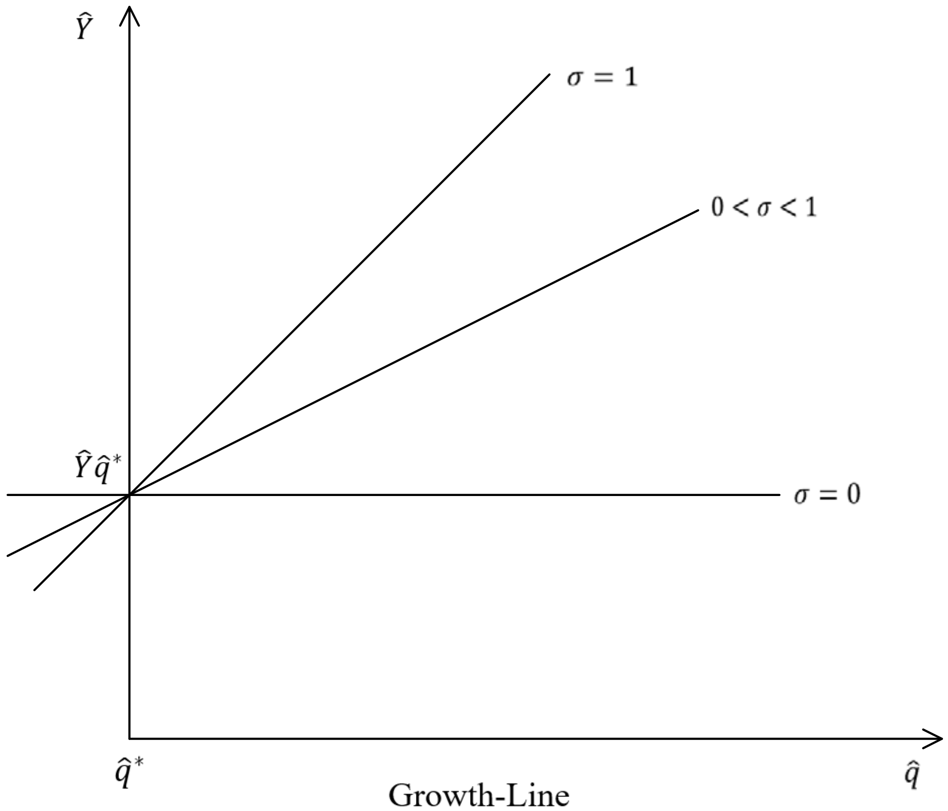
Hence, for a given initial share the inflation-line rotates counter clockwise with an increase in the energy price as also depicted in Figure 1. A constant positive rate of growth of the real domestic price requires a permanently increasing acceleration of inflation.

[4] The dot "." denotes the derivative of the variable with respect to time. Hence, $\dot{X} = \hat{X}/X$.



Inflation-Line

Figure 1



Growth-Line

Figure 2

A dynamic version of the balance-of-payments restriction (3) can be obtained once it is imposed that expenditures with imported energy are constrained to grow at a rate $\hat{Y}^*$ (possibly negative). This rate is assumed at this point exogenously given in the sense that it depends on the willingness of foreign lenders to further finance current account deficits. This in turn depends on the accumulated foreign debt and the growth rate of foreign demand for the country's exports. Taking natural logarithms and differentiating (3) with respect to time

$$\hat{Y}^* = \hat{q} + \hat{E} \qquad (15)$$

Expressed in terms of growth rates energy demand in (5) is given by

$$\hat{E} = \hat{Y} - \sigma\hat{q} \qquad (16)$$

Substitution of (16) into (15) yields

$$\hat{Y} = \hat{Y}^* - \hat{q}^* + \sigma\hat{q} \qquad (17)$$

which is the growth-line in Figure 2. Equation (17) states that output growth will be larger the larger is the rate of growth of foreign borrowing net of the rate of increase in imported energy prices $\hat{Y}^* - \hat{q}^*$ and the larger is the elasticity of substitution σ. Domestic price increases only enhance output growth if substitution is feasible or equivalently $\sigma > 0$. Otherwise the rate of growth is $\hat{Y}^* - \hat{q}^*$, which does not depend or the domestic price increase $\hat{q}$.

If we assume that domestic and international energy price increases are related by

$$\hat{q} = \gamma\hat{q}^* \qquad (18)$$

where γ is a policy[5] variable then (17) can be rewritten as

$$\hat{Y} = \hat{Y}^* + (\sigma\gamma - 1)\hat{q}^* \qquad (19)$$

This latter result shows that $\hat{Y} \lesseqqgtr \hat{Y}^*$ if and only if $\gamma \lesseqqgtr \frac{1}{\sigma}$. Hence, if $\hat{Y}^*$ is the rate of output growth prior to the increase in world energy prices the maintenance of growth requires that $\frac{1}{\sigma}$ times the international price increase be passed along to energy domestic prices. For σ within the range of 0.2 to 0.5 this means that γ is between 2 and 5. The impact upon inflation rates becomes undoubtedly a limiting factor.

IV. The Growth/Inflation Trade-Off

In the previous section we related the acceleration of inflation and the rate of growth to the level and the rate of change of energy real domestic price. For a given rate of change in international prices, the larger is the fraction γ that is passed on to domestic prices the larger are the rate of growth of the economy, except when substitution is not feasible ($\sigma = 0$) and the acceleration of inflation.

[5] Expressing (4) in growth rates, (18) is equivalent to the assumption that the subsidy factor $(1 - z)$ grows at the rate $(1 + \gamma)\hat{q}^*$ over time.

Eliminating γ from (13) and (19) we obtain a trade-off relationship between the rate of growth and the acceleration of inflation given by

$$\dot{\hat{P}} = \frac{s}{(1-s)\tau\sigma}\left[\hat{Y} - \hat{Y}^* + \hat{q}^*\right] \qquad (20)$$

and displayed in Figure 3.

For given initial energy shares the slope of the growth/inflation line is larger the larger is the degree of indexation of the economy (smaller τ) and smaller are the possibilities of energy substitution represented by the elasticity of substitution σ. The trade-off vanishes for $\sigma = 0$ as the growth/inflation line becomes vertical at the growth rate $\hat{Y}^* - \hat{q}^*$. In the latter case the increase in domestic energy prices $\gamma > 0$ only accelerates inflation without stimulating growth via substitution. To each point along the line there corresponds a value of the rate of domestic to international price increases γ. At any instant of time a combination of $\hat{Y}$ and $\dot{\hat{P}}$ can be chosen depending upon society's preference for growth vis-à-vis the acceleration of inflation as depicted by curve LL in Figure 3.

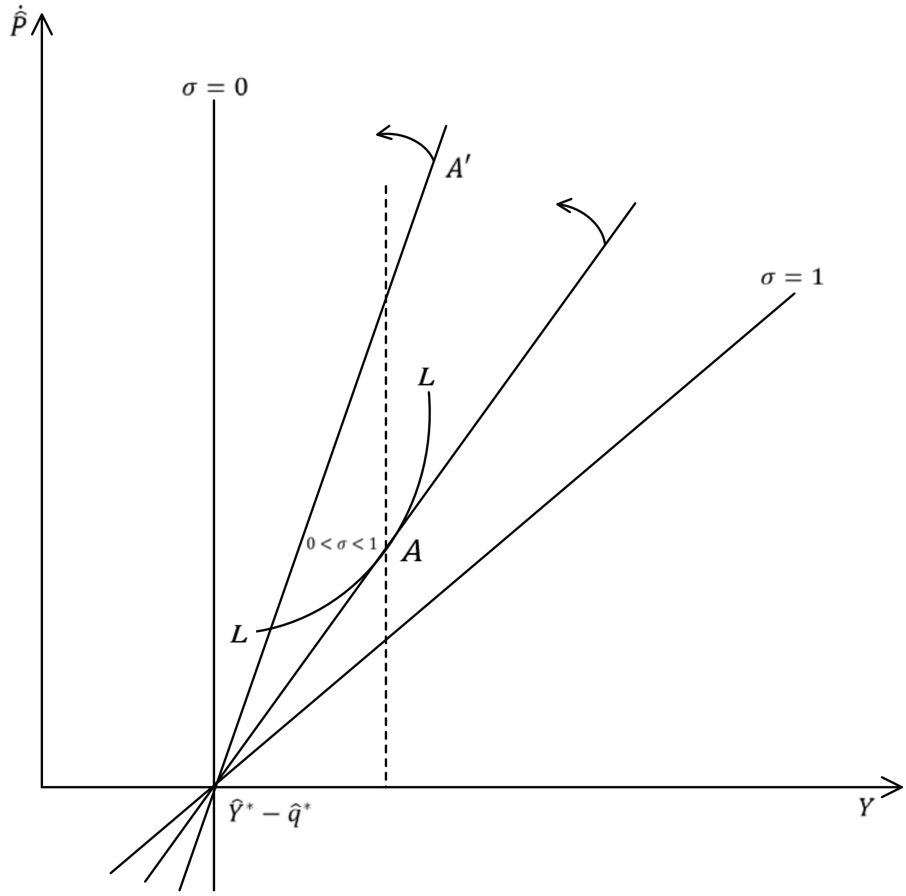
Observe that the trade-off relationship (20) is only stable for $\sigma = 1$ when a constant energy share s kceps the vulnerability of the economy unaltered by domestic real price changes. For $0 < \sigma < 1$ the growth/inflation line rotates counterclockwise as the international price increases are translated into domestic price increases ($\gamma > 0$). The only rate of growth compatible with stability of the rate of inflation ($\hat{P} = 0$) is $\hat{Y}^* - \hat{q}^*$ attained with $\gamma = 0$. To maintain a rate of growth superior to $\hat{Y}^* - \hat{q}^*$ society must be willing to accept larger and larger accelerations of inflation in the most likely case that $0 < \sigma < 1$.

The situation depicted in Figure 3 of a constant rate of increase of world prices does not give full dimension to what most oil-importing countries have faced. during the last decade. Firstly, increases in world energy prices have been in reality price shocks as represented by the step function,

$$\hat{q}^*(t) = \begin{cases} \hat{q}^*, & t = t_0 \\ 0\ , & t > t_0 \end{cases}$$

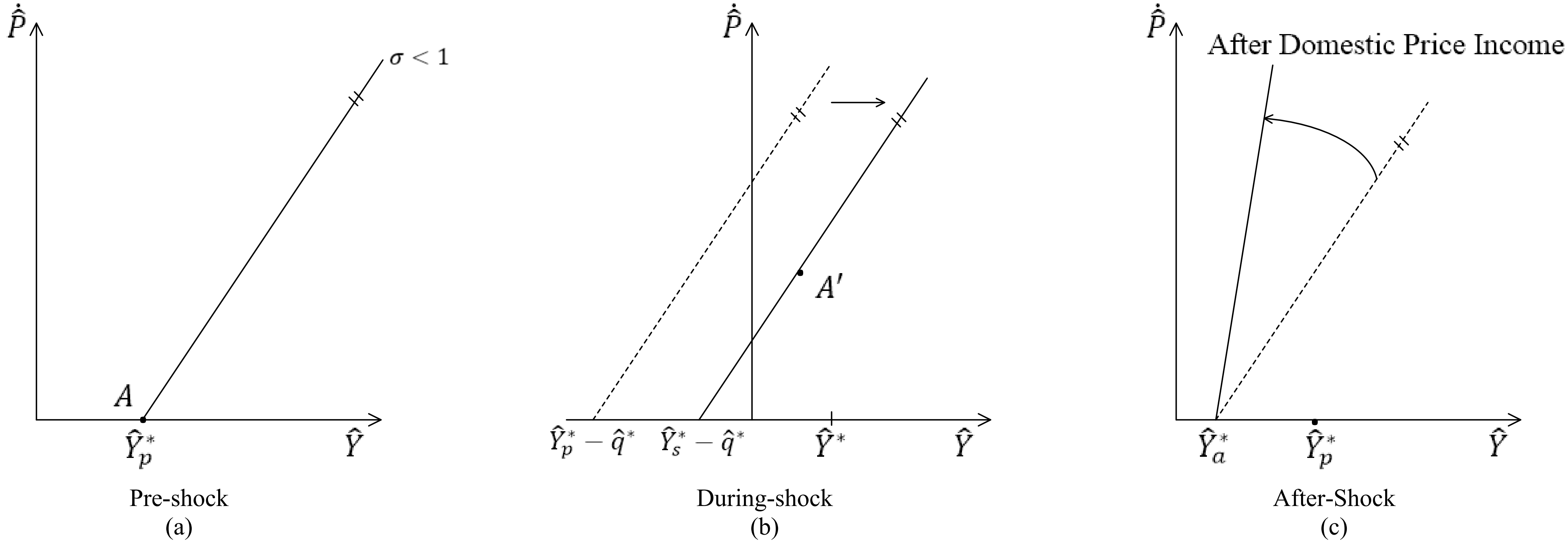
and secondly, the nature of the trade-off has been considerably more severe as the step increases have been many times larger than the rate of growth that prevailed at that instant of time. Figures (4-a) to (A-c) depict this situation for $0 < \sigma < 1$. At $t = t_0$, the horizontal intercept is negative showing that given the magnitude of the price increases experienced (up to 300) stability of the rate of inflation could at best be attained with negative growth at a very large rate. Besides its inflationary impact, a proportional increase in the energy domestic price ($\gamma = 1$) was no remedy as, within a reasonable range of substitution elasticities (0.2 to 0.6), maximum growth world still be negative. Even though the vu1nerability of the oi1-importing economies pre-shock was quite small (less than 5%) a more than proportional increase of the domestic price would imply in a significant acceleration of inflation (greater than 20%). As a consequence, most economies had to resort to increase $\hat{Y}^*$ from $\hat{Y}_p^*$ to $\hat{Y}_s^*$ as in Figure 4-b either using up any slack capacity in foreign borrowing or going into more costly and risky shorter-term loans. In practice the oil-importing economies reacted in the short-run through different combinations of slower growth, a larger foreign debt and a higher energy domestic real price such as point A' in Figure 4-b. Figure 4-c depicts one possibility for the after-shock period. The short-run step increase in the rate of foreign borrowing may reduce further borrowing capability from the pre-shock level $\hat{Y}_p^*$ to $\hat{Y}_a^*$ (possibly negative) and the growth/inflation line may become more vertical if at least a fraction of the increase in the world price is passed on to the domestic price.

In the next three sections we explore only the inflation and growth implication of varying energy domestic prices during an after-shock period ($\hat{q}^* = 0$). This assumes that the economy adjusts instantly to price-shocks mainly through some combination of a step increase in prices and a step increase in the foreign debt. In section eight we retake the discussion of the shock instant and relate it to our results.



The Growth/Inflation Trade-Off

Figure 3



Energy Price-Shock
Figure 4

V. Price Trajectories and Inflation

In section two it was demonstrated that, for an indexed economy, the acceleration of inflation is determined at each instant of time by the magnitude of the change in the real energy price. We consider now the promotion of price adjustments over a finite length of time. Given the current real price q_0 at $t = t_0$ and a terminal real price q_1 at $t = t_1$, different price trajectories $q(t)$ imply in different paths for the rate of inflation between t_0 and t_1 with real wages evolving accordingly. By (12) and (13) the greater the increase in real energy prices the larger is the decline in the real wage. If the relevant concern of society is the real wage and not employment, then alternative price trajectories can be analysed once an acceptable criterion expressed solely in terms of the rate of inflation is established. Growth and employment aspects of alternative price trajectories are discussed in the next section.

First we consider the terminal rate of inflation. This will be the case when society is only concerned with the terminal real wage as integrating both sides of (12) between t_0 and t_1 yields

$$w(t_1) = w(t_0) - \tau\left[\hat{P}(t_1) - \hat{P}(t_0)\right]$$

In this case the larger the terminal rate of inflation $\hat{P}(t_1)$ the smaller is the terminal real wage $w(t_1)$. The terminal rate of inflation can be determined by integration of (13) between t_0 and t_1 yielding

$$\hat{P}(t_1) = \hat{P}(t_0) + \frac{1}{\tau(\sigma-1)}\log\frac{1-aq_1^{1-\sigma}}{1-aq_0^{1-\sigma}} \quad (21)$$

The above result establishes a one-to-one relationship between the terminal real price and the terminal rate of inflation which is independent of the price path[6]. Consequently, the loss to society implied by the lower terminal real wage cannot be dampened by the trajectory. Furthermore, the terminal rate of inflation rises with the optimal price and the degree of indexation, which is inversely related to τ, and declines with the elasticity of substitution.

The conclusion above is not surprising if we recall that for indexed economies it is the acceleration of inflation $\dot{\hat{P}}$ as opposed to the rate of inflation $\hat{P}$ that tends to reduce real wages. Even though the terminal rate of inflation and the real wage do not depend upon the real energy price trajectory it does not follow that all paths are equally undesirable[7]. For most indexed economies in the process of changing energy prices abrupt variations in the rate of inflation have proved to be socially disturbing and to stimulate demand for more frequent wage adjustments or, in other words,

[6] The Euler equation vanishes for any solution trajectory.

[7] We do not consider in this section alternative objectives. For instance, if the terminal rate of inflation is the appropriate criterion concerning inflation and real wages, the reduction of the energy/output ratio favours a step increase in energy prices.

a higher degree of indexation[8]. This argument justifies the claim that for indexed economies real energy price trajectories have to be examined not only in terms of the instant rate of inflation but also in terms of the acceleration of inflation. While the former is inversely related to the real wage level, at any instant of time the latter accounts for changes in the real wage. The following loss function

$$L\left(\hat{P},\dot{\hat{P}}\right) = \hat{P}^{\alpha}\dot{\hat{P}}^{\beta} \qquad \beta > 1$$

represents with minimum simplicity the above assertion. The assumption that 3 is greater than unity implies that losses from the acceleration of inflation are incurred at increasing marginal costs.

An optimal energy price trajectory results then from the minimization of the integral loss from t_0 to t_1,

$$min \int_{t_0}^{t_1} \hat{P}^{\alpha}\, \dot{\hat{P}}^{\beta}$$

$$q(t_0) = q_0$$

$$q(t_1) = q_1$$

The Euler equation for this problem is, for small energy shares, given by[9]

$$\frac{\ddot{q}}{\dot{q}} = \lambda\frac{\dot{q}}{q} \qquad (22)$$

where $\lambda = \frac{\sigma-(1-\sigma)\alpha}{\beta}$. Solving the differential equation (22) yields

$$q(t) = (k_0 + k_1^t)^{\frac{1}{1-\lambda}} \qquad \lambda \neq 1 \qquad (23a)$$

$$q(t) = k_0 e^{k_1^t} \qquad \lambda = 1 \qquad (23b)$$

with k_0 and k_1 determined by the initial and terminal conditions q_0 and q_1 as

$$k_1 = \frac{q_1^{\lambda-1}-q_0^{1-\lambda}}{t_1-t_0}, \lambda \neq 1 \qquad (24a) \qquad\qquad k_0 = \frac{k_0^{1-\lambda}t_1-t_0q_1^{1-\lambda}}{t_1-t_0}, \lambda \neq 1 \qquad (24c)$$

$$k_1 = \frac{log\frac{q_1}{q_0}}{t_1-t_0}, \lambda = 1 \qquad (24b) \qquad\qquad k_0 = \frac{q_0^{t_1}}{q_1^{t_1}}\frac{1}{t_1-t_0}\lambda, \lambda = 1 \qquad (24d)$$

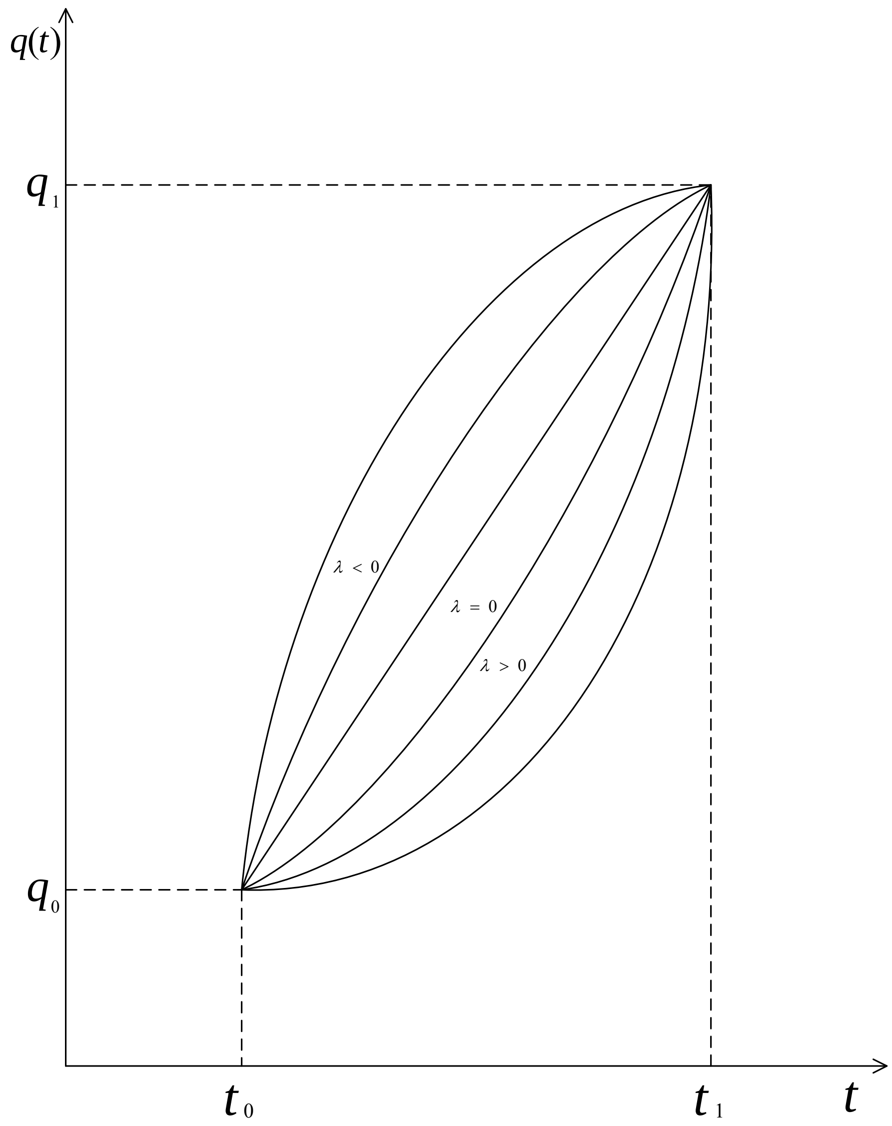
Figure 5 depicts the optimal price for different values of λ. When $\alpha = 0$, or the actual rate of inflation does not matter, the optimal price adjustment schedule is approximately linear when substitution possibilities are very limited ($\lambda, \sigma \cong 0$) and approximately exponential when substitution possibilities are very significant ($\lambda, \sigma \cong 1$). Between these two extremes of a constant price increase ($\lambda, \sigma = 0$) and a constant rate of price increase ($\lambda, \sigma = 1$) lie the solutions for $0 < \sigma < 1$. As one would expect the larger the elasticity of substitution the more can price increases be postponed. As substitution possibilities become larger the optimal trajectories tend to lower price increases in the beginning of the period and favour larger increases by the end of the planning horizon. This is due to the assumption of increasing marginal losses from an accelerating inflation. Still we can show by

[8] Formally, this suggests that in (12) τmight be a decreasing function of $\hat{P}$.
[9] The second order optimality condition is satisfied under the assumption that $\beta > 1$.

substitution of (23) into (13) that for small energy shares

$$\dot{\hat{P}} = \frac{ak_1}{(1-\lambda)\tau}(k_0 + k_1^t)^{\frac{\lambda-\sigma}{1-\lambda}}, \lambda \neq 1 \quad (25a)$$

$$\dot{\hat{P}} = \frac{ak_1}{\tau}, \lambda = 1 \quad (25b)$$



Energy Price Trajectories

Figure 5

This shows that the rate of inflation only grows linearly over time ($\dot{\hat{P}}$ is constant) for $\lambda = 1$ or $\lambda = \sigma$ which is the case for $\alpha = 0$. Otherwise the acceleration of inflation increases (decreases) over time for $\lambda > (<)\sigma$.

For negative values of the elasticity of the instant loss with respect to the rate of inflation α the optimal trajectories shift rightwards except for the limit case when $\sigma = 1$. The rationale for this solution stems from the observation that as the inflation rate gets larger the same acceleration of inflation is perceived as a smaller loss. Hence larger energy price increases can be further postponed towards the end of the period[10]. Conversely for positive values of α the optimal price trajectories shift leftwards for $0 \leq \alpha < 1$. If it is the case that at larger rates of inflation, or equivalently lower real wages, society perceives further acceleration as increasingly costly ($\lambda > 0$) than it is optimal to anticipate real energy price increases. These possibilities are also depicted in Figure 5.

The dependence of the optimal trajectories upon β, the elasticity of the instant loss with respect to the acceleration of inflation, is similar for positive and negative $\alpha's$. The larger is β the smaller are the anticipations and postponements of the energy price increase. Had we permitted $\beta = 0$ we would have a situation in which losses are associated only to the rate of inflation or equivalently the real wage level. In the most likely case that α is positive the solution is trivial. The rate of inflation should be kept constant and energy prices unchanged up to time t_1 when they should be adjusted through a step increase equal to $q_1 - q_0$.

VI. Price Trajectories and Growth

Consideration only of the rate and the acceleration of inflation led to optimal energy prices trajectories of the form (23). The growth pattern implied by such paths can be obtained by the conjunction of (23) and (17) giving

$$\hat{Y} = \hat{Y}^* + \frac{\sigma - k_1}{1-\lambda}\frac{1}{k_0 + k_1 t}, \qquad \lambda \neq 1 \qquad \text{(26a)}$$

and

$$\hat{Y} = \hat{Y}^* + \sigma k_1, \qquad \lambda = 1 \qquad \text{(26b)}$$

In the aftermath of an oil-shock, once the process of imported energy substitution is initiated, growth will be no smaller than $\hat{Y}^*$[11]. However, stable growth will only obtain for $\sigma = 0$ and $\lambda = 1$. When the loss does not depend upon the rate of inflation ($\alpha = 0$) these two cases corresponds respectively to Leontief and Cobb-Douglas production functions. Otherwise growth will increase smoothly during the interval for $\lambda < 1$ and decrease for $\lambda > 1$.

Alternatively, we may consider the case of an economy for which only output and growth matters. Traditionally alternative growth paths are analysed in terms of a utility function of output, income or consumption levels. However, it is not difficult to understand why, for the less developed oil importing countries, actual growth rates do play a major role in economic policy. As output growth is associated to employment growth, concern about growth is tantamount a concern about absorption of a growing labour force. Consequently, we want the benefits of a domestic energy price increase,

[10] The introduction of continuous discounting of future instant losses would also contribute to a postponement of energy price increases.
[11] As $k_1 = (q_1^{1-y} - q_0^{1-y})/(t_1 - t_0)$, we have $k_1 \gtreqless 0$ for $\lambda \gtreqless 1$.

for an economy subject to a balance-of-payments restriction, to be associated both with the output level and the growth rate. The benefit function is assumed, for the sake of simplicity, to be of the unit elasticity of substitution form.

$$\mathrm{U}(Y, \hat{Y} - \hat{Y}^*) = Y^{\varepsilon}(\hat{Y} - \hat{Y}^*)^{\eta} \qquad 0 < \eta < 1$$

The elasticity of the benefit with respect to the growth rate η is assumed to be less than unity. In this case higher growth rates are increasingly desirable but marginal benefits are decreasing. If the benefit of higher growth rates declines with the output level we would expect $\varepsilon < 0$.

The problem we want to solve is then

$$\max \int_{t_0}^{t_1} Y^{\varepsilon}\,(\hat{Y} - \hat{Y}^*)^{\eta}\, dt \qquad (27)$$

$$q(t_1) = q_1$$

$$q(t_0) = q_0$$

For small energy shares the Euler equation associated with (27) has the same form as (22),

$$\frac{\ddot{q}}{\dot{q}} = \left(1 - \frac{\varepsilon}{\eta\sigma}\right) - \frac{\dot{q}}{q}$$

The solution of the above differential equation[12] is equivalent to (23) with

$$\lambda = (\eta\sigma - \varepsilon)/\eta\sigma$$

We first consider the case in which $\varepsilon = 0$ or $\lambda = 1$. Then the optimal solution requires a constant rate of increase k_1 in energy real domestic prices as the trajectory is exponential by (23b), irrespectively of the values of σ and η. Also, if only instant growth matters by (26b) the resulting growth rate will be stable during the interval[13] for all values of the elasticity of substitution.

As we allow marginal benefits to decrease with the output level or, in other words, $\varepsilon < 0$, then $\lambda > 1$ and the optimal price trajectory in less than exponential as depicted in Figure 5. The larger is ε in absolute value the more can energy real price increases be postponed. Energy price increases and the growth rate become smaller in the beginning of the interval and larger towards the end of the horizon. Conversely larger values of the elasticities σ and η tend to anticipate the optimal price trajectory. As $\lambda < 1$ for $\varepsilon < 0$ the rate of output growth declines over the period.

As opposed to the rate of inflation when benefits are associated only to the output level ($\eta = 0$ and $\varepsilon > 0$) the optimal solution for $\lambda > 0$ consists of a step increase in energy prices in the beginning of the period ($t = t_0$). This will provoke a step increase in output at $t = t_0$ and growth at the rate $\hat{Y}^*$ thereafter.

[12] Second order optimality conditions are satisfied for $0 < \eta < 1$.

[13] This result is not surprising as the maxim and in (27) is the same in the minimization of the deviations of actual growth rates for $\hat{Y}^*$.

VII. Price Trajectories: Inflation and Growth

Inflationary and growth aspects of energy real price trajectories have been analysed separately in sections five and six. Optimal domestic energy price paths were determined for economies for which either inflation or growth matters. This is not realistic if we believe that the criteria interact or, in other words, that societies are willing to trade instantly a higher growth in employment for a loss in the real wage. To represent this possibility, we consider the loss function

$$\mathrm{L}(\hat{Y}, \dot{\hat{P}}) = \left(\hat{Y} - \hat{Y}^*\right)^{\delta} (\dot{\hat{P}})^{\beta} \qquad \beta > 1, \delta < 0$$

Under these assumptions instant losses increase with the acceleration of inflation and decline with the growth rate as curve LL in Figure 3 shifts to the northwest. Furthermore, for $\beta > 1$, marginal losses are increasing in $\hat{P}$ and the marginal rate of substitution

$$\frac{d\hat{Y}}{d\dot{\hat{P}}} = -\frac{\beta(\hat{Y} - \hat{Y}^*)}{\delta \dot{\hat{P}}}$$

is positive as desired. Convexity further requires that $\beta + \delta > 1$.

For the sake of simplicity, dependence upon the output level and the rate of inflation are not considered here with the knowledge from sections five and six, that a negative dependence upon both Y and $\hat{P}$($\varepsilon < 0$ and $\alpha < 0$) contribute to a further postponement of energy price increases.

A solution trajectory for $q(t)$ is then determined as before by the minimization of the integral loss,

$$\min \int_{t_0}^{t_1} (\hat{Y} - \hat{Y}^*)^{\delta} \, \dot{\hat{P}}^{\beta} dt \qquad (28)$$

$$q(t_1) = q_1$$

$$q(t_0) = q_0$$

The Euler equation for (28) is again under the assumption of small energy shares, similar to (22) with $\lambda = \sigma$,

$$\frac{\ddot{q}}{\dot{q}} = \sigma \frac{\dot{q}}{q} \qquad (29)$$

The solution[14] of (29) does not depend upon δ and β being given by

$$q(t) = (k_0 + k_1^t)^{\frac{1}{1-\sigma}} \qquad 0 < \sigma < 1 \qquad (30a)$$

$$q(t) = k_0 e^{k_1^t} \qquad \sigma = 1 \qquad (30b)$$

Observe that for $\sigma = 0$ the trajectory is irrelevant since as $\hat{Y} = \hat{Y}^*$ the instant loss is infinitely large unless $\dot{\hat{P}}$ is equal to zero.

Letting $\lambda = \sigma$ in (25) the trajectory of the rate of inflation is always linear with

[14] Second order optimality condition are satisfied for $\beta + \delta > 1$.

$$\dot{\hat{P}} = \frac{ak_1}{(1-\sigma)\tau} \qquad 0 < \sigma < 1 \qquad (31a)$$

$$\dot{\hat{P}} = \frac{ak_1}{\tau} \qquad \sigma = 1 \qquad (31b)$$

as shown in Figure 7. The rate of inflation at each instant of time is higher the smaller are the possibilities of substitution. The growth rate

$$\hat{Y} = \hat{Y}^* + \frac{\sigma k_1}{1-\sigma}\frac{1}{k_0+k_1^t} \qquad 0 < \sigma < 1 \qquad (32a)$$

$$\hat{Y} = \hat{Y}^* + k_1 \qquad \sigma = 1 \qquad (32b)$$

determined from (26) with $\lambda = \sigma$ decline smoothly over time unless $\sigma = 1$ in which case it is stable. These are displayed in Figure 8. At each instant of time output growth is larger the larger is the imports substitution elasticity.

VIII. Summary and Conclusions

In section two we demonstrated that an increase in imported energy real prices requires a reduction in the real price of the domestic factors production. The distribution of such real income losses may be a cause of conflict. The resolution of the distributive conflict is, however, essential to the effective promotion of imported energy substitution. For an economy facing a ba1ance-of-payments restriction substitution is crucial to prevent a decline in output and employment after an international price increase.

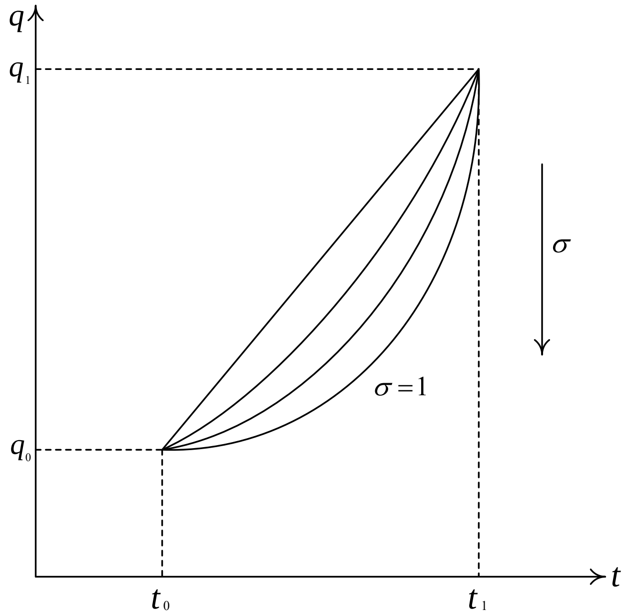
The dynamics of growth and inflation induced by the energy domestic price increase is characterized in section three. For a highly indexed economy it is shown that there is a positive linear relationship between the acceleration of inflation and the rate of change of real energy prices. On the other hand, the increase in real energy prices contributes to growth via import substitution. This allows us to establish a structural relationship between the acceleration of inflation and the rate of output growth for highly indexed economies under a foreign-exchange bottleneck in section four. The larger is the desired output growth rate the larger is the acceleration of the inflation rate and the decline, of the domestic factor price. Furthermore, it is shown that the trade-off is not stable for substitution elasticities less than unit. As the energy domestic price rises the economy becomes increasingly vulnerable to further price increases. The maintenance of an output growth rate over and above that determined from foreign borrowing capability corrected for the international price increase, generates an ever-growing acceleration of inflation. The nature of the trade-off experienced by the less developed oil importing economies is illustrated at the end of the section. It is an attempt to explain why most have relied union a considerable increase of the foreign debt. The cost upon inflation and output of an alternative route would have been significant and of little impact given the severity of the price-shocks.

Section five, six and seven concentrate upon an after-shock period when most economies have seen foreign borrowing capacity shrink. The adjustment of energy domestic prices can no longer be postponed as it becomes the main tool to dampen the decline in output growth. In section five we demonstrate that for indexed economies the terminal rate of inflation, at least as a first approximation, does not depend upon the energy real price trajectories, we conclude hence that the terminal rate of inflation was not an appropriate criterion to distinguish alternative paths. Having argued that for indexed economies nonlinear losses should be attributed to the acceleration of inflation, as it reduces real wages we determined optimal energy price paths taking into account both the rate and the acceleration of inflation. The resulting trajectories are smooth. The larger is the possibility of substitution the more can energy price increases be postponed over the period. However, if as the rate inflation gets higher society perceives further acceleration as an increasing loss, larger price increases should be anticipated.

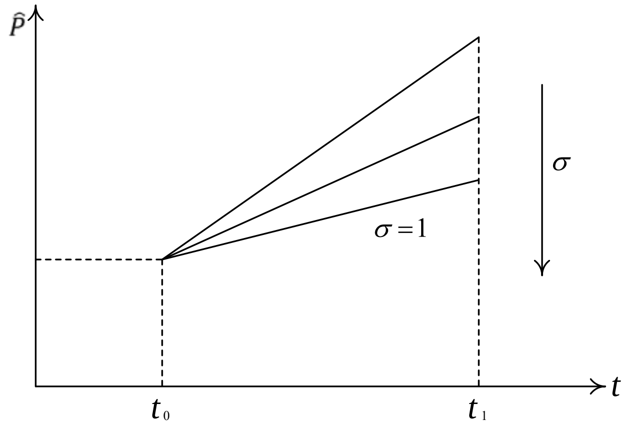
The output and growth implications of alternative energy real price trajectories were analysed in section six. A benefit function depending upon both the output level and its growth rate was established. The rationale for considering the instant growth rate stems from the observation that most less developed economies have high rates of unemployment and a permanently increasing population. We concluded that if only growth matters energy prices should always be increased at a constant rate to stabilize growth. If, however, as the output level increases further growth is perceived a smaller benefit then the rise in energy prices can be postponed.

The situation in which society is willing to trade higher output growth rates for larger accelerations of inflation is considered in section seven. The optimal price trajectories, in this case, depend only upon the substitution potential of the economy as represented by the elasticity of substitution. The rate of inflation will grow linearly over the time interval while the growth rate declines. As the process of imports substitution is initiated output growth is heavily enhanced falling thereafter.

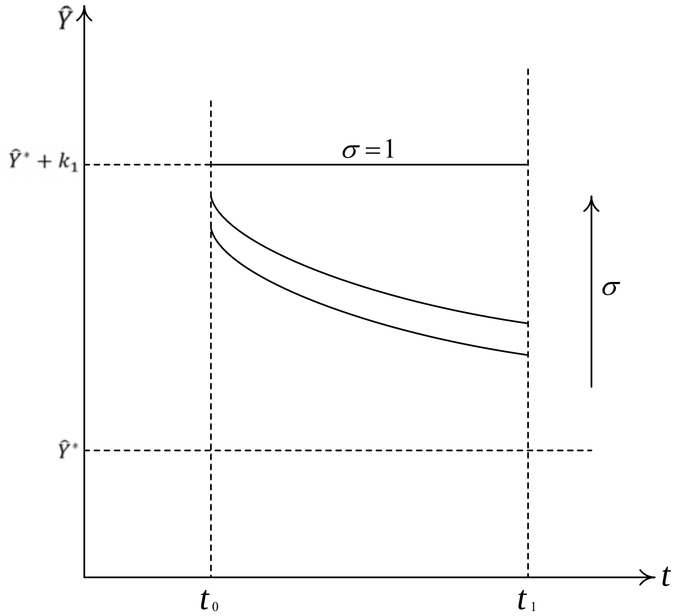
In the last three sections we concentrate on what we have named an after-shock period. We distinguish only two-stages the shock instant and the after-shock period. There should be no major difficulties in combining the results for the after-shock interval with the decisions relating to the moment of the international price-shock. If a fraction of the world price increase is passed on to consumers at that exact instant society must be willing to accept a step increase in the rate of inflation in exchange for the attenuation of a step decline in output. If society relies, at least partially, on foreign borrowing, the step increase in the foreign debt may reduce or even impede further growth in payments. Consequently, the minimum output growth rate within the after-shock period may fall below acceptable levels. The integration of the shock and after-shock period is certainly a topic for further research.



Optimal Trajectories
Figure 6



Inflation Rate
Figure 7



Growth-Rate
Figure 8

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