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The Division of Labour as a Search Process

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This paper is a theoretical exploration of the relationship between search procedures and division of labour. Instead of discussing the determinants of the optimal division of labour, it focuses on the search procedures by which the firm experiences with alternative organizations of the labour process. Changes in the division of labour result from an active effort of cost reduction in face of uncertainty as to the optimal organization of the labour process.

The paper is addressed to the question of explaining the historical prominence of search procedures based on job fragmentation. From Adam Smith's pin factory to Taylor's Scientific Management and Ford's assembly line, the experimentation with alternative organizations of the labour process was primarily confined to attempts at dividing and subdividing job categories. Recent trends in work organization, however, show that more general search procedures exist. Improvements in work organization can also be obtained by grouping and recombining job categories (wild, 1975; Davis and Cherns, 1975). The absence of any systematic attempt to innovate the work organization by resorting to job enlargement or job enrichment during most of capitalism's history forms the subject of this paper.

The historical record of minute job fragmentation has not escaped the attention of economists. The literature on the division of labour is already vast and still expanding. The justification for this paper lies not in the re-examination of portions of historical material but rather in the theoretical framework adopted. It differs from that utilized in the literature in two significant aspects.

First, no substantive hypotheses on the preference structure of the firm are offered. The firm is assumed to have a consistent and unambiguous preference structure so that it can always choose between two work organizations $D$and D'. But the model presented below is silent on why D is preferred to D'. The optimal division of labour is simply defined by the property that there is no other work organization preferred to it.

This lacuna is deliberate. The reasons by which D is preferred to D' may stem ftom a variety of factors: The Babbage principle of comparative advantage (Rosen,1978), the necessity of controlling the work process (Edwards,1979), purely technical constraints (Demyanyuk,1963) and the underlying cognitive processes (Piore,1980). Although these factors are not mutually exclusive, their historical articulations largely remain to be discovered. Moreover, their roles may vary in different historical periods. By not imposing a substantive hypothesis on firm's preference structure, it becomes possible to obtain general analytical results that justify the practice of introducing formal models for the understanding of historical processes. The usage of a formal model, it is hoped, can capture the kernel of seemingly isolated historical processes in a way that no context-specific presentation would be able to.

Second, the division of labour is viewed in this paper as a sequential search process. In each period of time the firm possesses a given work organization handed down by its own history.

Alternative work organizations are not presented to the firm *ab initio*, but have to be conceived of in the plane of thought and tried out in the plane of practice. Since experimentation is costly, the firm typically avoids abrupt transformations of the entire existing work organization. It experiences with alternatives that do not differ to a great extent from the prevailing work organization. The sequential search process terminates whenever all of the promising alternatives have already been tried out in practice.

The sequential search framework is supported by the historical evidence. The existing detailed descriptions of innovations in work organization coincide in emphasizing their cumulative, trial-and-error nature. In this regard, the development of the assembly line and mass production at Ford Motor Co. in the early 20th, century does not differ from the recent experiments on job enrichment and self-organization in USA and Europe (see Nevins,1954, ch. XV-XIX; Wild, 1975, ch. 6). In both cases, improvements in work organization came as an outcome of long, costly search activities. The postulate of a beforehand knowledge of the characteristics of the optimal work organization can hardly correspond to those conspicuous features of the historical evidence.

The advantages of the sequential search framework, however, are not confined to gains in descriptive realism. It suggests a plausible explanation for, and an evaluation of the costs of, adopting search procedures based exclusively on job fragmentation. It also illuminates the reasons by which the very adoption of this restrictive form of search procedure in the initial phases of capitalism paved the way for experiments based on more general search procedures in its late phases.

The paper is organized as follows. In section I, particular (PSP) and general (GSP) search procedures are defined and the sequential search framework expounded. Sections II and III elaborate a stylized representation of the labour process amenable to formal treatment. In section IV, cognitive aspects of PSP and GSP are discussed. Two different schemes of interpretation of the historical evidence are suggested. Sections V, VI and VII consist of exercises in trying to determine, at a high level of abstraction, the effectiveness of PSP relative to GSP. Several results are derived. One of them (Proposition 6 below) provides an analytical foundation for Adam Smith's dictum that the division of labour is limited by the extent of the market. Section VIII interprets the results obtained and states the argument of this paper fully. Section IX concludes briefly by pointing out one limitation of the argument presented in this paper.

I. The search problem: PSP versus GSF

At each period t, the firm possesses a given work organization D of the labour process. D is inherited from the past. Current output costs are $P = P(D)$.

The firm faces a Schumpeterian competition process (see Futia,1980) in which temporary supranormal profits are the reward for successful innovations in work organization. Since supranormal profits do not result from the imitation of innovations previously adopted by competitors, the firm is compelled to try our alternative work organizations whose *a priori* effects on costs are uncertain. The firm looks for a work organization $D' \neq D$ such that $P' < P$.

At each period t, the firm tries out a different work organization. If the candidate D' proves to be in fact preferable to D, then the firm substitutes D' for D. The search process resumes in period $t+1$ based on the fact that D' is now the existing work organization. If the candidate is not preferred to D, the firm continues to search for a better work organization in period $t+1$ keeping organization D in the mean time.

This sequential search may be clarified by the concept of search round. Suppose D is the existing work organization. The search round based on D is the process by which candidates to replace D are experienced with. Obviously, the search round may last just one period if the first candidate dominates D; if the search round lasts p periods, then p different work organizations are tried out in practice, the p^{th} being superior (or preferable) while the first $p-1$ being inferior to D according to firm's preferences. The search round based on D is over when a design D' superior to D is found. The search round based on D' then takes place. The sequential search terminates at D if the search round based on D is unsuccessful, i.e., if there is no work organization preferred to D.

There are λ different work organizations or designs D_i, $1 \leq i \leq \lambda$. Each yet untried design D_i offers a potential output cost of P_i with probability distribution $F_i(P_i, t)$. Probability distributions depend on t because expectations are revised in the light of accumulated experience. Designs that seemed ptromising at a given period may not look the same way few periods later and vice-versa.

Uncertainty as to the output costs under D_i can only be removed by trying it out in practice. There is no source of Information on the comparative efficiency of work organizations other than the experimentation process itself. The cost of testing D_i thoroughly depends on the extent to which D_i differs from the existing design D. If D_i does not differ significantly from D, implementation costs are not large because it is easy to modify D into D_i and vice-versa. Let $K_i(D)$ be the cost of implementing D_i given that D is the existing work organization. The net expected return of in period t is given by expression (1) bellows

(1) $$-K_i(D) + P\int_{-\infty}^{P} dF_i(P_i, t) + \int_{P}^{\infty} P_i dF_i(P_i, t)$$

The interpretation of (1) is immediate. If P_i turns out to be smaller than P, the firm adheres to D and output costs remain P. The search round based on D continues in period $t+1$. This event has probability $\int_{-\infty}^{P} dF_i(P_i, t)$. If turns out to be greater than P, the firm adheres to design D_i and output costs are given by P_i. The search round based on D_i initiates in period $t+1$. The expected output costs if this event happens is given by $\int_{P}^{\infty} P_i dF_i(P_i, t)$.

A necessary condition for work organization D_i to be tried out in the search round based on D is to have a net expected return greater than P. The condition is not sufficient because there may be more than one design with net expected return greater than P. This condition can be written as

(2) $\quad K_i(D) < \int_P^{\infty}(P_i - P)dF_i(P_i, t)$

(2) tends to rule out substantial alterations of the existing work organization. For if D_i were completely different from D, $K_i(D)$ would be large and the sign of the inequality (2) would be reverted unless the expected return differential (the right-hand side of (2)) were also sizeable. Changes in work organization tend to occur in a stepwise, gradual manner. The search process terminates at design D if there is no alternative design D_i with net expected return greater than $P(D)$ such that D_i is preferred to D. D is then called an equilibrium design.

The search problem is to find out an optimal strategy that selects the design to be tried out at each period so as to minimize the expected output costs over time. Strategies trace out division of labour paths. At the initial period t_i the firm relies heavily on the work organization handed down by tradition. Let D_1 be this initial design. D_1 is maintained until a superior design D_2 is found; in turn, D_2 is maintained until a superior design is discovered etc. The sequence $D_1, D_2, \dots, D_m$ describes the endogenous innovations undergone by the organization of the labour process. All of the designs in the sequence reflect the idiosyncrasies of firm's history with the exception of D_i. The sequence $D_1, D_2, \dots, D_m$ is called a division of labour path.

One would like to derive the properties of the division of labour path associated with the optimal strategy. Needless to say, it is necessary to put more structure into the above quite general problem to get more definite results. The remainder of this section is devoted to characterizing the search problem in a manner that illuminates the historical evidence on the division of labour.

We shall capture the cumulative, step-by-step nature of the innovations in work organization by imposing assumption A.1. below. Since implementation costs are directly related to the extent to which the existing design is modified, the firm tries out designs that do not depart significantly from the existing one. In A.1., the class $C(D)$ of designs consists of all of the designs obtainable either by dividing one job of D into two or by grouping two jobs of D into one. Subclass $C'(D)$ consists of only designs obtainable by the former method.

Assumption A.1.: The search round based on D is confined to designs either in $C(D)$ or in $C'(D)$. The search process terminates at design D if there is no design in $C(D)$ or in $C'(D)$ preferred to D.

In Assumption A.1., $K_i(D) = \infty$ if D_i is not in $C(D)$ or in $C'(D)$. The classes $C(D)$ and $C'(D)$ reflect the notion that the firm experiences with alternative organizations of the labour process by undertaking relatively small changes. If search is restricted to subclass $C'(D)$, the firm tries to improve the existing work organization by dividing one of its jobs into two. In each period, the firm splits up just one job of the existing organization into two and observes the efficiency in terms of cost reductions of this new arrangement of the labour process. Successful search in subclass $C'(D)$ results in increasing specialization and narrower job content. If search occurs in class $C(D)$, the firm may resort to job grouping as well. In each period, the firm either breaks down one job into two or combines two jobs into one. In contrast to $C'(D)$, successful search in $C(D)$ does not necessarily lead to minute division of labour.

The behaviour of the firm is said to conform to particular search procedures (PSP) or to general search procedures (GSP) depending on whether search occurs in class $C'(D)$ or $C(D)$ respectively. Historical evidence on the absence of systematic attempts to innovate the organization of the labour process by resorting to search procedures other than the fragmentation of jobs is interpreted as an adherence to PSP. In contrast, recent experiments on job enrichment reveal the adoption of GSP. Thus construed, the central question of this paper lies in explaining why PSP were preferred to GSP during the initial stages of capitalism.

It stands to reason that PSP are less effective than GSP. For the extension of the field of search from $C'(D)$ to $C(D)$ enhances the possibilities of finding out the optimal design D^*. To give a simple example, suppose D is the initial design handed down by tradition. If D^* is in $C(D)$ but not in $C'(D)$, PSP never succeed in approximating D^* irrespective of the extent to which the fragmentation process is carried out. The choice between PSP and GSP, however, is not entirely dictated by effectiveness considerations. It will be argued that PSP have advantages over GSP from the viewpoint of the cognitive aspects of discovery and problem-solving. Those heuristic advantages are more pronounced in the early phases of capitalism than in its late phases. Firms balance out heuristic advantages against the expected effectiveness loss in deciding the form of search procedure to be adopted. To the extent that heuristic considerations do not constrict completely firm's choice margin, ESP are likely to be adopted provided that the expected loss in effectiveness is not large. The calculation of the effectiveness of ESF relative to GSP in Proposition 10 below is the major analytical result of this paper.

II. The Labour Process

We begin by describing the labour process. It consists of n successive tasks which are denoted by integers $1, 2, \dots, n$. Tasks are successive in the sense that task k has to be performed before task k' whenever $k < k'$. Their order is invariant. Since the case $n = 1$ has no interest, $n \geq 2$ will be assumed throughout the paper.

The n tasks are dictated by the prevailing techniques or methods of production. Although the boundaries between tasks are partly a definitional matter, one can think of "tasks" as being the significant units from the point of view of job content. In other words, given the techniques of production and the state of perception of the labour process, the most narrowly defined jobs one can think of are one-task jobs. A specification of jobs comprising all of the n tasks is a work organization or a design. In Definition 1 below, designs are characterized as allocations of the n tasks to jobs.

Definition l: Let N be the set of n integers $1, 2, \dots, n$. A work organization or design is a partition of N into h sets, $1 \leq h \leq n$. Each set is said to be a job. Definition 1 has four important properties.

First, it rules out overlapping jobs. Since jobs are sets defined by partitions, they are pairwise disjoint, each task being allocated to a single job. Jobs have precise, unambiguous boundaries.

Second, it implies that jobs are defined by consecutive tasks. If both tasks k' and k'' belong to job J, $k' < k''$, then all of the tasks k such that $k' < k < k''$ also belong to J. Jobs are ordered in accordance with the ordering of the tasks: if J_x is the x^{th} set in the partition, then $x < x'$ implies $k < k'$ for all $k \in J_x$ and $k' \in J_x'$. Figure 1 below shows one possible job design for a 5–task production process. Dashed vertical lines indicate job frontiers. Definition 1 rules out the possibility of having one job consisting of tasks 1, 4 and 5, say. The two dashed vertical lines in Figure 1 indicate that the design has three jobs: $J_1 = J(1), J_2 = J(2, 3)$ and $J_3 = J(4, 5)$ where J_x is the x^{th} job and $J(.)$ stands for a job comprising tasks $(.)$.

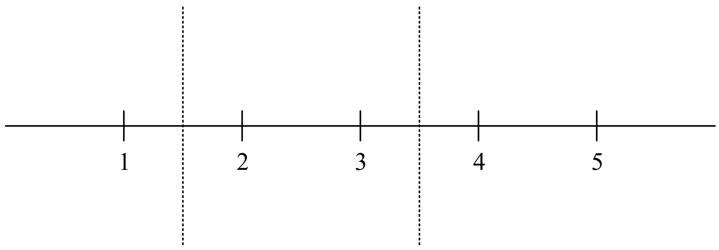


Figure 1

Third, Definition 1 leaves supervisory jobs out of account. This could be easily amended. A managerial or supervisory job can be analytically represented by the union of its supervised jobs. In contrast to work organization which are partitions of N, hierarchical structures thus construed are topologies for N[1]. Since hierarchical structures are not the subject of this paper, Definition 1 will be henceforth adopted.

Fourth, Definition 1 does not allow for changes in the methods of production. In Definition 1, different designs are different partitions of the same invariant set N. The last section of this paper

[1] (page 13). Hierarchical structures for a n-task production process can be defined as follows. For each partition of N, let $\sum_1$ be the collection of sets defined by that partition, $\sum_1$ describes the work organization D. Let $\sum_2$ be the collection of sets generated by the union of subcollection of sets in $\sum_1$. $\sum_2$ describes supervisory or managerial jobs. The span of control of each set in $\sum_2$ is given by its various components sets, i.e., the sub collection of sets in $\sum_1$. It is then trivial to verify that sets $\{\phi, \sum_1, \sum_2\}$ are a topology for N. A useful survey of theories and empirical evidence relating technology (the set N) and hierarchical organization (the topology for N) is given in Caves (l980), section II.

dwells upon this limitation.

There are two polar cases of designs. The $h = 1$ case occurs whenever there is a job that comprises all of the tasks. This case will be referred to as the zero division of labour design. The $h = n$ case occurs whenever the design is such that each job comprises only one task. This case will be referred to as the maximum division of labour design.

Classes $C(D)$ and $C'(D)$ of Assumption A.l. can be formally defined by imposing properties on partitions of N. In Definition 2 below, $C'(D)$ consists of designs obtainable by parcelling one job of D into two while $C(D)$ includes in addition designs generated by grouping two jobs of D into one.

Definition 2. Let D be a partition of N into h sets, $1 \leq h \leq n$. The class $C(D)$ consists of (a) partitions of N into $h - 1$ sets of which $h - 2$ are identical to sets in D and (b) partitions of N into $h + 1$ sets of which $h - 1$ are identical to sets in D. The subclass $C'(D)$ is confined to partitions in (b).

Designs can be represented by sequences of integers. The idea is simple. Designs consist of consecutive jobs. Information on the frontiers between jobs suffices therefore to determine the design unambiguously. If numbers were given to those frontiers, the design would be represented by a sequence of numbers. The R-representation of Definition 3 is a convenient way of introducing numerical representations for designs. R-representations will play a crucial, role in the demonstration of Proposition 10 below.

Definition 3. Let D be a design with h jobs J_x, $1 \leq x \leq h$. Define $\delta(x) = \{k \text{ such that } k \in J_x\}$. A R-representation of D is a sequence of h integers $\{\delta(1), \ldots, \delta(h)\}$.

There are two noticeable properties of R-representations. First, they are strictly increasing sequences. Second, $\delta(h) = n$ for any value of h. The zero division of labour design is represented as {n}; the R-representation of the maximum division of labour design is the sequence $\{1, 2, \ldots, n\}$. Proposition 1 below shows that R-representations are unique.

Proposition 1. There is one and only one R-representation for each design. Proof. Existence is obvious. Lemmas 1.1 and 1.2 demonstrate uniqueness.

Lemma 1.1. Distinct R-representations cannot refer to the same design. Proofs Let R and R' be two R-representations and D and D' be the associated designs. If R has more elements than R', then D has more jobs than D' and hence $D \neq D'$. Suppose R has the same number of elements as R' but that the x^{th} element of R differs from the x^{th} element of R'. Assume without loss of generality that $\delta(x) > \delta'(x)$. Then $\delta(x) \notin J'_x$ because $\delta'(x) = \max\{k \text{ such that } k \in J'_x\}$. Since $\delta(x) \in J_x, J_x \neq J'_x$ and hence $D \neq D'$.

Lemma 1.2. Distinct designs cannot have the same R-representation. Proof.: If D has more jobs than D', then R has more elements than R' and hence $R \neq R'$. Suppose D and D' have the same number of jobs but that the x^{th} job of D is different from the x^{th} job of D'. Define $\theta(x) = \min\{k \textit{ such that } k \in J_x\}$, $\theta'(x) = \min\{k \textit{ such that } k \in J'_x\}$. We have $\delta(x-1) = \theta(x) - 1$ and $\delta'(x-1) = \theta'(x) - 1$ for $x > 1$; $\theta(x) = \theta'(x) = 1$ for $x = 1$. Since jobs are sets defined by partitions, $J_x \neq J'_x$ entails either $\delta(x) \neq \delta'(x)$ or $\theta(x) \neq \theta'(x)$ or both. If $\theta(x) \neq \theta'(x)$, then $R \neq R'$. If $\theta(x) \neq \theta'(x)$, then $\delta(x-1) \neq \delta'(x-1)$ and again $R \neq R'$. QED.

This section is concluded by Proposition 2 below.

Proposition 2. Let $\lambda(h,n)$ be the number of designs with h jobs in a n-task production process. Let $\lambda(n)$ be the total number of designs for a n-task production process, $\lambda(n) = \sum_{h=1}^{n} \lambda(h,n)$. Then $\lambda(h,n) = \frac{(n-1)!}{(h-1)!(n-h)!}$ and $\lambda(n) = 2^{n-1}$. Proof.: By Proposition 1 it suffices to count the number of R-representations. Any strictly increasing sequence of h elements $\delta(i)$, $\delta(h) = n$, is a R-representation. Therefore, there are as many R-representations as there are different ways of arranging the $n-1$ integers $1, 2, \dots, n-1$ in strictly increasing sequences of $h-1$ elements. Let N' be the set of integers $1, 2, \dots, n-1$. Any subset of N' of size $h-1$ has only one increasing sequence of $h-1$ elements. Conversely, any increasing sequence of $h-1$ elements can be formed from the subset of N' consisting of its $h-1$ elements. It follows that the number of R-representations is equal to the number of subsets of size $h-1$ possessed by a set of $n-1$ elements, namely, the binomial coefficient $\binom{n-1}{h-1}$. The calculation of $\lambda(n)$ is trivial. Another proof: $\lambda(h,n)$ can be thought of as the number of ways of allocating (or placing) n tasks (or balls) into h jobs (or cells), jobs being non-void sets (or no cells remaining empty); Proposition 2 then follows from a lemma proved by Feller (1958, p. 37) for classical occupancy problems. QED.

Observe that $\lambda(1,n) = \lambda(n,n) = 1$ and $\lambda(h,n) \geq 2$ for $1 < h < n$. Apart from the two polar cases of the zero division of labour and the maximum division of labour designs, information on the number of jobs does not suffice to determine uniquely the features of the organization of the labour process. For there is always more than one design for any given number of jobs between 1 and n. As h varies from 1 to n, $\lambda(h,n)$ first increases monotonically, then decreases monotonically, reaching its maximum value at $h = \frac{n+1}{2}$ if n is odd and at both $h = \frac{n}{2}$ and $h = \frac{n}{2} + 1$ if n is even; moreover, $\lambda(h,n)$ is a symmetrical function of n. For $1 < h \leq n$, $\lambda(h,n)$ is an increasing function of n; in particular, the total number of admissible designs, $\lambda(n)$, is an exponential function of n.

The positive relation between n and $\lambda(h,n)$ for a given h is hardly surprising. For the greater the complexity of the labour process, crudely measured by the number of tasks, the greater the number of alternative modes by which it can be organized. The relations between h and $\lambda(h,n)$ for a given n

are less obvious. The fact that $\lambda(h,n)$ first increases monotonically and then decreases monotonically as h varies from 1 to n reveals the presence of a limitative effect. The initial monotonic increase starting off at $\lambda(1,n) = 1$ can be easily understood. The greater the number of jobs, the greater the flexibility in designing alternative organizations of the labour process. Yet increases in h tend to reduce the possibilities of designing alternative work organizations because the total number of tasks is fixed. The maximum division of labour design shows this limitative effect in its clearest forms there is just one way of organizing the labour process when there are n jobs and n tasks, namely, assign one task to each job. For small values of h, the limitative effect caused by a fixed value of n is not strong enough to compensate for the gains in flexibility brought forth by increases in h. Yet for large values of h the limitative effect predominates. It accounts for the monotonic decrease of $\lambda(h,n)$ until it reaches the value of 1 for $h = n$.

III. Synchronized designs

In this section we describe in more detail the organization of the labour process. By Proposition 2 we know that there are 2 distinct ways of organizing the labour process for a given n-task production process. A design was simply characterized above as an allocation of tasks to jobs. The labour process, however, is an organic unity in which a co-ordination scheme articulates the different job-holders. Co-ordination is needed whenever $h > 1$. The concept of synchronized design of Definition 4 offers a convenient analytical description of this co-ordination scheme.

Definition 4. design D is said to be synchronized if the complete description of the labour process as it proceeds in time is also, and at the same time, a complete description of all of the different individual jobs which are simultaneously performed at any period of time.

This definition of synchronized design corresponds, *mutatis mutandis*, to Hayek's definition of synchronized production processes (Hayek, 1941, p. 116). The concept of synchronized production has a long history in economic thought. It was clearly formulated by Marx: "... if we look at the workshop as a complete mechanism, we see the raw material in all stages of its production at the same time ... The different stages of the process, previously successive in time, have become simultaneous and contiguous in space". (Marx, 1977, p. 464). Schumpeter introduced it to describe production in the stationary state (Schumpeter, 1939, p. 40). Samuelson and von Weiszacker utilized it to give operational meaning to the labour theory of value (Samuelson, 1971; von Weizsacker,1971, part I). An example of Definition 4 is appropriate. Figure 2 below shows a synchronized two-job design. $T(0), T(1), T(2)$ indicate uniform time intervals.

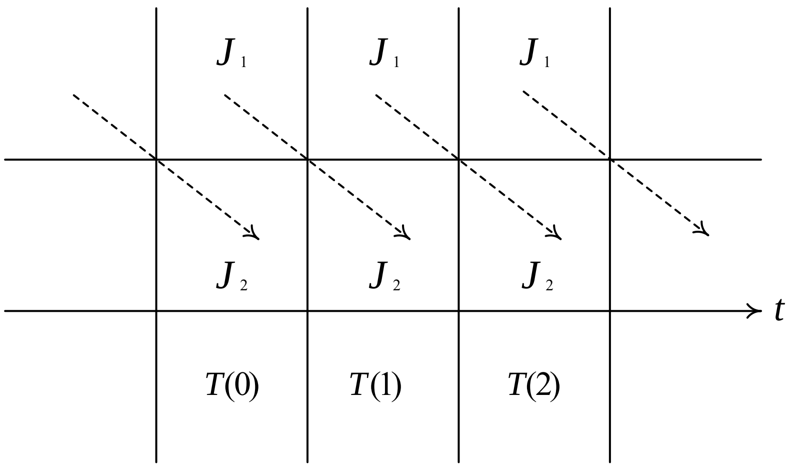


Figure 2

Diagonal dashed lines in Figure 2 describe the labour process as it proceeds in time. Workers holding jobs J_1 pass on the raw material to workers holding J_2 jobs. The transformation initiated in period $T(0)$, for instance, is completed in period $T(1)$. At the same time, a cross-section photograph reveals the two jobs being simultaneously performed. A photograph taken in period $T(1)$, for instance, shows J_2-workers completing the transformation process initiated in $T(0)$ and J_1-workers initiating the transformation process that will be completed in period $T(2)$. It is apparent that output is delivered each period in spite of the fact that the J_1 production process takes two periods to be completed. Synchronized work organizations thus make production schedules more continuous than they would otherwise be.

The example of Figure 2 displays the co-ordination scheme clearly. But it is silent on how the labour force is allocated to the two jobs J_1 and J_2. It follows from the definition of synchronized designs that no subset or group of workers should ever remain idle during the production process. In the example of Figure 2, the complete description of the labour process as it proceeds in time is $\{J_1; J_2\}$; for the design to be synchronized it is thus necessary that both J_1-workers and J_2-workers are fully employed in all periods. If there are too many J_2-workers relative to the number of J_1-workers, the flow of raw material, needed to keep all of the J_2-workers employed will fall short of the actual flow and vice-versa. We now turn to the allocation of the labour force among different jobs necessary to synchronize a given design D.

Let t_k stands for the labour time necessary to perform task k, $1 \leq k \leq n$. Abstracting from the time which is commonly lost in passing from one task to another, we define T_x, the time necessary to pefform job J_x, $1 \leq x \leq h$, as the total sum of the labour time requlred by its constituent tasks. Similarly, we define the total duration of the labour process $\bar{T}$ as the total sum of the labour time required by the n tasks: $\bar{T} = \sum_{k=1}^{n} t_k = \sum_{x=1}^{h} T_x$. Observe that in the zero division of labour design the single job has duration $\bar{T}$. Nonetheless, there will always exist one job with shorter working time whenever $h > 1$.

It is important to mark well the implications of these simple assumptions. First, the labour time necessary to perform each task individually considered is not affected by the way in which the labour process is organized. Second, the labour time necessary to perform a given job is a function solely of its constituent tasks and not of the overall labour design to which it belongs. Third, and perhaps more importantly, the total duration of the labour process does not vary from one work organization to another. That these assumptions axe restrictive is doubtless. They will serve to the purpose of characterizing the allocation of the labour force prevailing in the synchronized arrangement of the labour process sharply.

Based on these simple assumptions, we can determine the allocation of the labour force among the h jobs necessary to synchronize a given design D with $h > 1$ jobs. Some reflection shows that if no subset of workers is ever going to be idle, then relation (3) below have to hold:

$$(3) \qquad \frac{N_x}{T_x} = \frac{N_{x+1}}{T_{x+1}}, 1 \leq x \leq h - 1$$

Relation (3) provides the allocation rule we are seeking for. It compensates the extra time required by a given job devoting extra workers to it. N_x is the number of workers holding J_x jobs. If job J_1 takes two hours, say, while job J_2 takes just one hour, proper balance requires that there must be two workers holding J_1 jobs for each worker holding a J_2 job. If this proportion is obeyed, no idleness due to bottlenecks or internal maladjustments results. The velocity of throughput, to use Chandler's expression, is maximal. (Chandler, 1977, ch. 8). Observe that the rationale for (3) supposes all of the workers holding the same job J_x as being able to perform J_x in the same period of time T_x. In other words, (3) disregards the problem of exploiting the comparative advantages of workers in performing different tasks which has received attention in Rosen (1978).

Relation (3) corresponds to the "fixed mathematical relation or ratio which regulates ... the relative number of workers, or the relative size of group of workers, for each special function" that in Marx's view was characteristic of the division of labour under the system of manufacture. Marx's own numerical example of this "quantitative rule and ... proportionality for the social labour process" developed in the period of Manufacture conforms precisely to (3) above (Marx, 1977, p. 465)[2].

Relation (3), however, determines not only the allocation but also the minimum size of the

[2] (page 22). In Marx's words: "Different operations, however, require unequal lengths of time, and therefore, in equal lengths of time, yield unequal quantities of the specialized products. Thus if the same worker has to perform the same operation day after day, there must be a different number of workers for each operation; for instance, in type manufacture there are four founders and two breakers to one rubber; the founder casts 2,000 types an hour, the breaker breaks up 4,000 and the rubber polishes 8,000". (Marx, 1977, p. 465). That is, since the time necessary to cast a type if four times that necessary to polish it, there must be four founders to each rubber etc. In Marx's view, the discovery of this "iron law of proportionality" (idem, p. 476) was not dependent upon the special mental endowments of the capitalist (idem, p. 485, note 52); in some cases, "... a week's experience is enough to determine the proportion between the numbers of the 'hands' necessary for the various functions" (idem, p. 485).

labour force necessary to synchronize D. It will be seen in section V that this minimum size tends to increase along the division of labour paths generated by PSP, a result that provides an analytical foundation for Adam Smith's dictum that "the division of labour is limited by the extent of the market". (Smith, 1965, p. 17). In order to derive the minimum work force necessary to operate a given design D in a synchronized manner it is useful to write (3) as:

(4) $$\frac{N_x}{T_x} = \frac{N^*}{T^*(D)} \qquad 1 \leq x \leq h$$

where $T^*(D)$ is the minimum labour time required by a particular job in D, $T^*(D) = \min\{T_1, T_2, \dots, T_h\}$ and N^* is the number of workers allocated to the job requiring a labour time of $T^*(D)$ to be completed (obviously, there may be more than one such jobs). Since $T^*(D)$ is by definition the shortest labour time required by a particular job in D, there must be at least N^* workers holding any job J_x in design D. Observe that distinct designs may have the same T^* value; however, if two designs have different T^* values, they are, of necessity, distinct. By construction, $T^*(D) = \bar{T}$ for $h = 1$ and $T^*(D) < \bar{T}$ for $h > 1$; the smallest possible value of T^* is given by $\min\{t_1, t_2, \dots, t_n\}$, i.e., by the shortest working time required to perform any task individually considered. Relation (4) can in turn be written as:

(5) $$N_x = \alpha_x N^*, \; 1 \leq x \leq h$$

where

(6) $$\alpha_x = \frac{T_x}{T^*(D)}, \; 1 \leq x \leq h$$

with $\alpha_x \geq 1$ for all x, $\alpha_x = 1$ for at least one x.

Equation (5) gives the proportions governing the distribution of the entire labour force over the h jobs. Since the number N_x of workers holding any job J_x is a positive integer number, the minimum size of the labour force necessary to operate D in a synchronized manner, called $W(D)$, is given by:

(7) $$W(D) = \sum_{x=1}^{h} \alpha_x \bar{N}(D)$$

where $\bar{N}(D)$ is the smallest positive integer value of N^* such that $N_x = \alpha_x N^*$ is a positive integer for all x. Since $\sum_{x=1}^{h} T_x = \bar{T}$, we can derive a final expression for $W(D)$ using (6) and (7):

(8) $$W(D) = \frac{\bar{N}(D)}{T^*(D)} \bar{T}$$

Equation (8) shows that $W(D)$ may vary from one design to another because of either $T^*(D)$ or $\bar{N}(D)$ or both. A straightforward application of the definitions of $T^*(D)$ and $\bar{N}(D)$ shows that $W(D) = 1$ when $h = 1$, a quite trivial result indeed. When there is just one job, the minimum labour force consists of just one worker because no co-ordination problem arises. $W(D)$ is the minimum size of the labour force necessary to operate D in a synchronized manner. It stands to reason that any multiple M of $W(D)$ is compatible with the synchronized arrangement of D provided that $M\alpha_x\bar{N}(D)$

workers hold J_x jobs for all x, $1 \leq x \leq h$[3].

Before discussing the heuristic advantages of ESP, the subject of next section, two properties of synchronized designs will be noted.

First, observe that any design can be synchronized. No idleness occurs if the labour force is allocated in accordance with proportions (5). Georgescu-Roegen has pointed out that this capacity of doing away with idleness is typical of the factory system. The fact that agricultural production processes can hardly be uniformly staggered in time would thus suggest an asymmetry between innovations in work organization in agriculture and manufacture (Georgescu-Roegen, 1969; 1971, pp. 250-253).

Second, observe that the smaller $T^*(D)$ is, the more continous the production schedule becomes. The length of the time interval between output units as they come out of the labour process is given in a synchronized design by $T^*(D)$, the shortest labour time required by an individual job of D. If $T^*(D) < T^*(D')$ then production schedule is more continuous in D than in D' in the sense that the time interval between output units in D is shorter than in D'. To verify this property, it suffices to show that output is delivered at intervals of $T^*(D)$ length in any synchronized design D. The property is trivial when $h = 1$ because for the zero division of labour design $T^*(D) = \bar{T}$.

Suppose $h > 1$. Each group of workers holding J_x jobs pass on the raw material for the workers holding J_{x+1} jobs, $1 \leq x \leq h-1$. Final output is the outcome of job J_h. Let J_{x^*} be a job that takes $T^*(D)$ to be done. Then at time intervals of $T^*(D)$ length, the workers holding J_{x^*} jobs pass on the raw material to workers holding J_{x^*+1} jobs (provided that $x^* < h$) and receive the raw material already prepared by the workers holding J_{x^*-1} positions provided that $x^* > 1$. Since $h > 1$, x^* cannot be equal to 1 and to h at the same time. If proportions (5) are obeyed, the number of workers holding J_{x^*+1} and J_{x^*-1} jobs is such that the raw material is processed in intervals of $T^*(D)$ length in both jobs J_{x^*+1} and J_{x^*-1}. By a recursive argument, raw material is processed in $T^*(D)$-intervals in jobs J_{x^*+2} andJ_{x^*-2}, in jobs J_{x^*+3} and J_{x^*-3} etc. The property then follows from the fact that $h = x^* + b$ for some non-negative integer b.

This property of synchronized designs was encountered while commenting on Figure 2. The length of the uniform time intervals $T(0)$, $T(1)$ and $T(2)$ is $T^*(D) = \min\{T_1, T_2\}$. If there is no division of labour, i.e., if $h = 1$, then $T^*(D) = \bar{T} = T_1 + T_2$; the division of labour, in the sense of organizing the labour process in more than one job, may thus be said to increase the continuity of the production schedule[4]. It will be seen in Section V that along the division of labour paths generated

[3] (page 25). Marx observed correctly that "once the most fitting proportion has been established for the number of specialized workers in the various groups producing on a given scale, that scale can be extended only by employing á multiple of each particular group". (Marx, 1977, pp. 485-486; see also Babbage, 1971, pp. 211-213).

[4] (page 26). This fact has not, of course, escaped the attention of observers of the division of labour in capitalism. Marx

by PSP production becomes more and more continuous, a result that will interpreted in the light of recent theories on dualistic industrial structures.

IV. The heuristic advantages of PSP

During the first phases of capitalism, the firm relies heavily on traditional organizations of the labour process centred on crafts. Depending on the nature of the good produced, the firm may either bring several distinct crafts together or simply assimilate an already existing one. In both cases, technical knowledge is possessed by craftsmen. There is no book of blueprints describing how to carry on production. To conceive of alternative (and presumably more efficient) organizations of the labour process the firm must gather detailed information on production. Since technical knowledge is embodied in practical form in the skill and dexterity of craftsmen, the easiest way of obtaining information on production is the attentive analysis of craftsmen practice. (Some interesting comments on the role of observational techniques in parcelling out traditional job categories are given in Palma, 1971, ch. 1, section 4; for a discussion of Taylorism as an expropriation of craftsmen's practical knowledge, see Coriat, 1976, pp.115-120).

Analytical intellectual operations are thus a preliminary condition for envisaging alternative work organizations. To combine, group or divide differently the jobs of the existing work organization one has first to master the tasks they consist of. This resolution into simpler elements is an analytical intellectual operation. It is intellectual in that it occurs in the plane of thought; it is an operation in that it either anticipates or prepares the actual operations that will take place in the plane of practice. Once jobs are conceptually decomposed ln their minimum elementary tasks and the requirements for each task are carefully registered, it becomes possible to develop alternative organizations of the labour process.

The first heuristic advantage of ESP lies in economizing the scarce resource attention (Simon, 1978a). In PSP, the same analytical intellectual operations by which the firm grasps the know-how of craftsmen guide the search for more efficient work organizations. The careful examination of the detailed constitution of traditional job categories that occurs in the plane of thought begets the process of parcelling jobs out that occurs in the plane of practice. The same attention effort devoted to seize the practical knowledge of craftsmen generates alternative modes of organizing the labour process. In PSP, alternative work organizations are naturally suggested by a sufficiently probing observation of craftsmen's practice. The second heuristic advantage of PSP lies in rendering unnecessary the

quotes Stewart to show this effect on the continuity of production schedules: "By carrying on all of the different processes at once, which an individual must have executed separately, it becomes possible to produce a multitude of pins completely finished in the same time as a single pin might have been either cut or pointed" (Marx, 1977, p. 464, note 11).

elaboration of a different cognitive structure[5]. There is embedded in traditional crafts organization a cognitive structure that responds for a systematic, ordered way of perceiving and thinking of the labour process. PSP do not rupture with this cognitive structure. Crafts are divided and subdivided in unprecedented modes; but it is still craftsmen jobs as handed down by tradition that form the starting-point of the search process.

Marx referred to this fact when he characterized Manufacture as being based on a subjective analysis of the labour process into its constituent phases, subjective in the sense of being centred upon the subject, i.e., the craftsman (Marx, 1977, p. 501)[6]. Although enriched and differentiated by a clearer recognition of the elementary tasks the labour process consists of, the cognitive structure by which the labour process is apprehended in PSP does not differ from the one presiding traditional work organizations centred on crafts. Since innovations at the level of cognitive structures cannot be assimilated to routine processes, the adoption of ESP makes possible a considerable economy of intellectual effort. It is easier to conceive of modifications by parcelling out already structured job categories than by remoulding the entire work organization according to a different scheme.

The third advantage of ESP consists in contracting the field of search, Since $C'(D)$ is a subclass of $C(D)$, ESP reduce the complexity of the problem of finding alternatives superior to D. If the optimal design D^* were expected to be reached by ESP, the adoption of GSP would not be consistent with intelligent behaviour. For ESP would in this case provide a more selective (and hence less costly) way of scanning the space of admissible designs. The importance of selectivity in choosing search procedures is, of course, a *leitmotif* of Simon's theories of cognition: "Complexity is deep in the nature of things, and discovering tolerable approximating procedures and heuristics that permit huge spaces to be searched very selectively lies at the heart of intelligence, whether human or artificial". (Simon, 1978a, p. 12; see also Simon, 1977, 1978b, 1979).

This reduction of complexity can be easily calculated. Proposition 3 below shows that the difference in the number of designs between $C(D)$ and $C^*(D)$ is $h-1$. That is, the two fields of search coincide if D is the zero division of labour design and attain their maximal dlfference if D is the maximum division of labour design.

Proposition 3. Let D be a design with h jobs, $1 \leq h \leq n$. Let $H[C'(D)]$ be the number of designs in $C'(D)$ and $H[C(D)]$ the number of designs in $C(D)$. Then $H[C(D)] = n-1$ and $H[C'(D)] = n-h$.

Proof. We first calculate $H[C'(D)]$. D has h sets (jobs) J_x, $1 \leq x \leq h$. Set J_x has r_x elements

[5] (page 28). The concept of cognitive structure is central to Piaget's genetic epistemology. Flawell (1963) provides an excellent overall view of Piaget's theories up to about I960. Piaget (1975) offers the most complete treatment of the concept of cognitive structure available.
[6] (page 29). Marx contrasted the "purely subjective" organization of the labour process in manufacture with the "entirely objective" organization in large-scale industry (Marx, 1977, p. 508).

(tasks), $\sum_{x=1}^{h} r_x = n$. Each partition of into two sets generates a partition of N into $h + 1$ sets of which $h - 1$ are identical to sets in D and vice-versa. Since each set J_x has $r_x - 1$ partitions into two sets, it follows from Definition 2 that $H[{}'(D)] = \sum_{x=1}^{h}(r_x - 1) = n - h$.

To calculate $H[C(D)]$, observe that $C(D)$ consists of: (a) designs in $C'(D)$ and (b) designs generated by grouping two consecutive jobs of D into one. If D has h jobs, class (b) has $h - 1$ designs. Therefore, $H[C(D)] = H[C'(D)] + (h + 1) = n - 1$. QED.

The extension of the search field under GSP is constant. $H[C(D)]$ does not depend on properties of D. In contrast, the field of search under PSP is a function of the number of jobs in D. The larger h is, the smaller $H[C'(D)]$. If $h = 1$, $H[C(D)] = H[C'(D)]$ because the only possibility of experimentation is to divide the single job into two; if $h = n$, $H[C'(D)] = 0$ because it is no longer possible to carry out further the process of job fragmentation. The reduction of complexity entailed by the adoption of PSP occurs whenever $h > 1$, i.e., whenever the existing design already possesses some division of labour.

The selectivity of PSP can still be seen from another point of view, Let D, the existing design, have h jobs, $1 \leq h \leq n$. The maximum duration of the search round based on D is $n - h$ search periods for PSP and $n - 1$ search periods for GSP. That is, if D is not an equilibrium design, PSP diminish the lapse of time needed to find an alternative superior to D because they delimitate a narrower search field, Obviously, D may be an equilibrium design under PSP but not under GSP, a fact whose probability will be calculated and interpreted later. A further result on the selectivity of PSP will be given in Proposition 5 in the next section. It shows that the maximal proportion $U(n)$ of the 2^{n-1} admissible designs actually tried out under PSP is a sharply decreasing function of n. $U(n)$ is smaller than 0,1 of 1% for $n = 18$, the number of tasks in the manufacture of pins as reported by Adam Smith (Smith, 1965, p. 4).

The central argument of this paper can now be briefly presented. Of the three heuristic advantages of PSP, the first two pertain to specific historical circumstances whereas the third (the reduction of complexity in the search space) has general validity. Heuristic advantages of PSP as a whole tended to diminish as capitalism developed and cleared away the remains of traditional crafts organization. Since PSP are not as effective as GSP, one would not expect PSP to predominate in late capitalism as it did in its early phases. Two interpretative schemes will be suggested.

In the first scheme, the heuristic advantages of PSP did not constrict completely the margin of choice between alternative search procedures. Firms adhered to PSP because the expected effectiveness loss was not too large, relative to the heuristic advantages. The predominance of PSP in early capitalism resulted from a rational choice whose terms were progressively altered as capitalism unfolded.

In the second scheme of interpretation, cognitive considerations can alone explain the predominance of PSP. GSP only became feasible after the production process was mastered in its detains and a different cognitive structure emerged. Procedural rationality (Simon, 1976) rendered the adoption of PSP imperative in early capitalism. Their loss of effectiveness relative to GSP, which would be the type of search procedure resulting from substantive rationality, is interpreted as an unavoidable cost that bears the historical imprint of the circumstances under which capitalism took place.

In the next three sections, the relative effectiveness of F5P *vis-à-vis* GSP is calculated under quite general assumptions. The results obtained are then discussed in section VIII in the light of these two interpretative schemes.

One final point on the differences between PSP and GSP remains to be made. Piore (1980b) pointed out that the analytical operations by which the labour process is broken down and separated into its component tasks are only a part of the intellectual processes involved in the division of labour. Analytical intellectual operations are followed by the recombination or synthesis of tasks. Piore suggested that the intellectual processes by which alternative work organizations are envisaged can be characterized as an alternation or dialectic, moving back and forth between analysis and synthesis (Piore, 1980b, p. 76; see also Lakatos, 1978, on the alternation of analysis and synthesis in a different intellectual context). The argument illuminates the differences regarding intellectual processes between PSP and GSP.

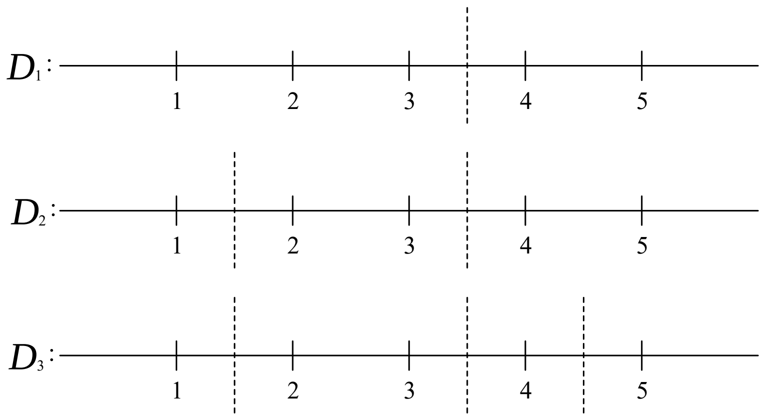
Suppose job J is to be fragmented into jobs J' and J''. It is necessary, first, to analyse J in terms of its constituent, elementary tasks and, second, to group part of the component tasks together to form J' and to group the remainder to form J''. Synthesis is thus at work in FSP, albeit confined to a partial reversing of analytical operations. In FSP synthetical operations are subordinated to analytical operations. By contrast, synthetical operations attain the status of an independent operation in GSP. Their domain of application is generalized. Suppose jobs J and $\bar{J}'$ are now to be combined to form $\tilde{J}$. Analytical operations decompose J and J' into their component tasks. Synthesis then groups them together. Instead of being limited to grouping tasks previously separated so as to form jobs more narrowly defined, synthesis is now extended to encompass the grouping and combination of distinct jobs of D. The extension of the search field from $C'(D)$ to $C(D)$ is accounted for by the generalization of the domain of application of synthetical operations. Analytical and synthetical operations are thus interwoven in both forms of search; their relations, however, are distinct in PSP and in GSP. Synthesis is founded on the reversibility of analytical operations in PSP while the prominence of analysis is no longer true of GSP. Analytical operations are a preliminary condition for envisaging alternative designs in both PSP and GSP; their difference regarding the intellectual processes involved stems from the role assigned to synthetical operations.

V. Division of labour paths

Different search procedures give rise to different division of labour paths. In Definition 5 below, PSP trace out standard paths while GSP generate non-standard paths.

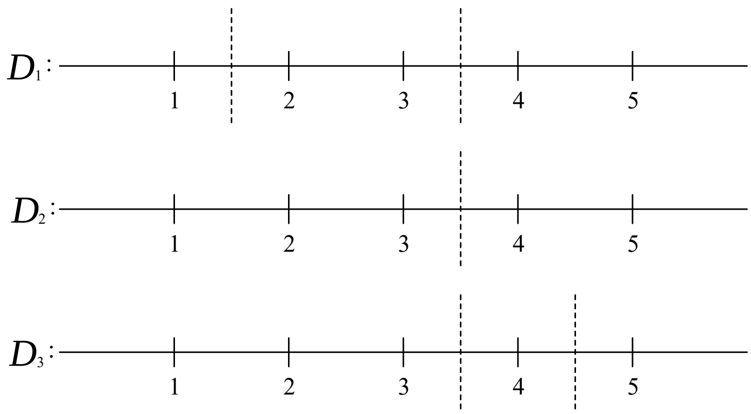
Definition 5. Let $D_1, D_2, \dots, D_m$ be a sequence of designs. The sequence is a standard (or non-standard) division of labour path of length m if D_{i+1} is in $C'(D_i)$ (or in $C(D_i)$) for $1 \leq i \leq m-1$. Paths are normalized if D is the zero division of labour design. A standard path of length n is said to be complete.

Designs D_1, D_2 and D_3 are an example of a standard path in a five-task production process. As in Figure 1, dashed vertical lines indicate job frontiers.



Along a standard path each design is generated from the previous one by splitting up just one job. Standard paths reflect cumulative movements towards work specialization. If D_i and D_j are in tjie same standard path, $j > i$, D_j can be generated from D_i by dividing and subdividing the jobs of D_i suitably. Complete standard paths have the starting point at the zero division of labour design and the ending point at the maximum division of labour design. Since $\lambda(h, n) > 1$ for $1 < h < n$ (see Proposition 2 above), two complete standard paths may coincide only in the first and last designs. Complete standard paths are normalized although the reverse is not necessarily true.

As an example of a non-standard path in a five-task production process, consider designs D_1, D_2 and D_3 below.



Standard paths are a special case of non-standard paths. While a standard path can have a maximum length of n, there is no such limitation binding on a non-standard path. Along a standard path the same design cannot appear more than once because there is a unidirectional movement towards job specialization. In contrast, the same design may appear more than once along a non-standard path because both job fragmentation and job grouping may occur. In the above example of a non-standard path consisting of the sequence D_1, D_2 and D_3 grouping occurs from D_1 to D_2 whereas fragmentation occurs from D_2 to D_3, the number of jobs first decreasing and then increasing. It follows from Proposition 2 that a non-standard path in which no design appears more than once can have a maximum length of 2^{n-1}; non-standard paths with design reappearance are just denumerable sequences D_1, D_2, ... Proposition 4 states a simple relation between standard and non-standard paths.

Proposition 4. Let D_i and D_{i+1} be any two designs in a non-standard path $D_1, D_2, \dots, D_m$, $1 \leq i \leq m-1$. Then either the sequence D_i, D_{i+1} or the inverse sequence D_{i+1}, D_i is a standard path.

Proof. If D_{i+1} is in $C'(D_i)$, the sequence D_i, D_{i+1} is a standard path. Suppose D_{i+1} is in $C(D_i)$ but not in $C'(D_i)$. Define $h' = h - 1$. By Definition 2, D_{i+1} is a partition of N into h' sets of which $h' - 1$ are identical to sets in D_i. Conversely, D_i is a partition of N into $h' + 1$ sets of which $h' - 1$ are identical to sets in D_{i+1}. Therefore, D_i is in $C'(D_{i+1})$ and the sequence D_{i+1}, D_i is a standard path. QED.

The intuitive content of Proposition 4 is obvious. Along a non-standard path, D_{i+1} is generated from D_i either by job fragmenting or by job grouping. If job fragmenting occurs, then D_i, D_{i+1} is a standard path of length 2. If job grouping occurs, two jobs of are collapsed into one job of D_{i+1}. By running the steps in the reverse order, D_i could be obtained from D_{i+1} by splitting up one of its jobs into two. The sequence D_{i+1}, D_i thus forms a standard path of length 2.

Since standard paths exhibit unidirectional movements towards job fragmentation, the number of designs that can be possibly experienced with along any standard path is limited. In Proposition 5 below, $\mathsf{U}(n)$ is the maximal proportion of the 2^{n-1} admissible designs that can be tried out along a standard path.

Proposition 5. $\mathsf{U}(n) = 1$ for $n \leq 3$. $\mathsf{U}(n)$ decreases monotonically with n for $n > 3$, $\lim_{n\to\infty} \mathsf{U}(n) = 0$.

Proof. The maximal proportion U obtains for a complete standard path $D_1, D_2, \dots, D_n$ in which all of the designs in $C'(D_i)$ are tried out before implementing D_{i+1}, $1 \leq i \leq n-1$. The path initiates at the zero division of labour design. We know from Proposition 3 that $C'(D)$ has $n - h$ designs if D has h designs. In the first round, the round based on the zero division of labour design D_1, the $n-1$ different two-job designs in $C'(D_1)$ are tried out. The last one in the experimentation order is implemented. In the next search round, the $n-2$ different three-job designs in $C'(D_2)$ are tried out.

The last one is implemented etc. The maximum number of designs that can be tried out along the path until the maximum division of labour design is found is given by the sum (9):

(9) $$1 + \sum_{h=2}^{n-1}[n - (h - 1)] = \frac{n^2 - n + 2}{2}$$

The proportion U is defined as the ratio of (9) to the total number of admissible designs 2^{n-1}:

(10) $$\mathrm{U}(n) = \frac{n^2 - n + 2}{2^n}$$

The calculation of $\lim_{n\to\infty} \mathrm{U}(n)$ is straightforward. QED.

The proportion $\mathrm{U}(n)$ given in (10) declines sharply as n increases: $\mathrm{U}(3) = 1$, $\mathrm{U}(4) = .87$, $\mathrm{U}(15)$ is already smaller than 1%. In Adam Smith's pin manufacture, $\mathrm{U}(18) = .001$ approximately; in Ford's model T chassis assembly line, $\mathrm{U}(45) = 0.563.10^{-10}$ (for detailed descriptions of the tasks, see Smith, 1965, p. 4, and Arnold and Faurote, 1972, pp. 140-150 for Ford's assembly line). The proportion $\mathrm{U}(n)$ reflects the maximum coverage of the total search space allowed for by PSP. The selectivity of PSP is striking. They guide search into only a tiny part of the total search space formed by the 2^{n-1} admissible designs. The more complex the labour process, complexity being crudely measured by the number of tasks, the more selective PSP are.

We can now demonstrate Proposition 6. Its bearings on Adam Smith's argument that the division of labour is limited by the extent of the market are unmistakable.

Proposition 6. Along a standard path in which all of the designs are synchronized, both the minimum number of workers and the continuity of the production schedule do not decrease. If the path is complete, they eventually increase.

Proof. Let $D_1, D_2, \ldots, D_m$ be the standard division of labour path, $m \geq 2$. The proof is based on equation (8) above. It is divided into three parts. Part (a) shows that $T^*(D_i) \geq T^*(D_{i+1})$ for $1 \leq i \leq m-1$. Part (b) uses this result to show that $W(D_i) \leq W(D_{i+1})$, $1 \leq i \leq m-1$. Part (c) completes the proof by showing that $T^*(D_1) > T^*(D_m)$ and $W(D_1) < W(D_m)$ whenever $m = n$.

To facilitate the proof, we introduce the following notation. Along a standard path D_{i+1} is obtained from D_i by fragmenting just one job of D_i. Let J_i be the job of D_i that is fragmented into J_1' and J_1'' of D_{i+1}. D_i has jobs $J_1, J_2, \ldots, J_h$ while D_{i+1} has in turn jobs $J_1', J_1'', J_2, \ldots, J_h$. Labour time is denoted by $T_1, T_2, \ldots, T_h$ for D_1 and $T_1', T_1'', T_2, \ldots, T_h$ for D_{i+1}, where $T_1' + T_1'' = T_1$. By definition, $T^* = \min\{T_1, T_2, \ldots, T_h\}$ and $T^{**} = \min\{T_1', T_1'', T_2, \ldots, T_h\}$. We also define the coefficients $\alpha_x = \frac{T_x}{T^*}$, $x = 1, \ldots, h$ and $\alpha_1' = \frac{T_1'}{T^{**}}$, $\alpha_1'' = \frac{T_1''}{T^{**}}$, $\alpha_1' = \frac{T_x}{T^{**}}$ for $x = 2, \ldots, h$. $\overline{N}$ is the smallest positive integer N such that $\overline{N}.\alpha_x$ is an integer for $x = 1, 2, \ldots, h$. Similarly, N' is the smallest positive integer N such that $N.\alpha_1'$, $N.\alpha_1''$ and $N.\alpha_x'$, $x = 2, \ldots, h$, are integers. As before, t_k stands for the labour time required by each task k individually considered, $k = 1, \ldots, n$. $\overline{T} = \sum_{k=1}^{k} t_k$ is the total production time.

Part (a). Let $T_0 = \min\{T_2, \ldots, T_h\}$. Since $T_1' + T_1'' = T_1$, we have $T^* = \min\{T_1' + T_1'', T_0\} \geq \min\{T_1', T_1'', T_0\} = T^{**}$. If $T_0 = T^{**}$, then $T^* = T^{**}$; if $T^{**} = T_1'$ or $T^{**} = T_1''$, then $T^* > T^{**}$.

Part (b). Using Part (a), write $T^* = cT^{**}$, c being a constant, $c > 1$. Then $\alpha_x' = c\alpha_x$ for $x = 2, \ldots, h$. Since both $N'\alpha_1'$ and $N'\alpha_1''$ are integers, $N'\alpha_1' + N'\alpha_1'' = N'c\alpha_1$ is an integer. Then $N'c\alpha_x$ is an integer for $x = 1, 2, \ldots, h$. Since $\alpha_x = 1$ for at least one x, $N'c$ is an integer. It follows that $N'c \geq \bar{N}$ because $\bar{N}$ is the smallest integer N such that $N\alpha_x$ is an integer for $x = 1, 2, \ldots, h$. Substituting the inequality $N'c \geq \bar{N}$ in the definition of $W(D)$ we obtain $W(D_{i+1}) = \frac{N'}{T^{**}}\bar{T} = \frac{N'c}{T^*}\bar{T} \geq \frac{\bar{N}}{T^*}\bar{T} = W(D_i)$.

Part (c). In a complete path $T^*(D_n) = \min\{t_1, t_2, \ldots, t_n\}$. Since $\min\{t_1, \ldots, t_n\} < \sum_{k=1}^{n} t_k = \bar{T}$, it follows that $T^*(D_n) < T^*(D_1) = \bar{T}$ because D_1 is the zero division of labour design. As to $\bar{N}(D)$ observe that $\bar{N}(D_1) = 1$ because the zero division of labour design has just one job; since $\bar{N}(D)$ is a positive integer, $\bar{N}(D_n) \geq \bar{N}(D_1)$. From the inequalities $T^*(D_1) > T^*(D_n)$ and $\bar{N}(D_1) \leq \bar{N}(D_n)$ we obtain $W(D_1) < W(D_n)$. QED.

Proposition 6 provides an analytical foundation for the relation between the division of labour, understood in the restrictive sense of increasing work specialization, and the “extent of the market” or the intensity of demand flow. Along a standard path jobs are progressively fragmented. The minimum number of workers needed to operate the design in a synchronized manner tends to increase along the path. If one compares the two polar modes of organizing the labour process, the zero division of labour design and the maximum division of labour design, then it is always true that the minimum size of the labour force is greater in the latter. Therefore, the corresponding output flow also increases. A low intensity of demand renders uneconomical the adoption of designs with too many jobs because specialization would then result in costly idleness. In this way the intensity of demand flow deters the development of the division of labour process.

If wages are paid in advance, i.e., prior to receiving the proceeds of output, then Proposition 6 implies that the minimum amount of capital needed to finance production tends to decrease along a standard path, being certainly greater in the maximum division of labour design than in the zero division of labour design, Marx referred to this fact as “...a law springing from the technical character of manufacture” (Marx, 1977, p. 480). It should be noted, however, that this “'law” does not spring from the strictly technical side of the labour process (the set N of invariant tasks) but rather from the properties of the synchronized arrangement of the labour process.

Proposition 6 is subject to limitations which have to be emphasized lest it should be misunderstood. First, an incremental increase in the division of labour does not inevitably modify $W(D)$ because along a standard path only the weak inequality $W(D_{i+1}) \geq W(D_i)$ holds. Thus, small changes in the division of labour do not depend necessarily upon previous alterations in demand flow to become feasible. Second, Proposition 6 states that the *minimum* number of workers eventually increases along a complete standard path. The extent of the market may be said to limit the division of labour only if demand intensity is such that a labour force of $W(D_i)$ workers brings about overproduction at least for one design D_i in the path. If demand intensity relative to the prevailing technology is such that $\bar{W}$ workers are to be employed, where $\bar{W}$ is a common multiple of $W(D_1), W(D_2), \ldots, W(D_m)$, then clearly the extent of the market puts no constraints whatsoever on the desired degree of the division of labour. Third, Proposition 6 does not suggest that dividing labour as finely as the market will allow is, of necessity, an intelligent behaviour. If demand intensity is such that both designs D_i and D_j can be operated in a synchronized manner, the choice between D_i and D_j depends upon their relative production costs as assessed by firm's preference structure, factors on which Proposition 6 has no bearings.

Proposition 6 also states that along a standard path the production schedule tends to become more continuous; it eventually becomes more continuous if the path is complete. This result suggests a cumulative process which might be of some relevance to the explanation of the behaviour of large enterprises in dualistic industrial structures. Piore (l980b) has added to Adam Smith's determinants of the division of labour another factor: the stability of the demand for output. The more stable demand is, the more encouraged would be enterprises to pursue even further the division of labour process. In modern industrial structure, large scale enterprises have the control of, and restrict themselves to, the stable segment of demand, the very capacity of separating out demand into a stable and an unstable portion being a crucial feature of dualism in product markets. Division of labour thus tends to be carried out to a greater extent in large scale enterprises than in small scale firms confined to the unstable segment of demand. Proposition 6 shows that increasing division of labour and more continuous production schedules come together. To the extent that large enterprises actually succeed in compartmentalizing demand, a more continuous production schedule reduces the size of inventories stock needed to adjust production and sales because the demand the enterprise faces is stable, uniformly distributed over time. In contrast, small-scale firms cannot scale down the ratio of inventories to sales because they face a relatively less stable demand. This asymmetry would exacerbate the dualistic tendencies inherent in modern industrial structure (Piore,1980a, 1980b).

Finally, we observe that more general paths do not exhibit a simple behaviour of $W(D)$ and $T^*(D)$. Along a non-standard path $D_1, \ldots, D_m$ the minimum number of workers $W(D)$ may increase, decrease, remain stationary or even display all of these movements as the firm innovates the work organization from D_i to D_m; the same is true of the continuity $T^*(D)$ of production. This can be easily seen by using Propositions 4 and 6. Let D_i and D_{i+1} be two designs in the non-standard path. By Proposition 4, either D_i, D_{i+1} or D_{i+1}, D_i is a standard path. By Proposition 6, $W(D_{i+1}) \geq W(D_i)$ and $T^*(D_{i+1}) \leq T^*(D_i)$ if D_i, D_{i+1} is a standard path whereas $W(D_{i+1}) \leq W(D_i)$ and $T^*(D_{i+1}) \geq T^*(D_i)$ if D_{i+1}, D_i is a standard path. The two possibilities taken together imply $W(D_{i+1}) \gtreqless W(D_i)$ and

$T^*(D_{i+1}) \gtrless T^*(D_i)$. This indeterminateness is magnified in paths of length $m > 2$ where $W(D)$ and $T^*(D)$ may first increase then decrease etc. It follows that no simple can be expected to hold between the "division of labour" and the "extent of the market" whenever the former is understood in a broad sense encompassing both job fragmentation and job grouping.

Proposition 6 pertains to paths in which all of the designs are synchronized. In the remainder of this paper, this restriction is relaxed. Propositions 7 to 10 below hold irrespective of whether designs are synchronized or not. Therefore, their validity is not contingent upon the several assumptions needed to prove Proposition 6.

VI. PSP and GSP equilibria

In this section we represent all of the possible standard and non-standard paths in terms of transition matrices $A(s)$ and $A(ns)$ respectively. In a transition matrix, $p(i,j)$ is the probability that design j will follow next to design i along the division of labour path. It will be seen that the assumptions on $A(s)$ necessary to ensure that paths converge to the optimum design are more restrictive than those one has to impose on $A(ns)$ to obtain the same optimality result.

As discussed before, we shall refrain from imposing a substantive hypothesis on the objectives of the firm. Preferences depend not only on firm's goals but also on the external environment. A design D may be deemed superior to D' in environment E and inferior in $E' \neq E$ without any concomitant modification in firm's objectives or goals. Major factors shaping the external environment include the intensity of demand, the prevailing degree and form of competition, exogenous waves of technical change and the resistance of labour to innovations. We shall take the environment as given in matrices $A(s)$ and $A(ns)$ below. Given the external environment, the preference structure of the firm is postulated to be complete (i.e., it is a complete coverage of the entire field of choice consisting of the 2^{n-1} admissible designs), consistent (i.e., the preference relation is transitive) and unambiguous (i.e., either D is preferred to D' or vice-versa). If the division of labour path followed by the firm terminates at design D, then D is the equilibrium of the firm in the given environment. Equilibria are optimal depending on whether or not the path terminates at the optimum design D^*, the optimality of D^* itself being conditional to the given characteristics of the external environment.

Since the search process takes time – i.e., a certain number of search "periods" – the firm can only attain the equilibrium position if the environment remains constant over the course of the search process. The latter may be properly viewed as a process of adaptation of the firm to the given outer environment; the equilibrium design expresses the culmination of the adaptation effort. We shall rule out in this and in the next section the possibility that the outer or external environment changes before the completion of the search process. This restriction seems to be a *sine qua non* condition of analytical tractability of the problem of determining the effectiveness of search procedures. This restriction will be discussed in the last section of this paper.

To facilitate the exposition, we index the 2^{n-1} admissible designs according to their R-representations. Let R and R' be two R-representations with h and h' elements respectively. The index v, $v = 1,\ 2, \dots, 2^{n-1}$ is constructed as follows: (a) if $h > h'$, then R has a greater index v than R'; (b) if $h = h'$, then R has a greater index v if R' precedes R in the lexicographical ordering. As an example of (b), the design with R-representation $\{2, 4, 5\}$ has a greater v-index than the design $\{1, 4, 5\}$ because $1 < 2$. Notice that the zero division of labour design has $v = 1$ and the maximum division of labour design has $v = 2^{n-1}$.

We first describe how the firm selects the design that is tried out in practice in each period t. Our description of the selection mechanism is compatible with a wide range of decision-making behaviour, instead of postulating explicit selection rules, we focus directly on the probability of selecting alternative designs. Each design v, $1 \leq v \leq 2^{n-1}$, has a probabilitiy $q(\bar{v}, v, t)$ of being tried out in period t. The existing design in period t is $\bar{v}$; obviously, $q(\bar{v}, \bar{v}, t) = 0$. During the search process, $\sum_{v=1}^{2^{n-1}} q(\bar{v}, v, t) = 1$; if the search process is over, $\sum_{v=1}^{2^{n-1}} q(\bar{v}, v, t) = 0$. The assumptions on firm's preference structure imply $q(\bar{v}, v, t) = 0$ if v has already been experienced with before period t. If the search process is guided by PSP, $q(\bar{v}, v, t)$ for all v not in $C'(\bar{v})$. Similarly, if the search process is guided by GSP, $q(\bar{v}, v, t) = 0$ if v is not in $C(\bar{v})$.

In each period t, one design v with $q(\bar{v}, v, t) > 0$ is tried out. Let v' be the selected candidate. If v' turns out to be inferior to $\bar{v}$, the search round based on $\bar{v}$ continues in period $t + 1$ with $q(\bar{v}, v', t + 1) = 0$. If v' Is found to be superior to $\bar{v}$, a search round based on v' initiates in period $t + 1$ with $q(v', v', t + 1) = 0$.

Probabilities q admit of several interpretations. $q(\bar{v}, v, t)$ is the probability that design v is experienced with in the search round based on $\bar{v}$ in period t. When $q(\bar{v}, v, t) = 1$, design v is regarded by the firm as being certainly more promising than the remaining candidates (i.e., the other yet untried designs in $C'(\bar{v})$ or in $C(\bar{v})$, depending on whether search is guided by PSP or by GSP). This superiority may result from strict principles of maximization. One can imagine that v was selected by the optimal Pandora's rule (see Weitzman, 1979). None the less, the superiority of v over the other candidates may reflect not only purely objective economic criteria (such as lower implementation costs K_v, higher expected returns in terms of cost reduction, shorter implementation time, economies of labour time etc.) but also political aspects such as the necessity of controlling the labour process (Edwards,1979). Subjective, idiosyncratic characteristics of decision-makers (such as prejudices, varying degrees of guessing rationality, inertia, mere stubbornness etc.) may also affect the selection of v crucially. Going to the opposite extreme of Pandora's rule, one can imagine that choice is contingent in the sense of being affected by events unrelated to the problem of selecting alternatives to $\bar{v}$, the probability q being in this case positive but less than one (see Nelson and Winter, 1978, for a defense of stochastic or contingent choice). The interpretation of q is dependent upon the type of decision-making behaviour one is willing to subscribe. Probabilities q oscillate between zero and one if choice is determinate; q assumes intermediary values to capture elements of choice contingency. The description of the selection mechanism in terms of probabilities q is thus robust to changes in the specification of the behavioural attributes of economic agents.

We turn next to the probability $p(\bar{v}, v)$ that any given design v is substituted for the existing design $\bar{v}$. If search is guided by PSP, $p(\bar{v}, v)$ is the probability that v will be the design next to $\bar{v}$ in the standard path; if search is guided by GSP, it is the probability that $\bar{v}$ will be followed by v in the non-standard path. Clearly, a necessary condition for $p(\bar{v}, v) > 0$ is v dominating $\bar{v}$ in firm's preferences. But even if v is superior to $\bar{v}$, $p(\bar{v}, v)$ may be less than one because other designs preferred to $\bar{v}$ may exist in $C'(\bar{v})$ or in $C(\bar{v})$. It is even possible that $p(\bar{v}, v) = 0$ for v dominating $\bar{v}$, an event that happens if there are too many other designs dominating $\bar{v}$ that are privileged by the selection mechanism.

Probabilities $p(\bar{v}, v)$ can be expressed formally. Observe initially that $p(\bar{v}, v) \geq q(\bar{v}, v, \bar{t})$, where $\bar{t}$ is the initial period of the search round based on $\bar{v}$. The reason is simple. Suppose v is not selected in period $\bar{t}$. Design v may still be tried out in period $\bar{t} + 1$ provided that the design selected in period $\bar{t}$ proved to be inferior to $\bar{v}$ in firm's preferences. If not selected again in $\bar{t} + 1$, v may still be selected in $\bar{t} + 2$ provided that the design selected in $\bar{t} + 1$ is not preferrred to $\bar{v}$ etc. Design v has as many chances of being selected as there are designs in $C'(\bar{v})$ or in $C(\bar{v})$ that are not preferred to $\bar{v}$. We know from Proposition 3 that there are $n - h$ designs in $C'(\bar{v})$ and $n - 1$ designs in $C(\bar{v})$ if $\bar{v}$ has h jobs. Of these $n - h$ designs (or $n - 1$ designs), R designs are not preferred to $\bar{v}$. If $R = n - h$ (or if $R = n - 1$), $\bar{v}$ is an equilibrium design and $p(\bar{v}, v) = 0$ for $v \neq \bar{v}$. If $\bar{v}$ is not an equilibrium design, $R < n - h$ (or $R < n - 1$) and the maximum duration of the search round based on $\bar{v}$ is $R + 1$ periods.

The search round based on $\bar{v}$ initiates at $\bar{t}$. Let $\beta(\bar{t} + d)$, d being a non-negative integer, be the probability that only designs not preferred to $\bar{v}$ are tried out in periods $\bar{t}, \bar{t} + 1, \ldots, \bar{t} + d$. Thus, $\beta(\bar{t})$ is given by the sum of probabilities $q(\bar{v}, v, \bar{t})$ over the R designs not preferred to $\bar{v}$; $\beta(\bar{t} + 1)$ is given by the sum of probabilities $q(\bar{v}, v, \bar{t} + 1)$ over the remaining $R - 1$ designs conditional to the probability that one design not preferred to $\bar{v}$ is selected in $\bar{t}$ etc. The probability $p(\bar{v}, v)$ is then given in (11) below, where we adopted the convention $\beta(\bar{t} - 1) = 1$:

(11) $$p(\bar{v}, v) = \sum_{d=0}^{R} \beta(\bar{t} + d - 1).\, q(\bar{v}, v, t + d)$$

Probabilities (11) are referred to as transition probabilities. (11) was derived under the assumption that v dominates $\bar{v}$; if v is not preferred to $\bar{v}$, we shall write $p(\bar{v}, v) = 0$ because v will not replace $\bar{v}$ even if selected to be tried out in practice. Clearly, if there is at least one v in $C'(\bar{v})$ (or in $C(\bar{v})$) preferred to $\bar{v}$ with a positive probability q of being selected, $\bar{v}$ will be eventually replaced by another design. In this case, we shall write $p(\bar{v}, \bar{v}) = 0$. The opposite case in which $\bar{v}$ dominates all of the designs in $C'(\bar{v})$ (or in $C(\bar{v})$) will accordingly be denoted by $p(\bar{v}, v) = 0$ for $v \neq \bar{v}$, $p(\bar{v}, \bar{v}) = 1$. Thus construed, transition probabilities $p(i, j)$ that j replaces existing design i in the division of labour path can be arranged in a matricial form.

The construction of the transition matrix is simple. It is convenient to order designs according to their v-indices. Let $A(h, h+r)$ be the $\lambda(h, n)$ x $\lambda(h+r, n)$ matrix of transition probabilities $p(i, j)$ from designs with h jobs to designs with $h + r$ jobs, $1 - h \leq r \leq n - h$, where $\lambda(h, n)$ is the number of designs with h jobs as calculated in Proposition 2. The transition matrix A below is a square matrix of dimensions 2^{n-1} x 2^{n-1}. The only submatrices $A(h, h+r)$ in A of dimension 1 x 1 are the corner submatrices $A(1,1), A(n, 1), A(1, n)$ and $A(n, n)$ because $\lambda(h, n) > 1$ for $1 < h < n$.

$$(12) \qquad \begin{bmatrix} A(1,1) & \cdots & A(1,n) \\ \vdots & & \vdots \\ A(n,1) & \cdots & A(n,n) \end{bmatrix}$$

Matrices $A(s)$ and $A(ns)$ of standard and non-standard paths respectively are defined by imposing properties on (12).

Definition 6. Matrix A in (12) has the following properties:

(a) $\sum_{j=1}^{2^{n-1}} p(i, j) = 1$ for all i; (b) $0 \leq p(i, j) \leq 1$ for all i and j; (c) if $p(i, j) > 0$ for some $j \neq 1$ then $p(i, i) = p(j, i) = 0$; (d) $p(i, j) = 0$ if i and j belong to the same submatrix $A(h, h)$ with $i \neq j$. In addition to properties (a)-(d), the matrix $A(ns)$ of non-standard paths obeys (e) $A(h, h+r) = 0$ for $|r| > 1$ while the matrix $A(s)$ of standard paths obeys in turn (e') $A(h, h+r) = 0$ for both $r > 1$ and $r < 0$.

Properties (a) and (b) require no comment. Property (c) reflects the definition of transition probabilities $p(i, j)$ as the probability that j will eventually replace i. If $p(i, j) > 0$ for some $j \neq i$, sooner or later design i will be substituted for another design j. It follows, first, that $p(i, i) = 0$ because the firm will not stay for ever with design i and, second, that $p(j, i) = 0$ because j is preferred to i and i cannot possibly replace j. Properties (a)-(c) taken together imply that $p(i, i) = 1$ if and only if $p(i, j) = 0$ for all $j \neq i$. Thus, diagonal elements in matrices $A(s)$ and $A(ns)$ are either zero or one.

Properties (e) and (e') derive from the assumption A.1. on classes $C(i)$ and $C'(i)$. If search is guided by GSP, possible candidates to replace the existing design i with h jobs are chosen among designs with $h + 1$ and $h - 1$ jobs; hence $A(h, h + r) = 0$ for both $r > 1$ and $r < 1$. Obviously, to have $h + 1$ or $h - 1$ jobs is a necessary but not sufficient condition for $p(i, j) > 0$. By Proposition 3, we know that any row in matrix $A(ns)$ has at most $n - 1$ non-zero transition probabilities $p(i, j)$ with $i \neq j$. Similarly, if search is guided by PSP, candidates are chosen only within the class of designs obtained by fragmenting one job of i; hence $A(h, h + r) = 0$ for both $r < 0$ and $r > 1$, a necessary but not sufficient condition for $p(i, j) > 0$ being j with $h + 1$ jobs. Proposition 3 tells us that the rows of submatrices $A(h, h + r)$ have at most $n - h$ non-zero transition probabilities $p(i, j)$ with $j \neq i$. Therefore the corner submatrix $A(n, n)$ in $A(s)$ obeys the property $A(n, n) = p(2^{n-1}, 2^{n-1}) = 1$, a fact of trivial interpretation: since class $C'(i)$ is generated by job fragmentation of the existing jobs of i, $C'(i)$ is empty when i is the maximum division of labour design.

Property (d) also follows from the characteristics of the search fields $C(i)$ and $C'(i)$. Designs having as many jobs as i are included neither in $C(i)$ nor in $C'(i)$. Off-diagonal elements in $A(h, h)$ are zero in both $A(s)$ and $A(ns)$. It follows that submatrices $A(h, h)$ are diagonal matrices with zeros and ones in the diagonal.

One example is helpful in visualizing matrices $A(ns)$ and $A(s)$. Consider a three-task production process. Proposition 2 shows that it admits of k designs. By Proposition 1 we know that R-representations are unique, R-representations being strictly increasing sequences of 1, 2 or 3 elements, the last one being $n = 3$. Therefore, the four designs are {3}, {1,3}, {2,3} and {1,2,3}. Their v-indices are 1, 2, 3 and 4, respectively; 1 has one job, 2 and 3 have two jobs, 4 has three jobs; 1 is the zero division of labour design and 4 is the maximum division of labour design. Matrices $A(ns)$ and $A(s)$ are given in (13) and (l4) for the case $n = 3$.

(13) A(ns) =

<table>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>1</td><td>A(1,1)</td><td colspan="2">A(1,2)</td><td>0</td></tr>
<tr><td>2</td><td rowspan="2">A(2,1)</td><td colspan="2" rowspan="2">A(2,2)</td><td rowspan="2">A(2,3)</td></tr>
<tr><td>3</td></tr>
<tr><td>4</td><td>0</td><td colspan="2">A(3,2)</td><td>A(3,3)</td></tr>
</table>

(14) A(s) =

<table>
<tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>1</td><td>A(1,1)</td><td colspan="2">A(1,2)</td><td>0</td></tr>
<tr><td>2</td><td rowspan="2">0</td><td colspan="2" rowspan="2">A(2,2)</td><td rowspan="2">A(2,3)</td></tr>
<tr><td>3</td></tr>
<tr><td>4</td><td>0</td><td colspan="2">0</td><td>A(3,3)</td></tr>
</table>

Matrices $A(ns)$ and $A(s)$ can also be stated directly in terms of transition probabilities. Dashed lines indicates the correspondence between (15) and (13), (16) and (14).

$$(15) \qquad A(ns) = \left[\begin{array}{c|cc|c} P(1,1) & P(1,2) & P(1,3) & 0 \\ \hline P(2,1) & P(2,2) & 0 & P(2,4) \\ P(3,1) & 0 & P(3,3) & P(3,4) \\ \hline 0 & P(4,2) & P(4,3) & P(4,4) \end{array}\right]$$

Matrices $A(s)$ and $A(ns)$ describe all of the possible standard and non-standard paths. Each path $D_1, D_2, \dots, D_m$ is a possible trajectory in matrices $A(s)$ or $A(ns)$, where v_i is the v-index of the i^{th} design in the path. The ex-ante probability of the path is then simply given by the product $\prod_{i=1}^{m-1} p(v_i, v_{i+1})$. A natural trajectory, in the sense used by Nelson and Winter (l977), is a path with ex-ante probability close to one. A path only terminates at design v_m if $p(v_m, v) = 0$ for $v \neq v_m$. The path is then said to converge to v_m and v_m is referred to as an equilibrium design.

$$(15) \qquad A(s) = \left[\begin{array}{c|cc|c} P(1,1) & P(1,2) & P(1,3) & 0 \\ \hline 0 & P(2,2) & 0 & P(2,4) \\ 0 & 0 & P(3,3) & P(3,4) \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Definition 7. A design v is a PSP (or a GSP) equilibrium if there exists at least one path $v_1, v_2, \dots, v_m, m \geq 1$, converging to v in $A(s)$ (or in $A(ns)$).

The optimal design v^* is obviously an equilibrium because it is preferred to all of the other admissible designs. Proposition 7 demonstrates that both standard and non-standard paths converge although it is silent on the designs they may converge to.

Proposition 7. All of the division of labour paths converge. Proof. Matrix A is a finite markovian chain. We need Lemma 7.1. *Lemma 7.1.*: In matrix A all of the states i are either absorbing or transient. Proof: If $p(i,i) = 1$, i is an absorbing state. Suppose $p(i,i) = 0$. Without loss of generality, separate all of the designs $j \neq i$ into two exclusive categories C^1 and C^2. State j is in C^1 if $p(i,j) > 0$, state j is in C^2 if $p(i,j) = 0$. If j is in C^1, then $p(j,i) = 0$ by property (c) of definition 6 above. If j is in C^1, i is preferred to j. Hence, $p(j,i) > 0$ if and only if j was experienced with before i; otherwise $p(j,i) = 0$. Then i is transient because the stochastic system never returns to it.

Since matrix A has at least one absorbing state, namely, the optimal design v^*, Proposition 7 follows from the fact that the probability of the stochastic system staying for ever in the set of transient states is zero for finite markovian chains (see Feller, 1958, p. 364). QED.

We shall say that search procedures are effective if they generate paths that converge to the optimum design v^*. To ensure effectiveness, one has to impose restrictions on transition matrices $A(ns)$ and $A(s)$. The uniqueness and optimality of the GSP equilibrium are assured by Assumption A.2. below.

Assumption A.2. In $A(ns)$, $p(v,v) = 0$ for $v \neq v^*$. Assumption A.2. implies that all of the non-standard paths converge to v^*. To verify this, it suffices to observe that non-standard paths converge (Proposition 7 above) and that convergence to any design v_m requires that $p(v_m, v) = 0$ for $v \neq v_m$.

If A.2. holds, the ex-ante probability of finding v^* by non-standard paths is one because any path leads to v^*. In other words, if v_m is an equilibrium, then v_m is the optimum v^*. Assumption A.2. ensures the effectiveness of GSP. By a similar reasoning, PSP are effective under A.3.

Assumption A.3. In $A(s)$, $p(v, v) = 0$ for $v \neq v^*$. Proposition 8 shows that the conditions under which PSP are effective are more restrictive than those necessary to entail the effectiveness of GSP.

Proposition 8. A.3. implies A.2. but A.2. does not imply A.3. Proof: Let $i \neq v^*$ be a design with h jobs. Define C^1 and C^2 as the set of designs $j \neq i, p(i,j) > 0$, such that j is in $A(h, h-1)$ and $A(h, h+r)$ respectively. A.3. holds if and only if C^2 is not empty. A.2. holds if and only if C^1 and C^2 are not both empty. If C^2 is not empty, $p(i,i) = 0$ in both $A(ns)$ and $A(s)$ whereas $p(i,i) = 0$ in A(ns) but $p(i,i) = 1$ in $A(s)$ if C^2 is empty but C^1 is not. It follows that A.3. is sufficient for A.2. although the reverse is not necessarily true.

This section is concluded by Proposition 9. It shows that if the optimum design does not coincide with the maximum division of labour design, there are at least two PSP equilibria; in this case, the maximum division of labour design is a non-optimal equilibrium. In Proposition 9, h^* is the number of jobs of the optimum design v^*.

Proposition 9. If $h^* < n$, there are at least two PSP equilibria. Proof. We need Lemma 9.1. Lemma 9.1. A necessary and sufficient condition for a design $\bar{v}$ to be an equilibrium is $p(\bar{v}, v) = 0$ for $v \neq \bar{v}$. Proof: Consider a path $v_1, v_2, \ldots, v_m$ converging to v_m. By definition, v_m is an equilibrium because $p(v_m, v) = 0$ for $v \neq v_m$. Since $m \geq 1$, the case $m = 1$ proves the Lemma.

In $A(s)$, the submatrix $A(n,n) = p(2^{n-1}, 2^{n-1}) = 1$. Therefore, $p(2^{n-1}, v) = 0$ for $v < 2^{n-1}$ by property (a) of definition 6. Then the maximum division of labour design is an equilibrium by Lemma 9.1. QED.

VII. The effectiveness of PSP relative to GSP

To assess the relative effectiveness of PSP we shall postulate A.2. but not A.3. throughout this section. All of the non-standard paths converge to v^* with probability one owing to A.2.. In contrast, some standard paths converge to v^* while others don't because A.2. is not sufficient to ensure the effectiveness of PSP (see Proposition 8 above). If a standard path converges to v_m, then v_m is, of course, preferred to all of the other designs actually experienced with along the path, but its optimality cannot be taken for granted. An equilibrium design under PSP may not be an equilibrium under GSP. By force of A.2., the equilibrium of the firm for the given outer environment under GSP is, of necessity, optimal whereas the equilibrium under PSP for the same given external environment may not be optimal.

Two related questions are in order. First, if a standard path converges to v_m, what is the probability Q that $v_m = v^*$? Second, what is the overall ex-ante probability P of finding out v^* following standard paths? This section is devoted to these questions. Answers are obtained under the condition that A.2. holds; they indicate therefore the (relative) effectiveness of PSP *vis-à-vis* GSP. The results of this section are interpreted in the next one.

Equation (17) answers the first question. Q is the probability that a PSP equilibrium is optimum. Let $v_1, v_2, \dots, v_m$ be a standard path, v_m having h jobs, $1 \leq h \leq n$ (by the definition of standard paths, $m = n$ for a complete path, $m < n$ otherwise). We know from Froposition 3 that $C'(v_m)$ has $n - h$ designs while $C(v_m)$ has $n - 1$ designs. If the path converges to v_m, then v_m is preferred to all of the $n - h$ designs in $C'(v_m)$; in other words, v_m is the best choice among the $n - h + 1$ designs. A non-standard path would only converge to v_m if v_m dominated the $n - 1$ designs in $C(v_m)$; in other words, if v_m were the best choice among the $n - 1 + 1 = n$ designs. Under A.2., v_m would then be the optimal design v^*. Since $C'(v_m)$ is a subclass of $C'(v_m)$, Q is given by (17):

(17) $Q = \frac{n-h+1}{n}$

Q decreases monotonically as h varies from 1 to n. $Q = 1$ for $h = 1$ because $C'(v)$ and $C(v)$ coincide when v is the zero division of labour design. For large values of n, Q can be approximated by:

(18) $Q \approx 1 - \frac{h}{n}$

Proposition 10 answers the second question mentioned above. P is the overall probability that the optimal design v^* will be reached by equi-probable standard paths starting off at the zero division of labour design. Let the optimal design v^* have h^* jobs, $1 \leq h^* \leq n$. $\rho(h^*)$ is the total number of normalized standard paths of length h^*, of which $\sigma(h^*)$ have v^* as their terminal design. P is given by the ratio $\frac{\sigma(h^*)}{\rho(h^*)}$.

Proposition 10. $P = \frac{1}{\lambda(h^*,n)}$. Proof. The proof has two parts. In part (a) we calculate $\rho(h^*)$ and in part (b) we calculate $\sigma(h^*)$.

Part (a). Let $v_1, v_2, \dots, v_{h-1}$ and $v'_1, v'_2, \dots, v'_{h-1}, v'_h$, $2 \leq h \leq n$, be two normalized paths. We know that $v_1 = v'_1$ because they both have the zero division of labour design as their starting-point. If $v'_i = v_i$ for $1 \geq i \leq h - 1$, the second path is said to be an extension of the first. Lemma 10.1 below is trivial and stated without proof. The calculation of $\rho(h^*)$ follows from Lemma 10.2 by recursion.

Lemma 10.1. Any normalized standard path of length m is an extension of a normalized standard path of length $m - 1$.

Lemma 10.2. Let $\rho(h)$ be the total number of normalized standard paths of length h. Then $\rho(h + 1) = (n - h)\rho(h)$. Proof: By Proposition 3, each normalized path, $v_1, v_2, \dots, v_h$ gives rise to

$n-h$ different extensions of the form $v_1, v_2, \dots, v_h, v_{h+1}$ where v_{h+1} is in $C'(v_h)$. Lemma 10.2. follows from the fact that each normalized standard path of length $h+1$ is an extension of some path of length h.

By a recursive reasoning, $\rho(2) = n-1$, $\rho(3) = (n-1)(n-2)$, $\rho(h^*) = \frac{(n-1)!}{(n-h^*)!}$.

Part (b). Let $\{\delta^*(1), \dots, \delta^*(h^*)\}$ be the R-representation of the optimum v^*. We want to calculate the number of normalized standard paths of the form $v_1, v_2, \dots, v_{h^*-1}, v^*$.

Lemma 10.3. If v' is in $C'(v)$, then the R-representation of v' has all of the elements of the R-representation of v plus one distinct of them. Proof: Suppose v has h sets. By Definition 2, v' has $h+1$ sets of which $h-1$ are identical to sets in v. Therefore, the R-representation of v' has $h-1$ elements identical to elements in the R-representation of v. Let J^1 and j^2 be the two sets of v' that are not identical to sets in v; then $\coprod_{i=1}^{2} J^1 = J$ for a set J in v. Let $\delta = \max\{k \text{ such that } k \in J\}$, $\delta^1 = \max\{k \text{ such that } k \in J^1\}$ and $\delta^2 = \max\{k \text{ such that } k \in J^2\}$. We have $\delta = \delta^2$; hence the R-representation of v' has all of the elements of the R-representation of v. Since $\delta^1 \in J$ and $\delta^1 < \delta$, δ^1 is distinct from any element of the R-representation of v and Lemma 10.3. is demonstrated.

Since v^* is in $C'(v^*_{h-1})$ for any standard path having v^* as its h^{th} design, the path being of course normalized, the R-representation of v^*_{h-1} is obtained from Lemma 10.3. by deleting one element of the R-representation of v^*, By construction, $\delta^*(h^*) = n$; hence only h^*-1 elements of the R-representation of v^* can be deleted. Therefore, there are $h-1$ designs v^*_{h-1} such that v^* is in $C'(v^*_{h-1})$, Each design v^*_{h-1} obtained by deleting one element of the R-representation of v^* has in turn $h-2$ designs v^*_{h-2} such that v^*_{h-1} is in $C'(v^*_{h-2})$. By a recursive argument, the number of standard normalized paths of the form $v_1, v_2, \dots, v^*$ is $\delta(h^*) = (h^*-1)!$

To prove Proposition 9 it suffices to recall that by Proposition 2 above $\lambda(h,n) = \frac{(n-1)!}{(h-1)!(n-h)!}$. QED.

Proposition 9 has interesting implications. First, $P = 1$ if either $h^* = 1$ or $h^* = n$. If $h^* = 1$, the optimal design is the zero division of labour design. $P = 1$ because there is no need to follow any path: the initial design is already optimal. If $h^* = n$, the optimal design is the maximum division of labour design. $P = 1$ because all of the paths of length n are complete paths; hence they converge to the maximum division of labour design. Looking at the case $h^* = n$ from another point of view, there is just one way of organizing the labour process when the number of jobs is equal to the number of tasks, namely, assign one task to each job. The precise path followed in accordingly irrelevant: any path leading to a n-job design necessarily reaches the optimal design.

Second, $P < 1$ for $1 < h^* < n$. The reason for this lies in the fact that there is always more than one way of organizing the labour process if $1 < h^* < n$ by Proposition 2 above. Some paths will then lead to D' while other paths will lead to non-optimal designs. By Proposition 10, the higher the number of alternative work organizations, the smaller the probability of reaching the optimal one. Since $\lambda(h,n)$ is a symmetrical function of h, P first decreases monotonically and then increases monotonically as h^* varies from 1 to n. The higher the number of tasks, a crude measure of the complexity of the labour process, the smaller the minimum value of P that obtains for a value of h^* equidistant between 1 and n. P is close to 100% if the optimal design is a minor variation of either the zero division of labour or the maximum division of labour design; but P can be as small as 11% (in the case $n = 18$ of Adam Smith's pin manufacture) or 2% (in the case $n = 45$ of Ford's model T chassis assembly line) if the optimum design lies equidistant between the two polar modes of organizing the labour process. The interpretation of these results is made in the next section.

VIII. Interpretations

It was seen in section IV that the attractiveness of PSP stems partly from characteristics of the historical setting on which capitalism developed and partly from factors grounded on the functioning of human mind. Two interpretative schemes were suggested. In the first scheme, firms set off heuristic advantages against expected effectiveness loss in deciding the form of search procedure to be adopted, In the second scheme, no choice was open to the firm. In this section, we discuss probabilities Q of equation (18) and P of Proposition 10 in the light of the historical evidence on the predominance of PSP in the early phases of capitalism.

In the first scheme of interpretation, the heuristic advantages of PSP do not constrain completely firm's choice. PSP are adopted under the limiting condition that their expected loss in effectiveness relative to GSP is not large. Thus, firm's expectations regarding the properties of v^* as well as the degree of belief in PSP play a fundamental role in the explanation of the historical proeminence of PSP.

It is analytically convenient to think of the initial organization of the labour process handed down by tradition as the zero division of labour design. The search process is animated by the presumption that the optimum design is distinct from the initial one. However, there is no information on the optimum from any source other than the experimentation process itself. The firm is in a state of complete ignorance with regard to the question of whether any given path will lead to the optimum or not. There is no ground for choosing between ex-ante equi-probable paths. None the less, it has expectations on the properties of the optimal design. These expectations may either derive from the observation of (presumably) similar search processes occurring in other firms or simply reflect *a priori* ideas on how best to organize the labour process.

Expectations are summarized by the ratio γ of h^* to n. If the firm expects the optimal work organization to be characterized by narrowly defined jobs, then γ is close to 1. For in this case the number of jobs should be large relative to the number of tasks. Conversely, γ is close to $1/n$ if the firm expects the optimal design not to depart from the traditional organization of the labour process significantly. For in this case the number of jobs should be small relative to the number of tasks. γ varies from $1/n$ to 1 as the expected value of h^* varies from 1 to n; γ is higher the more specialized the jobs of the optimal design are supposed to be. For large values of n, γ is close to zero if the firm has short-sighted or myopic expectations regarding v^* and γ is close to 1 if the optimum is supposed to be characterized by a minute parcelling of traditional job categories.

The expected effectiveness loss associated with PSP depends on the expectation parameter γ. If γ is either close to zero or to 1, the expected loss is not large. For P, the overall ex-ante probability of finding the optimum by equiprobable normalized standard paths, is in the neighborhood of 1 either in the case of myopic expectations (for γ close to zero implies h^* close to l) or in the opposite case in which the optimum is expected to be based on very specialized jobs (for γ close to 1 implies h^* close to n). In the presence of either type of expectation, the heuristic advantages of PSP would more than compensate for their expected effectiveness loss. The latter is rather small whenever the firm expects the optimum to be similar to one of the two polar modes of organizing the labour process.

The above argument explains the adoption of PSP in the initial periods of the search process. The firm adheres to FSP either because it expects the optimal work organization to be a minor variant of the traditional one or because it is convinced that the optimum consists in assigning a minimum number of tasks to each job. If results come out quickly, i.e., if an alternative superior to the existing design is regularly found after a few search periods, the firm would not cast doubts on the effectiveness of PSP. *Per contra*, suppose PSP fail in improving the existing design v (with h jobs) after a reasonable search effort. Apparently, v seems to be a serious candidate for the optimal design.

The firm may either continue to experience with designs in $C'(v)$ until the complete exhaustion of the entire search field formed by the designs in $C'(v)$ or change to GSP looking into designs that are in $C(v)$ but not in $C'(v)$. The firm is likely to revise its belief in the effectiveness of PSP according to the probability Q that v is, in fact, the optimal design v^*. In (18), Q is negatively affected by the ratio of h to n. The degree of belief in PSP tends thus to decrease along the standard path because h/n increases as the traditional job categories are progressively fragmented. In the initial periods of the search process, the confidence on PSP would remain high even in face of disappointing results whereas frustration in finding dominating alternatives would undermine the belief in the effectiveness of PSP in advanced moments of the search process.

These arguments can be weaved to suggest a plausible explanation for the predominance of PSP. During the initial phases of capitalism, PSP exhibited marked heuristic advantages from the cognitive viewpoint. Their expected effectiveness loss was not large because firms either had myopic expectations or were convinced at the outset that the optimal design involved a fine parcelling of traditional job categories. PSP were then adopted because heuristic advantages outweighed expected effectiveness loss. Once adopted, PSP would persist even in face of disappoint occasional results because the probability that a PSP equilibrium is a GSP equilibrium is high for designs that are a minor variants of the traditional work organization. The adherence to PSP in the initial phases of capitalism resulted from an act of rational choice, rationality being bounded by the limitations of human mind (Simon,1976) and shaped by specific historical circumstances (Lukes, 1977).

The very unfolding of the search process, however, placed limits to the initial superiority of PSP. On the one hand, their heuristic advantages became less marked. In particular, the necessity of apprehending the know-how of craftsmen disappeared as their practice was subjected to attentive observation. On the other hand, the confidence deposited on PSP became more sensible to poor results in the course of the search process. Finally, the prospects of further cost reductions by PSP tended to look dim as the process of job fragmentation was carried out to an extreme degree. As a consequence, the superiority of PSP *vis-à-vis* GSP was lessened. Thus, the development of the search process guided by PSP in the initial phases of capitalism cleared the way for the adoption of GSP in its late phases.

The second interpretative scheme offers a different explanation for the historical prominence of PSP. According to the second scheme, PSP were the only form of search procedure that firms could possibly resort to in early capitalism. The feasibility of GSP relied upon a prior apprehension of the labour process in its operative details and the emergence of a cognitive structure distinct from the one embedded in crafts, traditional work organization. As the search process guided by PSP unfolded, two effects took place. First, firms had an ever-improving grasp of the labour process. By mindfully heeding the elementary tasks and actively parcelling out traditional job categories, firms gradually gathered the relevant information on craftsmen practice. Second, the separation of tasks from the jobs in which they were originally engaged made possible to perceive them independently and hence to formulate a distinct cognitive structure (Piore, 1980b, p.76). As a consequence, the viability of GSP was an outgrow of the search process guided by PSP. The attempts to innovate work organization by GSP in late capitalism were made possible by the former dominance of PSP in its early phases. The second scheme of interpretation thus coincides with the first in viewing the actual adoption of GSP as being necessarily preceded by a period in which PSP predominated.

The meaning attached to probabilities Q and P changes in the second scheme of interpretation. Probability Q has no behavioural import because firms had no opportunity of choosing GSP in early capitalism.

The loss of effectiveness entailed by PSP is interpreted in this second scheme as a cost imposed by the historical backdrop out of which capitalism developed. This cost derives from the comparison of events under full rationality (which would recommend GSP) and bounded rationality (which is associated with PSP) provided that the latter is understood as reflecting not only the inner mind limitations but also the specific historical setting in which decisions are made.

In the second interpretative scheme, probability P may be viewed as indicating the presence of a social cost. To the extent that the optimum work organization as assessed by the firm's preference structure coincides with the social optimum, a thesis that Marxist analyses would not support, the social costs of choosing PSP are not large if the optimum is in the neighbourhood of either the maximum division of labour design or the zero division of labour design. The social cost would be substantial if the optimal design were not in the neighbourhood of either one of the two extreme polar modes of organizing the labour process. This social cost, be it large or not, was unavoidable in early capitalism. It could be only recuperated as GSP became viable in late capitalism, a process which resulted from the very pursuance of PSP in its initial phases.

IX. One limitation

The analytical results that support the argument of the previous section were derived under quite general assumptions. It is worthy of note that they are compatible (a) with any type of decision making behaviour; (b) with any substantive specification of the preference structure of the firm and (c) with any specification of the given external environment. This last point shows the partial equilibrium nature of our analysis. For an equilibrium design expresses the full adaptation of the firm to the given outer environment; since the environment itself is likely to change as a consequence of the cumulative search processes undertaken by firms, the above analysis cannot aspire to the status of a general equilibrium analysis. The hypothesis of constancy of the environment during the course of the search (or adaptation) process undertook by one firm considered in isolation is clearly faithful to the canons of partial equilibrium analyses. Yet one difficulty remains.

We presupposed that the n production tasks did not undergo any modification during the course of the sequential search process. In Definition 1, different designs are different partitions of the same invariant set N of tasks. To put it more strikingly, the technical methods of production are the same in the zero division of labour design (in which there is no specialization of work) and in the maximum division of labour design (in which workers are riveted to isolated tasks).

Of course, the attractiveness of different work organizations reflects the characteristics of the underlying given technology. The equilibrium of the firm resulting from the search process is an equilibrium relative to a given external environment. In accordance with a partial equilibrium framework, changes in the methods of production are subsumed under the alterations of the given outer environment. Thus, the above model allows for the effect of technical change upon the division of labour under the limiting condition that technical change itself is exogenous to the search process.

To the extent to which the forces generating technical change are independent of the actual course of the search process, the placing of technical change under the heading of alterations of the given external environment is justifiable.

Technical change, however, is to some extent an outgrow of previous innovations undergone by work organization. Three different accounts have been offered of the effect of the division of labour on technical change. In all of them, the search for superior work organizations induces changes in the technical side of the labour process.

The first was given by Adam Smith in the *Wealth of Nations*. Smith contended that "a great part of the machines made use of in those manufactures in which labour is most subdivided, were originally the inventions of common workmen who, being each of them employed in some very simple operation, naturally turned their thoughts towards finding out easier and readier methods of performing it" (Smith, 1965, p. 9). Adam Smith based his argument on the observation that men are much more likely to invent new methods of attaining any object "when the whole attention of their minds is directed towards that single object, than when it is dissipated among a great variety of things" (idem, p. 9). As a consequence, a minute division of labour would enhance the inventiveness of workers by concentrating their attention on a narrower span of tasks.

The Achilles' heel of Adam Smith's argument is, of course, that the resource attention is scarce or not relative to a given information flow (Simon, 1978a); a too specialized job in which the worker is confined to a small number of repetitive tasks can hardly be said to be the proper milieu for the flourishing of technical inventiveness, a point recognized by Adam Smith himself (see Smith, 1965, p.734-735; Marglin, 1976).

The second one was given by Marx in volume I of *Capital*. He argued that Adam Smith confused the differentiation of instruments of labour, in which workers themselves took an active part, with the invention of machinery, in which they had only a minor role (Marx, 1977, p. 468, note 19). Multi-purpose tools were suited for the variety of tasks subsumed under traditional job categories; they turned out to be needlessly generic when used to perform a limited range of tasks. In manufacture, the minute fragmentation of traditional job categories gave rise (a) to the adaptation of tools, previously used for many purposes, to new narrower ones and (b) to the specialization of tools in accordance with the characteristic features of the worker riveted to the narrower span of tasks. The first effect, the reshaping of tools consonant with the specific nature of the tasks they were confined to, was referred by Marx as differentiation while the second effect was referred to as specialization: "Manufacture is characterized by the differentiation of the instruments of labour – a differentiation whereby tools of a given sort acquire fixed shapes, adapted to each particular application – and by the specialization of these instruments, which allow full play to each special tool only in the hands of a specific kind of worker". (idem, p. 460). Both effects, however, were held by Marx to belong exclusively to the period of manufacture. Since the advent of large-scale industry, technical change ceased to be moulded by previous innovations in work organization (Marx, 1977, ch.15). In large-scale industry, technical change arises directly out of Science. It is the application of science to production which enables the machine to perform the same tasks previously performed by the worker. In contrast to the period of manufacture, in which invention occurred through the division of labour, in large-scale industry "invention ... becomes a business" (Marx, 1973, p. 704).

The third was given by Piore (1980b). In Piore's alternative rationale for the division of labour, the fragmentation of traditional job categories made possible to perceive their component tasks independently, thus enabling their combination or synthesis into new cognitive structures governed by a different and presumably more efficient logic. Innovations in work organization foster technical change because they provide "the process through which we escape the intellectual grip of existing forms of organization". (Piore, 1980b, p. 76). Job fragmentation unfetters the intellectual process of discovery to the extent to which it helps in visualizing combinations of tasks whose perception was concealed by the cognitive structure embedded in traditional work organizations centred on crafts. Piore's argument, however, does not imply either that technical change necessitates previous experiments in work organization to occur or that technical change follows necessarily after every experiment in work organization. Since the process involved in Piore's alternative rationale for the division of labour is essentially intellectual, Piore himself observed that it must not be physically embodied in the production process (Piore, 1980b, p. 77).

The common tenet of these three accounts of the effect of the division of labour upon technical change is that the latter is induced by the former. The presence of induced technical change has clear implications for the above model. It renders unsustainable the hypothesis of constancy of the methods of production (the set N) during the course of the search process. In all of the three accounts of induced technical change, however, the causality running from experiments in work organization to technical change is subject to qualifications which lessen considerably its force. The hypothesis of constancy may thus be justified on two grounds. First, because the arguments supporting induced technical change are not overwhelmingly persuasive. Second, because the quest for logical precision imposes costs in descriptive accuracy. As it often happens in modelling complex historical phenomena, some aspects of the subject at issue appear in their purest form only by dissociating in analysis factors that are indissolubly tied in historical experience.

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