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Abstract

In this note, I consider a setting in which an agent can exert costly unobservable effort towards two activities and is, a priori, uncertain about its ability to perform them. A (non-contractible) ability enhancing investment can be performed. The lack of commitment from a Principal concerned with (informational) rent extraction, and who is in control of both activities, induces the standard underinvestment outcome of Hold-Up problems. It is then shown that, if the two activities are assigned to two different Principals, ex-post incentives will be more powerful generating, from an ex-ante perspective, higher incentives for ability enhancing investment. The combination of higher ex-ante investments and ex-post incentives produces an outcome that is superior than a single Principal's outcome in terms of efficiency. This suggests that organizational structure, through its influence on the design of incentives over contractible variables, can play a key role is solving Hold-Up problems.

1 Introduction

Since the seminal works of Williamson (1985), Klein et al (1978), Grossman and Hart (1986), and Hart and Moore (1990) among others, Economists have drawn a big deal of attention to the study of (relationship-specific) investment decisions in a world in which contracts are incomplete, and to means to minimize potential inefficiencies in those decisions.

Regarding the second point, special emphasis has been placed on the ownership of assets as a mean to induce investment. Assets, the story goes, grants residual control rights over

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ex-post decisions to its owner. Hence, property rights is a mean to allocate ex-post bargain power which, by transferring ex-post surplus to one of the parties, provides incentives for ex-ante relationship specific investment. One issue that these papers take for granted is that all players are able to acquire the relevant assets, so the analysis focus exclusively on the surplus maximizing assignment of ownership: property rights over assets should be granted to the side of the market for which the investment is the most precious socially.

When – for various reasons such as credit constraints, limited liability, risk aversion, among others – the acquisition of assets is not feasible, one must consider alternative mechanisms to curb potential inefficient decisions in an incomplete contract world. Aghion and Bolton (1992), for example, suggest that in such case debt financing can work as a mechanism to allocate contingent control, inducing optimal decisions in states of bankruptcy.

Even if feasible, the granting property rights to *one* of the sides in a relationship necessarily reduces the *other* side's incentives to invest. In fact, this is what Grossman and Hart (1986), and Hart and Moore (1990) have pointed out as the main cost of vertical integration. It seems, therefore, that additional mechanisms should be sought to trim the resulting reduction in investment from the side of the party without propiertoship of assets.

In searching for an additional mechanism to stimulate relationship-specific investments, this note follows a somewhat similar path to the one in Aghion and Bolton (1992). I argue that an organizational structure *within* a firm, through its effect on the design of incentive schemes over contractible variables, serve as a mean to provide incentives for ex-ante investments in a situation in which the acquisition of assets is not feasible. In cases the unrestricted acquisition of assets *is* feasible an adequate organization structure may play a complementary role to the one played by ownership.

I analyze a setting in which an agent can exert costly unobservable effort towards two activities and is, *a priori*, uncertain about its ability to perform them. A non-contractible ability enhancing investment can be performed. The lack of commitment from a Principal concerned with (informational) rent extraction, and who is in control of both activities, induces the standard underinvestment outcome of Hold-Up problems. It is then shown that, if the two activities are assigned to two different Principals, ex-post incentives will be more powerful generating, from an ex-ante perspective, higher incentives for ability enhancing investment. The combination of higher ex-ante investments and ex-post incentives produces an outcome that is superior than a single Principal's outcome in terms of efficiency.

The reason why incentives are steeper in a dual structure is simple: the Principals have to compete for the agent's attention. This competition between Principals transfers some of the ex-post surplus to the agent, who then faces, from an ex-ante perspective, better incentives to invest. One sees that the mechanism through which efficiency gains can be attained in this note's setting is exactly analogous to the one implied by the Property Rights Theory. In fact, an organizational structure with more than one Principal can be seen as a credible way to commit to not extracting ex-post surplus from the investing party, much as the proprietorship of an asset guarantees to its owner participation in the ex-post surplus.[1]

This note is organized as follows. The next section describes the set-up of the model, and the timing of events. Section 3 solves the model for the two different organizational structures and compares their effects on the relevant variables. Section 4 discusses the results, and draws a brief conclusion. All results not in the text can be found in the appendix.

2 Model

The players in the model are an agent who can exert unobservable costly effort towards the production of two activities, $i = 1, 2$, and either one or two principals to which this agent report. In this note, we analyze two different organizational arrangements. In the first one, a single Principal is in charge of both activities, while in the second a different Principal is in charge.

Attached to activity $i = 1, 2$ is a contractible output

$$y_i = \theta + e_i,$$

[1] The practical difference between the two mechanisms can be significant however. Consider, for example, the case in which a middle manager in a large corporation can, by incurring some cost, specialize himself in a task that contributes to two different businesses of the corporation he works for. If such task is specific to the way this company manages its businesses (e.g., refers to a market that the company is the only one that has access to), the usual underinvestment outcome takes place. To solve this problem, the Property Rights Theory suggests that the middle manager should take over the relevant businesses, which may be, for a myriad of reasons, infeasible. The note suggests that an improvement can be attained by simply assigning two different bosses – one for each of the businesses – for the manager. While this can generate some obvious organizational costs (as, for example, it calls for different persons in charge of different but closely related areas), this is clearly a very practical way to provide incentives for specific investment. If the specific investment is very important, the benefits will offset the costs.

where θ is a a measure of the agent's ability to perform the activities 1 and 2, and $e_i \geq 0$ is a measure of the agent's effort towards activity i. The agent's ability is *a priori* unknown to all the players. More specifically, θ is assumed to be distributed according to a c.d.f. $F(.)$ over $[\underline{\theta}, \overline{\theta}]$ with corresponding density $f(\theta) > 0$. As it is standard., I assume that $\frac{1-F(\theta)}{f(\theta)}$ is decreasing in θ.

The agent, however, can – before engaging in production – invest to enhance its ability to perform the activities. Such investment is assumed to be observable by all parts but both non-contractible and relationship-specific: it does not affect the agent's outside option. The investment is assumed to make it more likely that the realization of the agent's ability to perform activities 1 and 2 is high. More specifically, I assume that for any two levels of investments $\overline{z} \geq z > z' \geq 0$,[2]

$$\frac{f(\theta|z)}{f(\theta|z')}$$

is increasing in θ so that higher realizations of ability are more likely if the agent invests more. To invest z, the agent incurs a private cost of $\frac{z^2}{2}$. The agent's ex-post utility function is given by

$$t - \frac{(e_1 + e_2)^2}{2},$$

where t is the total payment the Principals make to him. Note that the two types of effort are substitutes in the agent's preferences. This will be key for the results to follow.

A single Principal is assumed to care about the sum of profits generated by both activities while in the case there are two Principals, each of them is assumed to care solely about the profits generated by the activity they control. The timing of events is depicted in Picture 1. At time zero, the agent performs the relationship-specific investment. At period 1, he learns his ability and, in the one-Principal case, the set of contracts $\{y_1, y_2, t(y_1, y_2)\}_{y_1, y_2}$. is offered to the agent and he selects the contract that fits him better. In case there are two principals, two sets of contracts, $\{y_1, t_1(y_1)\}_{y_1}$ and $\{y_2, t_2(y_2)\}_{y_2}$ are offered. The agent *must* contract with both principals, i.e., the model is one of intrinsic common agency (Bernheim and Whinston (1986)). In period 2, he chooses privately the level of efforts towards production, and the level of production realizes. All payments are made in accordance in period 3.

[2] I take that $f(\theta|0) = f(\theta)$.

Figure 1 Here

3 The One Principal Case

In period 1, the Principal's problem, once the agents has invested z, is to solve the following program[3]:

$$\max_{y_1,y_2} \; E_\theta(y_1^{1P}(\theta) + y_2^{1P}(\theta) - t(\theta)|z)$$

$$s.t.$$

$$U(\theta) \equiv t(\theta) - \tfrac{(y_1^{1P}(\theta)+y_2^{1P}(\theta)-2\theta)^2}{2} \geq 0, \; \forall\theta \text{ (IR)}$$

$$U(\theta) \equiv t(\theta) - \tfrac{(y_1^{1P}(\theta)+y_2^{1P}(\theta)-2\theta)^2}{2} \geq t(\widehat{\theta}) - \tfrac{(y_1^{1P}(\widehat{\theta})+y_2^{1P}(\widehat{\theta})-2\theta)^2}{2}, \; \forall\theta, \widehat{\theta} \text{ (IC)}.$$

In words, his profit maximization problem is constrained by the fact that, no matter what is the realization of the agent's ability, the principal has to match the agent's outside option (IR). It must also be the case that the agent is better off reporting his true ability rather than any other one to the Principal (IC).[4] An application of the Envelope Theorem (Milgrom and Segal, 2002) along with a single crossing condition that the agent's utility satisfies allows one to replace these constraints by

$$U(\theta) = U(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} 2(y_1^{1P}(\tau) + y_2^{1P}(\tau) - 2\tau)d\tau, \tag{1}$$

and $y_1^{1P}(\theta) + y_2^{1P}(\theta)$ being non-decreasing in θ. Using (1), one can see that

$$t(\theta) = U(\underline{\theta}) + \frac{(y_1^{1P}(\theta) + y_2^{1P}(\theta) - 2\theta)^2}{2} + \int_{\underline{\theta}}^{\theta} 2(y_1^{1P}(\tau) + y_2^{1P}(\tau) - 2\tau)d\tau.$$

Substituting this in the objective function, integrating by parts, and noting that participation is guaranteed whenever the agent with the lowest ability gets non-negative utility, the Principals's program becomes

$$\max_{\{y_1(\theta),y_2(\theta)\},U(\underline{\theta})} E_\theta(y_1^{1P} + y_2^{1P} - \frac{(y_1^{1P} + y_2^{1P} - 2\theta)^2}{2} - [U(\underline{\theta}) + 2\frac{(1 - F(\theta|z))}{f(\theta|z)}(y_1^{1P} + y_2^{1P} - 2\theta)]$$

[3] Here I use the fact that $e_i = y_i - \theta$, and normalize the agents outside option to zero.

[4] Note that I make use of the Relevation Principle.

$$s.t.\ U(\underline{\theta}) \geq 0,\ y_1^P(\theta) + y_2^P(\theta) \text{ non-decreasing in } \theta\ .$$

Ignoring the constraints and maximizing the objective pointwise, one has $U(\underline{\theta}, z) = 0$, $y_1^{1P}(\theta, z) = y_2^{1P}(\theta, z) = \frac{1}{2} + \theta - \frac{(1-F(\theta|z))}{f(\theta|z)}$. Compared with an efficient outcome[5], one sees that to minimize the rents left to the agent, the Principal distorts downward the required level of output. This is a standard feature of models with asymmetric information., what is non-standard in the model is that such distortion ex-post affects the ex-ante incentives for the agent to invest.

More specifically, at period 0, anticipating the Principal's choice of outputs and, consequently, his ex-post utility, the agent invest so to maximize

$$E_\theta(\int_{\underline{\theta}}^{\theta} 2(y_1^{1P}(\tau, z) + y_2^{1P}(\tau, z) - 2\tau) d\tau | z) - \frac{z^2}{2} =$$

$$\int_{\underline{\theta}}^{\overline{\theta}} 2(y_1^{1P}(\theta, z) + y_2^{1P}(\theta, z) - 2\theta) \left[\frac{1 - F(\theta|z)}{f(\theta)}\right] f(\theta)\, d\theta - \frac{z^2}{2}. \tag{2}$$

It is clear that, due to the ex-post downward distortion of output, the agent cannot fully rip the benefits of his investment and therefore underinvests (compared to what is socially optimal) so that the standard Hold-Up problem ensues.

4 Two-Principal Case

With two Principals, matters are slightly more complicated in period 1. The main source of complication is the fact the Revelation principle does not apply in a common agency setting. However, as shown by Martimort and Stole (2002), an extension of the Taxation Principle (see, Salanie, 1997) – the Delegation Principle –, applies and the whole equilibrium set can be computed using a fairly simple methodology.

The main idea is to consider individually each of the principal's problem for a fixed set of contracts offered by the other. In such case, under some assumptions that have to be checked

[5] Efficiency here requires that, for any z,

$$y_1(\theta) + y_2(\theta) = 1 + 2\theta.$$

in equilibrium, the methodology used in the single board case fully applies and the problem reads exactly as a single principal's one. More specifically, let $\{y_2(z), t_2(y_2(z))\}_{y_2}$ be a fixed set of contracts offered by the second Principal for a given level of investment by the agent. The agent will choose among them the one that maximizes his utility. As a consequence, it is as if Principal 1 had to deal with an agents with preferences given by

$$t_1(y_1) + \phi(y_1, z, \theta), \tag{3}$$

where $\phi(y_1, z, \theta) = \max_{y_2} t_2(y_2(z)) - \frac{1}{2}[y_1 + y_2(z)) - 2\theta]^2$. Therefore, for a fixed set of contracts offered by Principal 2, Principal 1's problem is exactly the same as the one of a single principal deciding *only* on y_1 and facing an agent with preferences described by (2) In particular, the Revelation Principle fully applies in such a case and attention can be restricted to Direct Mechanisms of the form $\{y_1(\widehat{\theta}), t_1(\widehat{\theta})\}_{\widehat{\theta}}$.

Defining $\Phi(\theta) = \max_{\widehat{\theta}} t_1(y_1(\widehat{\theta}) + \phi(y_1(\widehat{\theta}), \theta)$, and using the Envelope Theorem, incentive compatibility is, under the assumption that $\phi_{\theta y_1} \geq 0$, now equivalent to

$$\Phi(\theta) = \Phi(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} 2\left[y_1(\tau) + y_2(y_1(\tau), z, \tau) - 2\tau\right] d\tau \tag{4}$$

and $\frac{dy_1(\theta)}{d\theta} \geq 0$. It is important to notice that the single crossing condition ($\phi_{\theta y_1} \geq 0$) needed to replace the incentive compatibility constraints by the above two conditions is now endogenous: it depends on the set of contracts offered by Principal 2, and has to be checked in equilibrium. Ignoring this issue for now, and proceeding exactly in the same fashion as before (i.e., integrating condition (3) by parts and substituting $t_1(\theta)$ in the objective function, as well as imposing $\Phi(\underline{\theta}) = 0$ as it minimizes the payments to the agent and guarantees the satisfaction of the participation constraints), the Principal 1's problem becomes

$$\max_{y_1} E(y_{1-}\phi(y_1, z, \theta) - 2\frac{(1 - F(\theta|z))}{f(\theta|z)}(y_1 + y_2(y_1, z, \theta) - 2\theta).$$

The first order necessary condition for optimality is given by

$$1 - (y_1 + y_2(y_1, z\theta) - 2\theta) - 2\frac{(1 - F(\theta|z))}{f(\theta|z)}[1 + \frac{dy_2(y_1, z, \theta)}{dy_1}] = 0. \tag{5}$$

The problem for Principal 2 is analogous and yields a similar first order condition. In a symmetric equilibrium, $y_1 = y_2 = y$, and, as shown in the appendix, $\frac{dy_2(y_1(\theta),\theta)}{dy_1} = \frac{\dot{y}}{\dot{y}-2}$[6]. A

[6] For a derivation, see the appendix.

symmetric equilibrium solves the differential equation

$$1 - (2y^{2P} - 2\theta) - 2\frac{(1 - F(\theta|z))}{f(\theta|z)}[1 + \frac{\dot{y}^{2P}}{\dot{y}^{2P} - 2}] = 0.$$

with boundary condition $y^{2P}(\overline{\theta}, z) = \frac{1}{2} + \overline{\theta}$. The convexity of the agent's cost in exerting effort implies that outputs for activity 1 and 2 are substitutes in his preferences, i.e., $\frac{dy_2(y_1,\theta)}{dy_1} < 0$. This particular feature, as shown by Martimort (1992) has as an important implication

Proposition 1 *(Martimort (1992) For any given $z \in [0, \overline{z}]$, there is a unique Symmetric Equilibrium in the subgame played by the Principals. Such equilibrium is the solution of*

$$1 - (2y^{2P} - 2\theta) - 2\frac{(1 - F(\theta|z))}{f(\theta|z)}[1 + \frac{\dot{y}^{2P}}{\dot{y}^{2P} - 2}] = 0.$$

with boundary condition $y^{2P}(\overline{\theta}, z) = \frac{1}{2} + \overline{\theta}$.

In addition to the uniqueness result in Proposition 1, the substitutability of outputs in the agent's preferences has also another important implication. From (4), one can see that, when deciding on his output target, Principal 1 perceives the reduction in the output that the agent will be demanded from Principal 2, $\frac{dy_2(y_1,z,\theta)}{dy_1}$, as an additional benefit. One then has

Proposition 2 *For any given $z \in [0, \overline{z}]$ and activity $i = 1, 2$, the output demanded from the agent in a dual structure, y_i^{2P} , will be strictly higher than in a unitary structure, y_i^{1P}.*

4.1 Investments Decisions in a Two-Principal Case

Proposition 2 implies that, from an ex-post perspective, the output will be closer to the first best in a common agency set-up. As a consequence, since the agent's informational rents are increasing in the demanded output, the result also implies that, for any given level of investment z, the agents' ex-post utility will be strictly higher in a two-principal arrangement. It seems, therefore, that, in principle, the agent faces steeper incentives upon deciding to invest and, consequently, picks a higher z in a dual structure.

This argument, however, ignores the effect of the agents choice of investment in the outputs themselves. A higher investment, by enhancing the agent's ability, makes it more likely that a high θ is drawn. Therefore, for any given θ, a higher z increases the likelihood that a Principal is facing a type higher than θ. The provision of informational rents to the agent becomes costlier, and, therefore, output is reduced.

As it turns out, this (perverse) effect of a higher investment on ex-post output is more critical in an unitary structure. The reason why this is the case is again the fact that outputs are substitutes in the agent's preferences. Each Principal anticipates that a higher z will lead to a reduction in the output demanded from the other Principal. Hence, in response to such effect of an increase in z, the Principals reduces their output in a less aggressive fashion when compared to an unitary structure. It then follows

Proposition 3 *The agent will invest more in a setting in which he reports to two Principals. Hence, making the agent report to two Principals strictly improves efficiency by both increasing the demanded targets and by inducing more investment from the agent.*

5 Concluding Remarks

This note aimed to argue that, if one wants to stimulate relationship-specific investment, not only the assignment of property rights over assets but also the design of the organizational structure *within* a firm is of relevance in instances of contractual incompleteness. A well designed organizational structure, through its impact on the design of incentives over contractible variables, determines the parties' ex-post participation in the total surplus, which in turn affect incentives for ex-ante investment.

The results are: an organizational structure in which the agent reports to two principals results in (i) more output being demanded from the agent for a *given* level of investment, (ii) a lower sensitivity of the principal's choice of output to changes in the specific-investment made by the agent, and, as a consequence of (i) and (ii), (iii) higher incentives for the agent to invest in the relationship.

An interpretation of those results is that, through some competition between its different sections, an organization may be able to credibly "bring the market inside the firm", and make some types of relationship-specific investments – i.e., those that do not affect significantly the agent's outside option – *less* specific from the point of view of whom takes the decision to invest.

While there are well known limitations to trying to replicate the market's functioning inside a firm – such as the fact that some of the incentives inside an organization have to be self-enforced (Baker et al (2001, 2002)), and that part of decisions are authority-based (Simon (1951), Aghion and Tirole (1988)) – , an organizational structure that assigns different Principals to different activities partly accomplishes this task. This may serve as a mean to provide incentives for ex-ante investments in situations in which either the unrestricted acquisition of assets – the remedy for the Hold-Up problem suggested the Property Rights Theory – is not feasible, or two-sided specific investments are important.

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6 Appendix

This appendix derives first an expression for $\frac{dy_2(y_1,\theta)}{d_1}$ when evaluated at the equilibrium, and proves Proposition 3.

Toward accomplishing the former, note that the first order condition for $\max_{y_2} t_2(y_2) - \frac{1}{2}\left[y_1 + y_2 - 2\theta\right]^2$ reads

$$t_2'(y_2) - [y_1 + y_2 - 2\theta] = 0. \tag{6}$$

By the implicit function theorem,

$$\frac{dy_2}{dy_1} = \frac{1}{t_2''(y_2) - 1}.$$

Using the fact that (a) must hold for all θ in equilibrium, one can differentiate it to find, using symmetry,

$$t_2''(y_2)\dot{y}(\theta) - 2[\dot{y}(\theta) - 1] = 0,$$

so that $t_2''(y_2(\theta)) = 2\frac{[\dot{y}(\theta)-1]}{\dot{y}(\theta)}$ and we have the expression in the text.

Proof of Proposition 3: To show the result, one just needs, evoking Topkis (1998), to show that the objective function of the following parameterized program has an objective function with Increasing Differences in (z, x), where $x \in \{0, 1\}$:

$$\max_{z\in[0,\overline{z}]} \Omega\left(x, z\right) = x[\int_{\underline{\theta}}^{\overline{\theta}} 2(2y^{2P}(\theta, z) - 2\theta)[1 - F(\theta|z)]d\theta] +$$

$$(1-x)[\int_{\underline{\theta}}^{\overline{\theta}} 2(2y^{1P}(\theta,z)-2\theta)[1-F(\theta|z)]d\theta]-$$

$$\frac{z^2}{2}.$$

Trivially, checking for Increasing Differences for such function is equivalent to checking whether

$$\int_{\underline{\theta}}^{\overline{\theta}} 4[y^{2P}(\theta,z)-y^{1P}(\theta,z)][1-F(\theta|z)]d\theta]d\theta$$

is increasing in z. The derivative of this expression with respect to z is

$$-\int_{\underline{\theta}}^{\overline{\theta}} 4[y^{2P}(\theta,z)-y^{1P}(\theta,z)]F_z(\theta|z)]d\theta]d\theta+$$

$$\int_{\underline{\theta}}^{\overline{\theta}} 4[\frac{dy^{2P}(\theta,z)}{dz}-\frac{dy^{1P}(\theta,z)}{dz}][1-F(\theta|z)]d\theta]d\theta.$$

The first term is unambiguously positive as $y^{2P}(\theta,z) > y^{1P}(\theta,z)$ for all $\theta \in [\underline{\theta},\overline{\theta})$, and $F_z(\theta|z) < 0$. As for the second term, note first that

$$\frac{dy^{1P}(\theta,z)}{dz} = -\frac{d\frac{[1-F(\theta|z)]}{f(\theta|z)}}{dz} \equiv -H_z(\theta|z).$$

For any given θ, the output demanded in equilibrium in a dual structure is implicitly defined by

$$F(y,\theta,\dot{y}(\theta),z) \equiv 1-(2y^{2P}-2\theta)-2\frac{(1-F(\theta|z))}{f(\theta|z)}[1+\frac{\dot{y}^{2P}}{\dot{y}^{2P}-2}]=0.$$

By the implicit function theorem, one sees that

$$\frac{dy^{2P}(\theta,\dot{y}(\theta),z)}{dz} = -\frac{\frac{\partial F}{\partial z}}{\frac{\partial F}{\partial y}} = -H_z(\theta|z)\left[1+\frac{\dot{y}^{2P}}{\dot{y}^{2P}-2}\right].$$

Hence,

$$\frac{dy^{2P}(\theta,z)}{dz}-\frac{dy^{1P}(\theta,z)}{dz} = -H_z(\theta|z)\left[\frac{\dot{y}^{2P}}{\dot{y}^{2P}-2}\right] > 0,$$

and the result follows.■

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