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On the Long-Run

Inflation-Unemployment Trade-Off

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Economic thinking on inflation has changed drastically in the last ten years as the natural rate hypothesis, originally advanced by Milton Friedman and Edmund Phelps, became orthodoxy and a textbook staple. This, in a sense, has been a return to a pre-keynesian mode of thought. Keynes' General Theory was an attempt to explain why the economic system "seems capable of remaining in a chronic condition of sub-normal activity for a considerable period without and marked tendency towards recovery or towards complete collapse" (p. 249). Introduction of the old non-vertical Phillips curve in the keynesian model, following A. W. Phillips' path breaking 1958 article, did not affect this basic notion that long-rum equilibrium may occur at different levels of activity. The only difference here is that each equilibrium position of the economy is characterized by a distinct rate of inflation which has to be accommodated by an appropriate rate of growth of the money supply. With the natural rate hypothesis, however, we came all the way back to the neoclassical notion that macroeconomic equilibrium is possible only at the normal 'natural' level of activity.

This victory of the natural rate hypothesis over the traditional keynesian view seems to have been mostly a result of the strong seduction that the rational behavior axiom exerts over economists. The main thrust of Friedman and Phelps against the prevailing keynesian orthodoxy was to show that a long-run inflation-unemployment tradeoff would be possible only if people suffered from permanent money illusion, failing to fully incorporate inflationary expectations in their behavior. This, however, cannot occur in an economy where agents engage in rational maximizing behavior, regardless of the fact that some stickiness in wages and prices may occur in the short run. That is to say, the rational behavior makes the neoclassical long-run result implied by the natural rate hypothesis inevitable even in a typically keynesian model.

The purpose of this paper is to have another look at this proposition. Is it really impossible to generate a long-run inflation-unemployment tradeoff in a model of the economy where permanent money illusion is not present? In order to answer this question, we will first formulate, in section 1, a model that is consistent with the natural rate hypothesis, and then, in section 2, show how it can be modified to reject it. In this second section we will be particularly interested in the consequences of allowing for endogenous capital stock adjustments as a result of macroeconomic disturbances. The question of empirical relevance is briefly examined in section 3, and the paper is closed by some final remarks in section 4.

1. A Natural Rate Model

We start by considering a simple model of the production-employment sector of the economy

that is consistent with the natural rate hypothesis. The model follows the keynesian tradition, as established by authors such as Kerneth Boulding[1], Don Patinkin, Robert Clower, Axel Leijonhufvud, Robert Barro and Herschel Grossman, of assuming short-run rigidity of wages and prices, which implies that the economy may operate under non market clearing conditions in the short run.

Suppose there is an aggregate production function

(1) $Y_t = F(K_t; N_t); F_1' > 0, F_2' > 0$

relating real aggregate output Y_t to the capital stock K_t and actual employment N_t. Aggregate labor demand, N_t^d, is a decreasing function of the real wage, V_t, and an increasing function of the capital stock:

(2.a) $N_t^d = D(V_t, K_t); D_1' < 0, D_2' > 0$

(2.a) $F_N'(K_t, N_t^d) = v_t; F_N' > 0, F_{N_1}' < 0$

Aggregate labor supply is positively related to the real wage:

(3) $N_t^S = S(V_t); S' > 0$

Note that in the short period employment, labor demand and labor supply may all be different. Also, in order to keep the argument as simple as possible, we are assuming a stationary economy, in which there is no technological progress and zero population growth. This is a standard simplifying procedure in macroeconomic analysis, that normally come together with the assumption of a fixed capital stock:

(4) $\mathbf{K_t = \overline{K}}$

In a stationary economy the unemployment rate, however defined, must be a stable inverse function of employment

(5) $u_t = L(N_t); L' < 0$

Let the disequilibrium behavior of the economy be described – as in Patinkin and Solow-Stiglitz – by a wage equation:

(6) $\widehat{W}_t = a(N_t - N_t^S) + \widehat{P}_t^e$

and a price equation:

(7) $\mathbf{\widehat{P}_t = b(N_t - N_t^d) + \widehat{W}_{t-1}}$

where $\widehat{w}_t$ is the rate of nominal wage increase, $\hat{p}_t$ and $\hat{p}_t^e$ are the actual and expected rates of inflation,

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[1] As far as we know Kenneth Boulding has never received the credit he deserves for being a precursor, in the early fifties, of the modern disequilibrium literature.

and a and b are positive constants. We assume that expectations on the rate of inflation are formed adaptively on the basis of the previous year rate:

(8) $\hat{P}_t^e = \hat{P}_{t-1}$

This, however, should not be understood as a denial of rational expectations, since we want to restrict our analysis to the case of an economy where all changes in the rate of inflation are completely unanticipated. In such an economy a short-run inflation-unemployment tradeoff is always present, and this is the best context on which to examine the question of the existence of an analogous long-run tradeoff. If there is no short-run tradeoff, it is quite obvious that there is no long-run tradeoff either.

To close the model, remember that the rate of change of the real wage is defined by:

(9) $\hat{V}_t = \hat{W}_t - \hat{P}_t$

We assume, in the keynesian tradition, that real output is determined in the goods market, so that it can be taken as exogenously given in the present discussion which is focused on the labor market. This means that we have enough equations to determine our nine variables $K_t, N_t, N_t^d, N_t^s, u_t, \hat{w}_t, \hat{p}_t, \hat{p}_t^e$ and v_t (or $\hat{v}_t$).

To show that this model does not reject the natural rate hypothesis, let us note first that there is a single value of real output – call it the "natural" level of output – which is consistent with equilibrium in the labor market, that is $N_t = N_t^d = N_t^s$, and hence – from (6) and (7) – with inflation equilibrium, a situation where the rate of inflation stays constant over time. This corresponds to an intersection of the supply and demand for labor schedules as in point *M* of Figure 1. Note that there are also unique "natural" levels for employment, real wage and unemployment rate corresponding to this equilibrium position (though not for the inflation rate).

Consider now the consequences of a government policy that maintains real output permanently bellow its natural level. From (1) this implies that employment is also bellow its natural level and that consequently the economy must be on a position that is off its supply and demand for labor curves. In terms of Figure 1, if N_1 is the employment level resulting from the government policy, the economy will be somewhere in the line-segment connecting points *D* and *S*. From (6) and (7) it is clear that the consequence is a continuous bottomless decline in the rates of price and wage inflation. Contrariwise, if government policy keeps employment permanently above its natural level, the consequence must be a continuous acceleration of inflation. It follows that in this economy equilibrium is possible only when real output is at its natural level: the long-run Phillips curve is vertical at the natural unemployment rate and the natural rate hypothesis is verified.
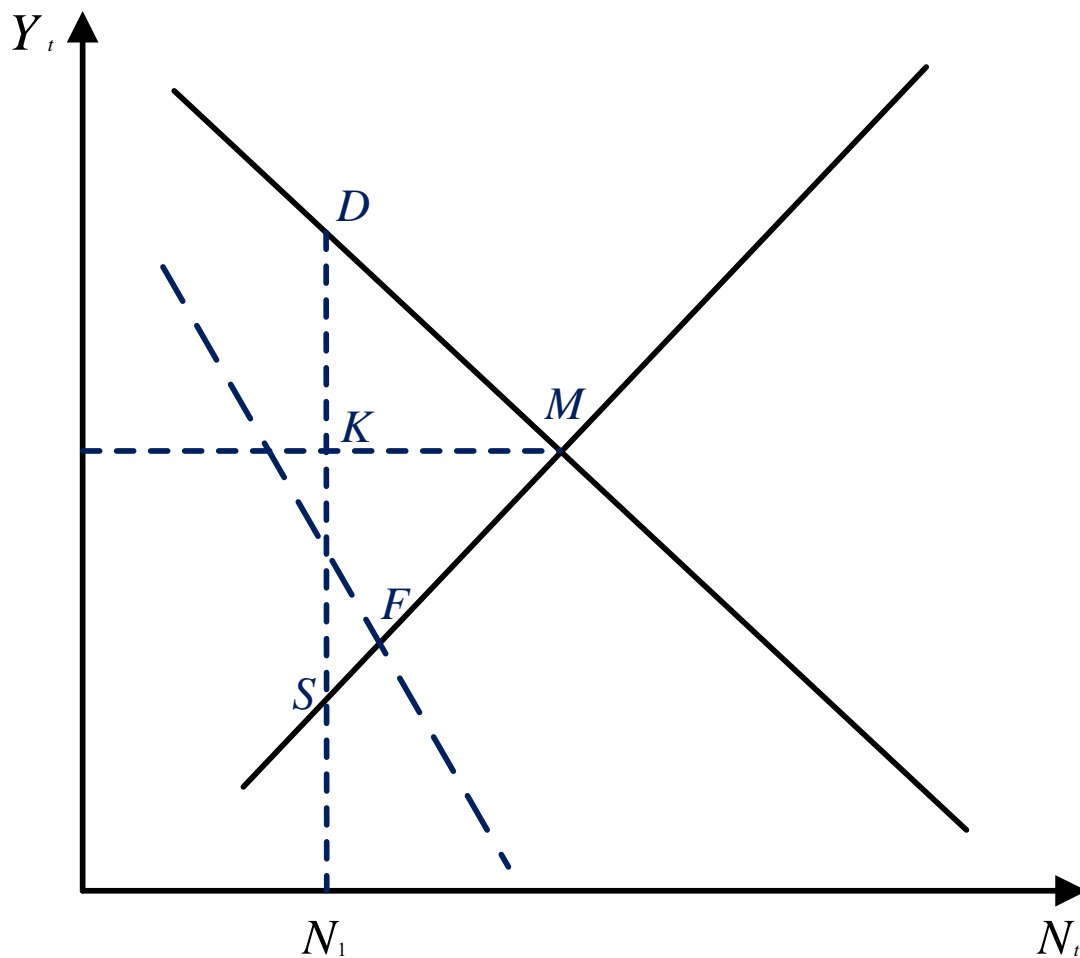
Figure 1

2. A Model without Natural Rate

So far we have followed the standard practice of the inflation literature, assuming a stationary economy in which there is no technical progress and no population growth. This, of course, is just a harmless simplifying hypothesis, as it does not exclude the possibility of trade cycles generated by exogenous disturbances or policy mismanagement, and as most of the results obtained under it can be easily translated into a growing economy context in terms of deviations from long-run trends.

What is not harmless, however, is the equally conventional assumption of a fixed capital stock, which we also adopted in the previous section. This seems to be a sort of conditioned reflex in the inflation literature, whose origin is probably to be found in the old tradition of dealing with inflation as something that can be fully explained in the context of a short-run macro-model of the economy. This was so in the old quantity theories as well as in Keynes' General Theory, and was not modified by subsequent developments such as the Philips curve. Apparently, however, nobody cared to check

the hypothesis when, a decade ago, it became fashionable to add inflation expectations to the Phillips curve equation.

It should be obvious that if expectations, generated by a process that involves time-lags, have a role to play in the determination of inflation, it is no longer valid to restrain the discussion to a short period context. In particular, if we want to study how the economy moves from one long-run equilibrium position to another in response to some exogenous unanticipated disturbance, we must consider a transition period of definite time span for which it seems quite arbitrary to assume a priori a fixed capital stock. The correct approach in this case, as in trade cycle analysis, is to deal explicitly with changes over time in both fixed and working capital.

What happens if the model of the previous section is reconstructed without the assumption of a fixed capital stock? Suppose, following Barro and Grossman, that firms in our economy have the objective of maximizing the market price of equity shares, and that this is equivalent to maximizing profit per unit of capital. Note however that firms also face a sales constraint determined by the exogenously given level of aggregate demand. Hence, assuming constant returns to scale in production, aggregate real profit per unit of capital is:

(10) $\frac{Q_t}{K_t} = F\left(1, \frac{N_t}{K_t}\right) - V_t \frac{N_t}{K_t}$

which has to be maximized subject to the restriction:

(11) $F(K_t, N_t) = Y_t$

where Y_t is an exogenous parameter. The necessary conditions for a maximum are simply:

(12) $F_N(K_t, N_t) = V_t; \; F'_{N_1} > 0, F'_{N_2} < 0$

together with (11). It follows – by solving (12) for K_t – that the stock of capital which firms as a whole want to hold at any moment can be expressed as a function of employment and the real wage:

(13) $K_t^* = B(N_t, V_t); \; B'_1 > 0, B'_2 > 0$

Comparison of (2) and (12) snows that:

(14) $K_t = B(N_t^d, V_t)$

which means that the existing stock of capital is optimal whenever employment equals labor demand at the current real wage.

Taking net investment to be a fraction h of the gap between desired and actual capital stock, we have:

(15) $K_t = K_{t-1} + h(K_{t-1}^* - K_{t-1}) = K_{t-1} + h[B(N_{t-1}, V_t) - B(N_{t-1}^d, V_t)]$

which shows that the stock of capital will be decreasing when employment is less than labor demand, stationary when employment equals labor demand, and increasing when employment exceeds labor demand. One way of modifying the model of the previous section so that it will allow for endogenous adjustments of the capital stock in response to macroeconomic disturbances is to replace equation (4) by this last equation[2].

Determination of q_t is possible only with a complete specification of the macro-model. Assuming a money-capital economy, as in Tobin (1969), this would require five additional equations:

(16) $\mathbf{F_k(K_t, N_t) = R_t}$; $\mathbf{F_k' < 0, F_{k_2}' > 0}$

where is the marginal efficiency of capital relative to reproduction cost,

(17) $r_t q_t = R_t$

where r_t is the real rate of return on capital,

(18) $\frac{M_t}{P_t} = L^*(Y_t, \hat{p}_t^e, r_t q_r K_t); L_1^{*\prime} > 0, L_2^{*\prime}, L_3^{*\prime} < 0, L_4^{*\prime} > 0$

where M_t is the money supply,

(19) $I_t = K_t - (1 + d)K_{t-1}$

where I_t is real aggregate investment and d is the depreciation rate,

(20) $C(Y_t) + I_t + G_t = Y_y$

where $C(Y_t)$ is the consumption function and G_t is real government expenditure.

Note that the analysis of this paper implicitly assumes that policy variables such as M_t and G_t are being passively; adjusted all the time so as to maintain a constant level of real output.

It is easy to show that in this revised version of the model, in which capital is no longer necessarily fixed, the natural rate hypothesis is rejected. Suppose the economy is initially at full equilibrium in point M of Figure 1, with both the rate of inflation and the capital stock being constant over time. Consider now the consequences of a government policy that produces a permanent reduction in real output. In a first period, while the stock of capital is still fixed, employment falls to say, and the economy operates on a position that is off its supply and demand for labour schedules, such as point K in Figure 1. From (6) and (7) it is clear that the consequence is a continuous decrease

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[2] Note that a more realistic specification of the investment function would have to take the fraction h as variable, for example: (15.a) $K_t = K_{t-1} + h(q_t)(K_{t-1}^* - K_{t-1}) = h' > 0$. where q_t is the value of capital relative to its replacement cost, as in Tobin (1959). Hence, if p_t is the price of currently produced goods (that can be used for consumption or investment), $q_t p_t$ is the market price of existing capital goods.

in the rate of inflation. In the present case, however, this is not a bottomless decrease, as it was in the case of the fixed capital model of last section. Equation (15) tell us that in such a position the stock of capital must also be decreasing over time, and this has two consequences: it raises employment, as capital is substituted by labor in the production of the fixed level of real output, and' it simultaneously reduces the aggregate demand for labor at any given real wage. In terms of Figure 1 this means that the economy is at a position on a vertical line segment joining the supply and demand for labor schedules, such as *DS*, that is moving to the right over time, while the demand for labor schedule is simultaneously moving to the left. The obvious implication is that eventually a new equilibrium position will be reached, in a point such as F in Figure 1 where the employment level is greater than but smaller than the one that existed at the initial equilibrium position. With the reduction in the capital stock a raw aggregate demand for labor schedule – shown by the broken line in Figure 1 – will intersect the labor supply schedule exactly at point *F*, and both the rate of inflation and the stock of capital will again be constant over time.

What all this means is that in the model of this section there is nonatural equilibrium position for the economy. Long-run macroeconomic equilibrium is possible at any level of real output.

3. Is It Empirically Relevant?

Our main result can now be summarized as follows. The natural rate hypothesis does not hold when in a conventional macro-model of the economy we allow for the possibility of adjustments over time in the capital stock as a response to changes in aggregate demand. If, in a stationary economy, one starts from a given initial equilibrium level of real output and reduces aggregate demand, both the stock of capital and the level of employment will shrink over time until a new equilibrium position is found. In the transition phase the rate of inflation will be reduced while the unemployment rate increases, generating a long-run inflation-unemployment tradeoff.

An objection could be raised at this point that although adjustments over time in the stock of capital will in theory always lead to a long-run inflation – unemployment tradeoff when changes in the rate of inflation are completely unanticipated, as a practical matter these adjustments are so slow when compared with price adjustments that they can be safely disregarded. This, of course, is an issue that can only.be settled empirically. We would like to emphasize, however, chat adjustments in the quantity of capital in response to aggregate demand disturbances mu;; very well occur without a long lag by means of changes in inventories, postponement of replacements or changes in the rate of capital

accumulation in the case of a growing economy[3].

It should also be noted that lack of strong response of mark-ups to excess demand will tend to accentuate the long-run inflation-unemployment tradeoff. The empirical evidence seems to indicate that although an excess demand term is usually significant in econometric estimations of the price equation, the effect is nevertheless of small magnitude, with most of the variance in price inflation being accounted for by changes in standard unit labor cost. To figure out the consequences of this, let us consider the polar case where prices are wholly cost-determined and the b coefficient in equation (7) is zero. Suppose the economy finds itself on a point off its demand and supply of labor curves, such as point K in Figure 1, after a permanent reduction in real aggregate demand. It is obvious that in the present case the subsequent decline in the rates of wage and price inflation will be followed by a continuous fall of the real wage, until the economy reaches a position on its labor supply curve, such as point S in Figure 1. After this stage, however, as the economy goes on operating on its labor supply curve, there will no longer be any force affecting the rate of inflation (the fact that the economy is still off its demand for labor curve has no effect on mark-ups in the present case). Hence the rate of inflation will stabilize before a new equilibrium position is reached, and possibly in a relatively short period of time, even though the complete adjuslme.it of the quantity of capital to the new level of real aggregate demand may take a long time.

In a second phase of the adjustment process, the rate of inflation will be constant as the economy moves upwards along its labor supply curve, as from point S to point F in Figure 1. It is clear then that, in the case of cost-determined prices, adjustments in the rate of inflation may not be sufficient to restore long-run equilibrium after a real demand shock: the economy will also need some capital stock adjustment to do the job.

A second possible objection to our argument may be raised on the basis of existing econometric evidence. Suppose there is a linear relationship between employment and the unemployment rate:

(21) $N_t = n - cu_t$

with n and c being positive constants, and define u_t^* as the unemployrnent rate corresponding to the labor supply of each period, that is:

(22) $N_t^s = n - cu_t^*$

Substituting in (6) and (8), and adding the stochastic error term e_t, we obtain a Phillips equation:

(23) $\widehat{w}_t = acu_t^* - acu_t + \hat{p}_{t-1} + e_t$

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[3] This, incidentally, suggests that the long-run Phillips tradeoff is likely to be steeper in a rapidly growing economy than in a stagnant one.

The reason why this equation becomes consistent with a long-run inflation-unemployment tradeoff when we allow for capital stock adjustments is that in this case u_t^* is an endogenous variable and the equilibrium condition $u_t = u_t^*$ does not by itself define the equilibrium value of the unemployment rate. Econometric estimation of equations similar to (23), however, have not apparently rejected the hypothesis of a constant u_t^* – which appears as the intercept term in estimated equations – and this would seem to suggest that capital stock adjustments are irrelevant for the analysis of inflation.

The answer to this objection is that we have an error in variable problem when we try to estimate (23) with the assumption of a constant u_t^*. We know – from (22) and (3) – that u_t^* must be negatively correlated with the real wage, and we may assume that the real wage behaves prociclically, being positively correlated with the previous period rate of inflation (this could be a consequence, for example, of a weak response of mark-ups to excess demand). It follows that we will obtain a good fit for a regression such as:

$$(24)\ u_t^* = \bar{u} - k\hat{p}_{t-1} + z_t$$

where $\bar{u}$ and k are positive constants and z_t is an stochastic error term. Substitution into (23) leads to:

$$(25)\ \widehat{w}_t = ac\bar{u} - acu_t + (1 - k)\hat{p}_{t-1} + (e_t + acz_t)$$

which shows that estimation of (23) with the restriction of a constant u_t^* will produce a significant intercept terra but also an estimate of the coefficient of the expectation term that is biased to be less than its true unity value. This seems quite consistent with the existing econometric evidence.

4. Final Remarks

This paper has shown that the natural rate hypothesis is not a logical consequence of the rational behavior axiom. It demonstrated that a long-run inflation-unemployment tradeoff can be generated in a conventional model of the economy, under the assumption that there is no permanent money illusion, when explicit consideration is given to the possibility of endogenous adjustments in the stock of capital in response to changes in real aggregate demand. Whether this is an empirically relevant proposition is an open issue that goes beyond the objectives of the present paper. It may be that the natural rate hypothesis can still be rescued as a useful first order approximation to reality, but this has to be proved.

Rejection of the natural rate hypothesis, on the other hand, has some interesting implications that can only be hinted here. First, there is a large literature on the choice of the optimal combination

of inflation and unemployment that became obsolete with the vertical Phillips curve but would have to be resurrected if the natural rate hypothesis were abandoned. Second, the notion that the long-run inflation- -unemployment tradeoff depends on the investment function implies, for example, that a more accentuated tradeoff will appear when a given reduction in real output is accompanied by an increase in the real interest rate – as when only monetary policy is used, than when it is accompanied by a decrease in the real interest rate – as when fiscal policy is combined with monetary policy. Hence the problem of the relative effectiveness of monetary and fiscal policies would need reevaluation. Third, and finally, it should be noted that the long-run tradeoff is a result of two dynamic processes: the adjustment of the rate of inflation and the adjustment of the capital stock. This means that time is a factor that must be taken explicitly into account in order to determine, for example, the increase of unemployment that is associated in the long-run with a given reduction of the rate of inflation. It is quite likely that the unemployment rate will be smaller at the end of a stabilization program that produces a fast reduction in inflation than at the end of an alternative gradualist stabilization-program that engenders a slow fall of inflation of the same magnitude. These, however, are interesting questions that we must leave as tasks for future research.

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