

TEXTO PARA DISCUSSÃO

No. 611

Characterization of equilibria at  
which the advantageous gift  
phenomenon occurs

Yves Balasko



# 1. Characterization of equilibria at which the advantageous gift phenomenon occurs

We still have two countries or consumers (i.e.,  $m = 2$ ), and finite but arbitrarily large numbers of goods and factors. Consumer  $i$  with  $1 \leq i \leq 2$  is endowed with the factor bundle  $\omega_i \in \mathbb{R}_{++}^\ell$ . Let  $\omega = (\omega_1, \omega_2) \in \mathbb{R}_{++}^\ell \times \mathbb{R}_{++}^\ell$  be the endowment vector in factors.

**Definition 1** (Transfer problem). *The transfer problem or advantageous gift phenomenon occurs at the equilibrium  $(q, p, \omega)$  in the Heckscher-Ohlin model if there exists an endowment vector  $\omega'$  in a neighborhood of  $\omega$  and such that  $\omega_1 < \omega_2$*

The issue is to characterize the equilibria  $(q, p, \omega)$  such that a gift of goods from consumer 1 to consumer 2 implies an increase in consumer 1's utility at equilibrium.

**Proposition 2.** *The equilibrium  $(q, p, \omega)$  satisfies the advantageous transfer property if and only if its index is equal to  $-1$ .*

*Proof.* Let  $\Delta\omega_1 \in X = \mathbb{R}_{++}^k$  be any gift of resources from consumer 1 to consumer 2. The favorable gift property is satisfied if the utility of consumer's 1 equilibrium allocation is higher despite the gift of resources.

In order to study this problem, I use the equivalence with the exchange model with  $k$  factors and, for that model, I use the dual formulation.

In the price income space where the coordinates are the price vector  $q \in S$  and wealth  $w_1$ . The section manifold  $B(r)$  is a curve going from the hyperplane  $w_1 = 0$  to the hyperplane  $w_1 = q \cdot r$ . By giving up resources, consumer 1's set  $A(\omega_1)$  is below the original hyperplane. The utility increases along  $B(r)$  if the tangent to  $B(r)$  make the right angle. This is a question of orientation of the hyperplane  $A(\omega)$  and of the tangent line to  $B(r)$  at  $x$ .

A basis for the hyperplane  $A(\omega)$  consists of the  $\ell - 1$  vectors  $(\omega_1^1, 1, 0, \dots, 0)$ ,  $\dots$ ,  $(\omega_1^{\ell-1}, 0, \dots, 0, 1)$ . These vectors are perpendicular to the vector  $(1, -\omega_1^1, \dots, -\omega_1^{\ell-1})$ . The orientation of this set of  $\ell$  vectors is defined by the sign of the determinant

$$\det \begin{bmatrix} \omega_1^1 & 1 & 0 & \dots & 0 \\ \omega_1^2 & 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ \omega_1^{\ell-1} & 0 & 0 & \dots & 1 \\ 1 & -\omega_1^1 & -\omega_1^2 & \dots & -\omega_1^{\ell-1} \end{bmatrix}$$

This determinant is equal to  $(-1)^{\ell-1}(1 + (\omega_1^1)^2 + (\omega_1^2)^2 + \dots + (\omega_1^{\ell-1})^2)$  and has the sign of  $(-1)^{\ell-1}$ .

The index of the equilibrium  $(q, \omega)$  is equal to  $+1$  or  $-1$ , the sign being the opposite of the sign of  $\det J(q, \omega)$ . The index of a no-trade equilibrium is equal to  $+1$ .

A necessary and sufficient condition for the advantageous gift of resources is therefore that the index of the equilibrium in factors is negative.

Incidentally, for such endowments, the number of equilibria is odd, equal to the odd number  $2n+1$ . Of these  $2n+1$  equilibria,  $n+1$  have an index equal to  $+1$  and  $n$  have their index equal to  $-1$ . Advantageous gift occurs at these equilibria.  $\square$

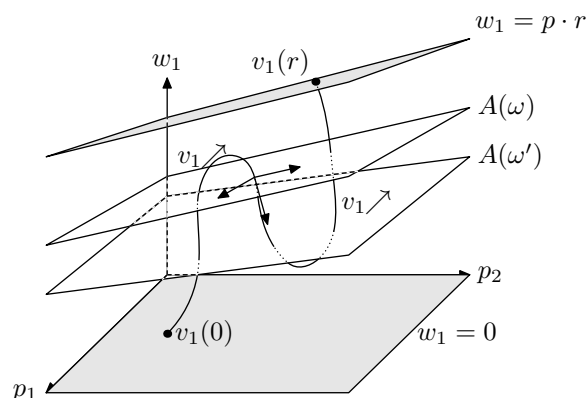


Figure 1: Index and advantageous gift

Another remark is that the advantageous gift phenomenon in the case of two consumers (or, equivalently, of two countries) can occur only if the endowment vector has multiple equilibria, which requires that the volume of factor trade is large enough because equilibrium is unique for small volumes of trade.

Departamento de Economia PUC-Rio  
Pontifícia Universidade Católica do Rio de Janeiro  
Rua Marques de São Vicente 225 - Rio de Janeiro 22453-900, RJ  
Tel.(21) 31141078 Fax (21) 31141084  
[www.econ.puc-rio.br](http://www.econ.puc-rio.br)  
[flavia@econ.puc-rio.br](mailto:flavia@econ.puc-rio.br)