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Eduardo Zilberman
Vincius Carrasco
Pedro Hemsley

DEPARTAMENTO DE ECONOMIA
www.econ.puc-rio.br

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Eduardo Zilberman
Department of Economics
PUC-Rio

Vinicius Carrasco
Department of Economics
PUC-Rio

Pedro Hemsley
Department of Economics
UERJ

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Abstract

In a repeated unobserved endowment economy in which agents negotiate long-term contracts with a financial intermediary, we study the risk-sharing implications of the interaction between incentive compatibility constraints (due to private information) and participation constraints (due to one-sided commitment). In particular, we assume that after a default episode, agents consume their endowment and remain in autarky forever. We find that once they are away from autarky today, if the probability of drawing the highest possible endowment shock is sufficiently small, the optimal contract prevents agents from reaching autarky tomorrow and, thus, from being "impoverished". Moreover, an invariant cross-sectional distribution of life-time utilities (or values) exists. A numerical example shows that the mass of agents living in autarky can be zero in the limit.

Keywords: risk sharing contracts; private information; one-sided commitment.

JEL Classification: D31, D82, D86.

*Zilberman (corresponding author): zilberman@econ.puc-rio.br. Carrasco: vnc@econ.puc-rio.br. Hemsley: pedro.hemsley@uerj.br.

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1 Introduction

This paper studies optimal risk sharing contracts in an endowment economy with private information and one-sided commitment. Each risk-averse agent is endowed with a sequence of perishable goods, distributed identically and independently over time and agents. A long-term contract can be signed with a risk-neutral financial intermediary (or principal). Private information imposes incentive compatibility constraints, as in Thomas and Worrall [1990]. One-sided commitment introduces a set of ex post participation constraints. As in Thomas and Worrall [1988] and Kocherlakota [1996], we assume that after repudiating the contract, agents consume their endowment and then remain in autarky forever. In contrast with those papers, we assume that the principal can credibly commit to the long-term contract.

Focusing on both private information and one-sided commitment allows this paper to encompass two important contributions of the existing literature. On the one hand, Thomas and Worrall [1990] study the implications of private information for risk sharing in an endowment economy assuming full commitment. On the other hand, Ljungqvist and Sargent [2012]'s version of Thomas and Worrall [1988] only considers one-sided commitment in the context of complete information. When private information is the sole contracting friction, the principal spreads continuation values in order to provide incentives. The so-called immiseration result arises, in which the continuation values become arbitrarily negative with a probability of one. In sharp contrast, when commitment is instead one-sided under complete information, continuation values increase over time, reaching a finite limit in finite time when full insurance is achieved.[1]

Our model also echoes some earlier work by Phelan [1995], who considers both incentive and participation constraints, although with a different modeling of participation, in particular, ex-ante participation constraints, which simply impose a lower bound on the set of possible continuation values. Hence, they do not depend on current realizations of the endowment. In contrast, we consider ex post participation constraints that allow

[1]Thomas and Worrall [1988] informally mention this result, although they perform their analysis assuming that the principal can also renege on the contract.

agents to consume their endowment after a default episode.[2] Despite some similarities with previous contributions, the interaction between ex post participation and incentive compatibility constraints, which is the focus of this paper, has novel implications.

Our main result (Proposition 1 in Section 4) states that once away from the autarky state today (i.e., once agents are promised a value higher than the value of autarky), if the probability of drawing the highest possible endowment shock is small enough, the optimal contract prevents agents from reaching the autarky state tomorrow. In other words, the optimal contract prevents agents from being "immiserated" (or "impoverished") in the sense that the optimal contract does not deliver the value of autarky tomorrow, which is the greatest lower bound on the set of feasible continuation values.

Moreover, our numerical simulations suggest that the mass of agents living in autarky can be zero in the limit, even if the probability of drawing the highest possible endowment shock is bounded away from zero. In particular, we provide a simple numerical example showing that once an agent is away from the autarky state, if a long sequence of the lowest possible endowment shock is realized, continuation values associated with this sequence converge to a value strictly above the value of autarky in finite time. This is in sharp contrast to the immiseration result.[3]

To prove Proposition 1, we state two intermediate lemmas that characterize the optimal contract at the autarky state. These lemmas are also useful to develop some intuition behind the main result. First, we find that if an agent is at the autarky state and has access to financial markets, the financial intermediary cannot spread continuation values to provide incentives unless the highest realization of the endowment is drawn.[4] In other words, if the agent draws any realization of the endowment other than the highest possible one, then he remains stuck in autarky. Second, we find that at the autarky state, some

[2]In related contexts, Hertel [2004] and Broer et al. [2017] consider both private information and limited commitment (or enforcement). Broer et al. [2017] study consumption risk sharing in a similar environment with persistent shocks, public insurance and ex ante participation constraints, whereas Hertel [2004] studies risk sharing contracts between two risk-averse agents, as in Kocherlakota [1996]. Other papers that study the interaction between private information and limited commitment, but in different contexts, include Sleet and Yeltekin [2001] and Ales et al. [2014].

[3]When private information is the sole friction in the model, Phelan [1998] argues that the crucial assumption to generate the immiseration result rests on preferences.

[4]This result follows directly from the restrictions on continuation values and transfers that arise from the constraints in the recursive problem.

intertemporal trade occurs between the financial intermediary and the agents who draw the highest endowment shock. Hence, the autarky state is non-absorbing.[5]

The intuition behind our main result is as follows. In a problem with asymmetric information, the possibility to spread continuation values is a profitable tool to provide incentives. But at the autarky state, the principal cannot spread continuation values for the agents who have drawn endowment shocks (their types) other than the highest one. If the probability of obtaining the highest possible endowment shock is small enough, then autarky is a persistent state. In this case, the impossibility to spread continuation values for lower types becomes excessively costly. Hence, the slope of the value function of the principal becomes positive in the neighborhood of the autarky state, which prevents him from promising the value of autarky.

Moreover, we show that the constraint that can prevent agents from being "immiserated" is the participation constraint that causes the agent hit by the lowest income shock to be indifferent between living in autarky and honoring the contract. Furthermore, we show that none of the states reached is absorbing. In particular, Proposition 2 shows that a non-degenerate invariant cross-sectional distribution of life-time utilities (or values) exists. These results contrast Ljungqvist and Sargent [2012]'s version of Thomas and Worrall [1988], who show that the continuation values converge to an absorbing state, pinned down by the participation constraint of the highest type.

Finally, our results also differ from Phelan [1995], who shows that the lower bound on the set of continuation values is a recurrent state. Phelan [1995] also shows that a non-degenerate invariant distribution exists. Because the main difference between this paper and Phelan [1995] is the presence of ex post participation constraints, we solve numerically for the optimal contract with and without these constraints to highlight their role in the model. We rely on a simple numerical example to emphasize that ex post participation constraints are crucial to prevent agents from being "immiserated". Without them, the mass of agents at the autarky state, which is the greatest lower bound on the set of feasible continuation values, is positive in the limit.

[5]This result is reminiscent of the literature on dynamic risk sharing contracts with private information. In general, whenever a lower bound on continuation values is present, it is not an absorbing state. See, for example, Atkeson and Lucas [1995] and Wang [1995], among others.

The remainder of this paper is organized as follows. Section 2 presents the model. Section 3 states some intermediate results. Section 4 studies the dynamics of the model. Section 5 discusses some numerical results. Finally, Section 6 concludes.

2 The Model

We consider an economy in which many infinitely lived ex ante identical agents can sign a single long-term contract with a financial intermediary (or principal). In each period, an agent is endowed with θ units of a perishable consumption good (his type). We assume that type θ is private information drawn from the set $\{\theta_1 < \theta_2 < ... < \theta_n\}$, with $n \geq 3$. In particular, we assume that endowment shocks are independently and identically distributed over agents and time, with $\pi_j = \text{prob}(\theta = \theta_j) > 0$, $j = 1, ..., n$, such that $\sum_{j=1}^{n} \pi_j = 1$.

Each agent derives utility from a consumption stream $\{c_t\}_{t=0}^{\infty}$. Preferences are separable over time, such that the discounted instantaneous utility at t is denoted by $\delta^t u(c_t)$, where $\delta \in (0, 1)$ is the discount factor. We assume that u is strictly increasing, strictly concave, twice continuously differentiable and bounded above, i.e., $\sup u(c) < \infty$. Finally, we normalize life-time utility by the factor $(1 - \delta)$:

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t u(c_t).$$

This normalization facilitates the proof that the value function of the principal, which maps life-time utilities (or values) to life-time profits, is strictly concave for a δ that is high enough, a result we use in Section 4 to derive a Lagrange functional for the problem.

The financial intermediary is risk neutral with free access to credit markets, where he can borrow and lend at a constant risk-free interest rate given by $\frac{1}{\delta} - 1$. Hence, the agents and the principal discount the future at the same rate, an assumption we relax in Appendix C.2. The financial intermediary can credibly commit to a long-term loan contract designed to maximize his life-time profits, which are also normalized by the factor $(1 - \delta)$. In particular, at $t = 0$, he offers a long-term contract to agents that promises a given normalized life-time utility (or value) v_0. In contrast, agents can walk away from the contract at any time at a cost of living in autarky forever.

The optimal contracting problem can be written recursively. Suppose an agent enters a period with a given promised value v. Hence, for each θ_j, the contract assigns a transfer b_j to the agent, which can be negative, and a promised continuation value w_j that the contract must honor in the beginning of the next period.

For a given value v promised to the agent at the end of the previous period, the contract $\{b_j, w_j\}_{j=1}^n$ offered by the financial intermediary must respect four restrictions. First, the financial intermediary must honor the last period promised value. To do so, the expected value of the contract must be equal to v. Hence, the promise-keeping constraint reads as follows:

$$\text{(PK)} \qquad \sum_{j=1}^{n} \pi_j [(1-\delta)u(\theta_j + b_j) + \delta w_j] = v.$$

Second, since θ is private information, agents can misreport their endowment shocks. Incentive compatibility requires that:

$$\text{(IC)} \qquad (1-\delta)u(\theta_j + b_j) + \delta w_j \geq (1-\delta)u(\theta_j + b_k) + \delta w_k, \text{ for all } j, k.$$

Third, we assume that agents cannot commit to honor the contract. Once an agent reneges on the contract, he is excluded from the financial market and forced to remain in autarky forever. Hence, the contract must respect the following participation constraints:

$$\text{(PC)} \qquad (1-\delta)u(\theta_j + b_j) + \delta w_j \geq (1-\delta)u(\theta_j) + \delta w_{aut}, \text{ for all } j,$$

where $w_{aut} = \sum_{j=1}^{n} \pi_j u(\theta_j)$ is the normalized expected life-time utility of living in autarky forever.

Many contributions in the literature also assume this specific form of ex post participation constraints.[6] In particular, the nature of the punishment, i.e., living in autarky forever,

[6] A non-exhaustive list includes Thomas and Worrall [1988], Kocherlakota [1996], Attanasio and Ríos-Rull [2000], Kehoe and Levine [2001], Ligon et al. [2002], Krueger and Perri [2006], Krueger and Perri

is crucial to derive our analytical results. This outside option entails a lower bound w_{aut} on the set of continuation values, a constraint that will be used to derive some results in the next sections. Indeed, both (PK) and (PC) imply that:

$$v = \sum_{j=1}^{n} \pi_j [(1-\delta)u(\theta_j + b_j) + \delta w_j] \geq \sum_{j=1}^{n} \pi_j [(1-\delta)u(\theta_j) + \delta w_{aut}] = w_{aut}.$$

Hence, $w_j < w_{aut}$ for some j violates the next-period promise keeping constraint. Finally, note that no contract can provide incentives by promising values higher than $w_{max} = \sup u(c_t) < \infty$.

Therefore, given a promised utility v, the principal solves

$$W(v) = \max_{\{b_j, w_j \in [w_{aut}, w_{max}]\}_{j=1}^{n}} \sum_{j=1}^{n} \pi_j \left[-(1-\delta)b_j + \delta W(w_j)\right]$$
$$\text{subject to (PK), (IC) and (PC),}$$

where W is the value function of the principal.

This problem encompasses two important contributions from the literature. Ljungqvist and Sargent [2012]'s version of Thomas and Worrall [1988] leaves (IC) out of the problem. Similarly, under additional assumptions on u, Thomas and Worrall [1990] solve the case in which (PC) is absent, which implies that the set of constrains $w_j \geq w_{aut}$, for all j, is also absent from the problem.[7]

Phelan [1995] also considers participation constraints in a model of risk sharing with private information, although with a different modeling of participation. In particular, he assumes that the decision to repudiate the contract is taken at the beginning of the period, or equivalently, one-period contracts are enforceable. The principal can also renege on the contract at a fixed cost. As a result, participation constraints translate into a lower and an upper bound on the set of continuation values.[8] If he assumed that

[2011], Tian and Zhang [2013] and Laczó [2014]. See also Gobert and Poitevin [2006] who allow for savings in a model of dynamic risk sharing with limited commitment.

[7] To be precise, neither contribution normalizes life-time utility and profits by $(1-\delta)$. It is easy to verify that this normalization is innocuous, not altering the results in Ljungqvist and Sargent [2012] and Thomas and Worrall [1990].

[8] In his model, this lower bound is endogenized by assuming that the outside option is the value of signing a long-term contract with another financial intermediary, which is determined in equilibrium. The main messages of the paper, i.e., some risk-sharing occurs and a non-degenerate distribution of values

the outside option of the agents is to live in autarky forever, then $w_j \geq w_{aut}$ for all j would follow. Therefore, the program above without (PC) but with the set of constrains $w_j \geq w_{aut}$, for all j, is akin to the setup in Phelan [1995].[9]

3 Characterization at $v = w_{aut}$

In the next subsections, we state some intermediate lemmas that are used to prove our main result (Proposition 1 in Section 4). These lemmas might be of interest by themselves. The first subsection explores the interaction among (PK), (IC) and (PC) to show that at $v = w_{aut}$, unless the highest endowment shock is realized, the principal cannot spread continuation values to provide incentives. The second subsection shows that at $v = w_{aut}$, some intertemporal trade occurs between the financial intermediary and agents who draw the highest shock. Hence, although autarky impairs the amount of risk sharing that can be achieved, it is not an absorbing state. Some risk sharing occurs at $v = w_{aut}$.

3.1 The Interaction Among (PK), (IC) and (PC)

In this subsection, we argue that the interaction among promise keeping (PK), incentive compatibility (IC) and ex post participation (PC) constraints has the potential to substantially limit the amount of risk sharing in this economy.

A standard result in the literature states that (IC) and the concavity of u impose restrictions on the transfers that can be made by the principal. In particular, $b_{j-1} \geq b_j$ and $w_{j-1} \leq w_j$ for $j \geq 2$. In words, in order to provide incentives, transfers must weakly decrease with income, whereas continuation values must weakly increase. The following auxiliary lemma states that the interaction among (IC), (PC) and the concavity of u imposes further restrictions on the transfers that can be made by the principal. It considers the case in which (PC) is binding at one type, which must eventually happen along the

exists in the limit, would follow if this lower bound were treated exogenously.

[9] In contrast with this paper, Phelan [1995] simplifies his framework along two dimensions. First, for ease of exposition, he considers an economy with $n = 2$. Second, he assumes constant absolute risk aversion preferences.

optimal contract path if π_n is small enough, as we show below.

Lemma 1. *If $i < k$ ($i > k$) and (PC) is binding at k, then $b_i \geq 0$ ($b_i \leq 0$). In addition, if (PC) is not binding at i, then $b_i > 0$ ($b_i < 0$).*

Proof. (PC) and (IC) imply that

$$\begin{aligned}(1-\delta)u(\theta_k) + \delta w_{aut} &= (1-\delta)u(\theta_k + b_k) + \delta w_k \geq (1-\delta)u(\theta_k + b_i) + \delta w_i \\ &\geq (1-\delta)[u(\theta_k + b_i) - u(\theta_i + b_i)] + (1-\delta)u(\theta_i) + \delta w_{aut}.\end{aligned}$$

The concavity of u and $i < k$ ($i > k$) implies that $b_i \geq 0$ ($b_i \leq 0$). Finally, if (PC) is not binding at i, the last inequality is strict, which completes the proof. □

In words, if (PC) is binding at a given type, say k, then every type below (above) it must receive positive (negative) transfers. The relevant part of the lemma for the rest of the analysis is that $b_i \geq 0$ if $i < k$ and (PC) is binding at k. Note that Lemma 1 does not make use of (PK). If we consider the role of (PK), even further restrictions on the contract offered by the principal apply. In fact, recall from the previous section that (PC) and (PK) imply that $w_j \geq w_{aut}$.

The rest of this subsection and the next characterize properties of the optimal contract at $v = w_{aut}$. This is useful for two reasons. First, it allows us to show that a variant of the immiseration result, in which agents get stuck in autarky forever, does not follow (Lemma 3 in Section 3.2). In other words, some risk sharing occurs at $v = w_{aut}$. Second, it allows us to derive conditions under which the optimal contract does not assign the value of autarky as a continuation value (Proposition 1 in Section 4). In this case, our numerical simulations show that the mass of agents living in autarky in the limit can be zero, which is in sharp contrast with the immiseration result.

The next lemma states that if an agent was promised the value of autarky in the previous period, then he remains in autarky unless hit by the highest realization of the endowment, θ_n. Moreover, transfers to the highest type are weakly negative, $b_n \leq 0$.

Lemma 2. *If $v = w_{aut}$, then $w_j = w_{aut}$ and $b_j = 0$ for $j = 1, ..., n-1$. Moreover, (PC)* *also binds at $j = n$ and $b_n \leq 0$.*

Proof. Note that (PC) is binding for all j. Otherwise, (PK) would imply that

$$w_{aut} = \sum_{j=1}^{n} \pi_j[(1-\delta)u(\theta_j + b_j) + \delta w_j] > \sum_{j=1}^{n} \pi_j[(1-\delta)u(\theta_j) + \delta w_{aut}] = w_{aut},$$

which yields a contradiction. Since (PC) is binding at $j = 1, ..., n$, then $w_j \geq w_{aut}$ implies $b_j \leq 0$. In addition, (PC) binding at $j = n$ and Lemma 1 imply that $b_j \geq 0$ for $j = 1, ..., n-1$. □

Lemma 2 shows that the interaction among (PK), (IC) and (PC) has a severe implication for risk sharing at $v = w_{aut}$. Unless the highest endowment shock is realized, the principal cannot spread continuation values to provide incentives. Although the proof is straightforward, the intuition rests on understanding how different combinations of the constraints in the problem restrict transfers and continuation values. At autarky, (PK) forces (PC) to be binding for all types. But if (PC) is binding for all types, given that $w_j \geq w_{aut}$, transfers must be weakly negative for all types. What makes the highest type special is the absence of another type with positive mass that has the incentive to mimic him downward. Lemma 1 shows that the incentive of a binding type ($j = n$ in this case) to mimic lower types forces transfers to be weakly positive for these smaller types. Hence, the only choice variables that are not pinned down by the set of constraints at $v = w_{aut}$ are $b_n \leq 0$ and $w_n \geq w_{aut}$.[10] Finally, note that Lemmas 1 and 2 do not rely on the objective function (or preferences) of the principal. They are still valid, for example, if the discount factor of the principal differs from that of the agents.

The presence of (PC), which is the key difference between this paper and Phelan [1995], is crucial to prove Lemma 2. In the absence of (PC) from the problem but imposing that $w_j \geq w_{aut}$, which is akin to his setup, the principal could spread continuation values for lower types without violating the remaining constraints.[11] This does not mean that it

[10] One can show that the optimal contract at $v = w_{aut}$ maximizes the principal's profits from transacting with $j = n$, $-(1-\delta)b_n + \delta W(w_n)$, subject to (PC) binding at $j = n$, $(1-\delta)u(\theta_n + b_n) + \delta w_n = (1-\delta)u(\theta_n) + \delta w_{aut}$.

[11] Indeed, fix $w_n > w_{n-1} > ... > w_1 \geq w_{aut}$, and let b_1 be free to adjust. Then, pick $b_2, ..., b_n$ (as "functions" of b_1) such that downward incentive compatibility constraints are binding, i.e., $(1-\delta)u(\theta_j + b_j) + \delta w_j = (1-\delta)u(\theta_j + b_{j-1}) + \delta w_{j-1}$. Hence, standard results imply that other incentive compatibility

would be always optimal to do so. We provide a numerical example below showing that even in the absence of (PC) from the problem, the principal could find it optimal to set $w_j = w_{aut}$ for $j = 1, ..., n-1$, but only if π_n is high.

An immediate implication of Lemma 2 is that autarky is a highly persistent state if π_n is small, as expected in unequal societies. In particular, one may conjecture that $\pi_n = 0$ makes autarky an absorbing state. This conjecture is wrong. A close inspection of the proof of Lemma 2 reveals that $\pi_n > 0$ is crucial to conclude that (PC) is biding at n, and thus, to apply Lemma 1. In fact, by assuming that $\pi_n = 0$, there would be redundancy in the analysis, as $j = n-1$ would play the role of the highest type in practice.[12]

In Appendix A, we consider a small departure from this environment that makes autarky an absorbing state. In particular, we drop the assumption that the endowment θ takes values in a finite space, and we assume, instead, that θ takes values in a compact set, say, $[\underline{\theta}, \overline{\theta}]$. In principle, one may also allow for both discrete and continuous types. As the arguments in this section and Appendix A suggest, for autarky to be an absorbing state, the type space must be connected near its upper bound, say, $\overline{\theta}$. Indeed, when $v = w_{aut}$, the interaction among (PC), (IC) and (PK) forces all discrete and almost all continuous types to remain in autarky, including those slightly below $\overline{\theta}$.

Lastly, we consider in the online appendix the case of history-dependent endowments. In particular, θ takes values in a finite space following a first-order Markov process. Then the value of autarky w^j_{aut} varies with the previous realization of the income shock θ_j. We show that Lemma 2 remains valid if $w^n_{aut} \leq w^j_{aut}$ for $j = 1, ..., n-1$. That is, if the value of autarky after the highest income shock is smaller than after any other shock. We note that this sufficient condition is restrictive as it implies strong negative correlation at θ_n.

constraints are not violated. Finally, keeping in mind that $b_2, ..., b_n$ are "continuous" and "monotone" in b_1, adjust b_1 such that (PK) is satisfied at $v = w_{aut}$.

[12] This claim is not straightforward. Since $\pi_n = 0$, the financial intermediary's profits and (PK) do not depend on w_n and b_n. Hence, one needs to show that there exist values for w_n and b_n that preserve (IC) and (PC). Given that $b_{n-1} \leq 0$ by Lemma 2, the concavity of u implies that this is accomplished by setting $w_n = w_{n-1}$ and $b_n = b_{n-1}$.

3.2 Risk Sharing

Despite the severe implication of Lemma 2, the following proposition shows that some risk sharing occurs at $v = w_{aut}$. In fact, some intertemporal trade occurs between the financial intermediary and the agents who draw θ_n. In particular, agents transfer part of their endowment to the financial intermediary, $b_n < 0$, in exchange for a promise of life-time utility above the autarky value, $w_n > w_{aut}$.

Lemma 3. *If $v = w_{aut}$, then $w_n > w_{aut}$ and $b_n < 0$.*

Proof. Suppose $v = w_{aut}$, and consider the following contract, which is slightly different from autarky. At some t, the principal receives $\varepsilon > 0$ if the highest endowment is realized. At $t+1$, upon the realization of the highest endowment in the previous period, the agent receives $\xi > 0$ in all possible states. In the remaining periods and contingencies, no transfers occur. Set ε and ξ, such that:

$$(1-\delta)u(\theta_n - \varepsilon) + \delta \sum_{j=1}^{n} \pi_j u(\theta_j + \xi) = (1-\delta)u(\theta_n) + \delta \sum_{j=1}^{n} \pi_j u(\theta_j).$$

Clearly, this contract satisfies incentive compatibility and participation constraints. Moreover, the agent is indifferent between this contract and autarky forever. Take a first-order Taylor approximation at the equation above around $\varepsilon = 0$ and $\xi = 0$. Thus,

$$-(1-\delta)u'(\theta_n)\varepsilon + \delta \sum_{j=1}^{n} \pi_j u'(\theta_j)\xi = 0 \iff \varepsilon = \frac{\delta}{1-\delta} \frac{\sum_{j=1}^{n} \pi_j u'(\theta_j)}{u'(\theta_n)} \xi > \frac{\delta}{1-\delta}\xi.$$

The inequality follows from $u'' < 0$ and, thus, $\frac{\sum_{j=1}^{n} \pi_j u'(\theta_j)}{u'(\theta_n)} > 1$.

The principal's net revenue obtained in this contract, $\pi_n[(1-\delta)\varepsilon - \delta\xi]$, is positive for both ε and ξ small enough. Hence, autarky is not optimal. Finally, Lemma 2 implies that $b_n < 0$ and $w_n > w_{aut}$. □

In words, despite the severe implication of Lemma 2, some risk sharing occurs in this economy. A variant of this result is also present in Thomas and Worrall [1990], who consider an extension in which both incentive compatibility and participation constraints interact. In particular, they assume that both the principal and the agents may renege on the contract and, thus, are subject to participation constraints. They show that under high enough discount factors, some risk sharing always occurs. Lemma 3 considers one-sided commitment instead and is valid for all values of δ. More generally, an inspection of the proof reveals that Lemma 3 is still valid if we assume different discount factors, and the discount factor of the principal is lower.

This result also echoes Phelan [1995], who considers an economy with $n = 2$, constant absolute risk aversion (CARA) preferences, and another modeling of the participation constraints described above. In this case, he shows that some intertemporal trade occurs when the promised value is at its lower bound. In contrast, Lemma 3 holds for generic preferences as long as u satisfies strict concavity.

In related contexts with asymmetric information, whenever a lower bound on continuation values is present, other papers derive similar implications. Wang [1995], Atkeson and Lucas [1995] and Hertel [2004], for example, show that this lower bound is not an absorbing state, and a non-degenerate invariant distribution exists (something we discuss in the next section).

Despite similarities with previous contributions, Lemmas 2 and 3 say something novel. Once in autarky, except for the highest type, the principal cannot spread continuation values to provide incentives. Therefore, the agent only leaves autarky if the highest possible realization of the endowment θ_n is drawn, which happens with probability π_n. If π_n is small, as expected in many unequal economies, then autarky is a highly persistent state. These two reasons, the impossibility of properly providing incentives and high persistence, imply that autarky is a costly state. In the next section, we explore this implication for the dynamics of the optimal contract.

4 Dynamics

In this section, we argue that for $v > w_{aut}$, the optimal contract prevents agents from reaching autarky tomorrow if the probability of drawing the highest realization of the endowment π_n is small enough. This is a direct implication of the fact that the autarky state becomes costlier as π_n gets smaller.

We also assume that δ is high enough. This assumption is used in the online appendix to show that the value function of the principal, W, is strictly concave. Except through the strict concavity of W, the results in this paper do not rely on this assumption. Hence, any other set of assumptions that guarantees strict concavity of W would be sufficient.

The online appendix derives a Lagrange functional for the principal's problem. We show that one can attach Lagrange multipliers to the constraints (PK), (IC), (PC) and $w_j \geq w_{aut}$ and derive the set of optimality conditions.[13] Let μ, $\lambda_{j,j-1}$, $\lambda_{j,j+1}$, ς_j and $\delta\xi_j$ be the Lagrange multipliers associated with (PK), (IC) that prevents j from mimicking $j-1$, (IC) that prevents j from mimicking $j+1$,[14] (PC) and $w_j \geq w_{aut}$, respectively. The Lagrangian reads:

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{j=1}^{n} \pi_j \left(-\left(1-\delta\right) b_j + \delta W\left(w_j\right)\right) + \mu \left[\sum_{j=1}^{n} \pi_j \left(\left(1-\delta\right) u\left(\theta_j + b_j\right) + \delta w_j\right) - v\right] + \\
& + \sum_{j=2}^{n} \lambda_{j,j-1} \left[\left(1-\delta\right) u\left(\theta_j + b_j\right) + \delta w_j - \left(\left(1-\delta\right) u\left(\theta_j + b_{j-1}\right) + \delta w_{j-1}\right)\right] + \\
& + \sum_{j=1}^{n-1} \lambda_{j,j+1} \left[\left(1-\delta\right) u\left(\theta_j + b_j\right) + \delta w_j - \left(\left(1-\delta\right) u\left(\theta_j + b_{j+1}\right) + \delta w_{j+1}\right)\right] + \\
& + \sum_{j=1}^{n} \varsigma_j \left[\left(1-\delta\right) u\left(\theta_j + b_j\right) + \delta w_j - \left(1-\delta\right) u\left(\theta_j\right) - \delta w_{aut}\right] + \sum_{j=1}^{n} \delta\xi_j (w_j - w_{aut}),
\end{aligned}
$$

with $\lambda_{1,0} = \lambda_{n,n+1} = \lambda_{0,1} = \lambda_{n+1,n} = 0$.

Note that we do not account for the constraints $w_j \leq w_{max}$ in the Lagrangian. A simple argument shows that these constraints are never binding along the optimal contract path. Indeed, the value function $W(v)$ is bounded below by the normalized life-time profits when the principal pays a constant amount for all types in all periods, i.e., $-\bar{b}(v) \leq W(v)$,

[13] To do so, we define a relaxed version of the program by allowing the principal to choose a joint distribution probability over transfers and continuation values. In other words, we convexify the program so that the maximization problem is well-defined and can be cast as a Lagrange functional. Then, we show that W is differentiable and, if δ is high enough, strictly concave. Hence, the solution of the relaxed version must be deterministic and, thus, feasible within the original program.

[14] A standard result states that the concavity of u implies that it is sufficient to account for local upward and downward (IC) constraints.

where $\bar{b}(v)$ solves $\sum_{j=1}^{n}\pi_j u(\theta_j+\bar{b})=v$. Analogously, $W(v)$ is bounded above by the normalized life-time profits generated by the first-best unconstrained contract, which guarantees full insurance by assuring constant consumption for all types in all periods, i.e., $W(v)\leq\sum_{j=1}^{n}\pi_j[\theta_j-\bar{c}(v)]$, where $\bar{c}(v)=u^{-1}(v)$. Since $w_{max}=\sup u(c)<\infty$, these bounds above imply that

$$\lim_{v\to w_{max}} W(v)=\lim_{v\to w_{max}} W'(v)=-\infty.$$

Consequently, if $v<w_{max}$, it is never optimal for the principal to set $w_j=w_{max}$ for some j.

This result is used in the next proposition to show that $W'(w_{aut})$ becomes arbitrarily large as π_n gets arbitrarily small. This guarantees that for each $v>w_{aut}$, there exists π_n small enough such that the optimal continuation value is interior, i.e., $w_j\in(w_{aut},w_{max})$. In other words, once away from the autarky state today, the optimal contract prevents the agent from reaching it tomorrow. We are ready to state our main result.

Proposition 1. *For each $v>w_{aut}$, there exists $\underline{\pi}(v)$ such that $w_j>w_{aut}$ for $j=1,...,n$ and for all $\pi_n<\underline{\pi}(v)$.*

We sketch the proof below and fill in the details in Appendix B. After manipulating the first-order conditions of the Lagrangian above with respect to w_j, substituting $\mu=-W'(v)$ (envelope theorem; see Milgrom and Segal [2002]), and using Lemmas 2 and 3 to evaluate the resulting equation at $v=w_{aut}$, one obtains:

$$W'(w_{aut})=W'(w_n)+\frac{1}{\pi_n}\sum_{j=1}^{n}\left(\varsigma_j+\xi_j\right),$$

where w_n, with a slight abuse of notation, is the optimal continuation value for type-n at $v=w_{aut}$. Also, the multipliers are evaluated at $v=w_{aut}$.

At $v=w_{aut}$, we show that the optimality conditions imply that $\lim_{\pi_n\to0}\sum_{j=1}^{n}\left(\varsigma_j+\xi_j\right)>0$. Intuitively, in the absence of binding participation constraints, a well-known result states that the principal spreads continuation values in order to provide incentives. Since $v=w_{aut}$, then $w_1<w_{aut}$ would violate one of the participation constraints of the

problem. Moreover, the strict concavity of W, $w_n \in (w_{aut}, w_{max})$ and $\lim_{v \to w_{max}} W'(v) = -\infty$ imply that $W'(w_n) > -\infty$. Since this result is valid for all distributions of $\{\pi_j\}_{j=1}^{n}$, including those with $\pi_n \to 0$, then $\lim_{\pi_n \to 0} W'(w_n) > -\infty$. Hence,

$$\lim_{\pi_n \to 0} W'(w_{aut}) = \infty.$$

This kind of "Inada condition" guarantees that for each $v > w_{aut}$, in a neighborhood of $\pi_n = 0$ (i.e., for all $\pi_n < \underline{\pi}(v)$), the optimal contract prevents agents from being "immiserated" (or "impoverished") tomorrow, in the sense that the optimal contract does not deliver the value of autarky, which is the greatest lower bound on the set of feasible continuation values.

Intuitively, in a problem with asymmetric information, the possibility to spread continuation values is a profitable tool to provide incentives. At $v = w_{aut}$, the financial intermediary cannot vary continuation values unless the agent draws the highest endowment θ_n. If the probability of such event, π_n, is small enough, autarky is a persistent state, which makes the impossibility to spread continuation values for lower types markedly costly. In this case, the slope of the value function of the principal W' becomes positive in the neighborhood of $v = w_{aut}$, as a v slightly above w_{aut} allows the principal to vary continuation values for lower types. Hence, the principal chooses $w_j > w_{aut}$ for all j whenever this choice is feasible, which it is for $v > w_{aut}$ but not for $v = w_{aut}$.

Due to the immiseration result in the absence of (PC) from the problem, (PC) or $w_j \geq w_{aut}$ must eventually bind at least at one j. An immediate implication of Lemma 1 and Proposition 1 is that as long as a small enough π_n implies $w_j > w_{aut}$ for all j, (PC) cannot bind at $j > 1$.[15] Therefore, the constraint that can prevent agents from being "immiserated" is the one that makes the lowest type, $j = 1$, indifferent between living in autarky forever and honoring the contract.

Another immediate implication of the steps in the proof of Proposition 1 is that due to

[15] Indeed, suppose that (PC) binds at some $k > 1$; then, Proposition 1 and (IC) imply that

$$(1-\delta)u(\theta_k) + \delta w_{aut} = (1-\delta)u(\theta_k + b_k) + \delta w_k > (1-\delta)u(\theta_k + b_j) + \delta w_{aut}, \text{ for all } j \neq k.$$

Hence, $b_j < 0$ for $j \neq k$. But Lemma 1 implies that $b_j \geq 0$ for $j = 1, ..., k-1$, a contradiction.

the need to spread continuation values given the presence of (IC) in the problem, none of the states reached is absorbing.[16] In particular, $w_n > v$ for all $v \in [w_{aut}, w_{max})$.[17] Importantly, this result does not rely on π_n being small enough.

These two implications contrast with Ljungqvist and Sargent [2012]'s version of Thomas and Worrall [1988], who leave (IC) out of the problem. In this case, v converges to a finite value, an absorbing state that makes the highest type $j = n$, rather than the lowest type $j = 1$, indifferent between living in autarky forever and honoring the contract.

Note that $\underline{\pi}$ depends on v. Our proof is silent on whether or not there exists a fixed $\underline{\pi}$ (independent of v) such that for all $v > w_{aut}$ and for all $\pi_n < \underline{\pi}$, then $w_j > w_{aut}$ for all j. If this stronger result is valid, in the limit, the mass of agents living in autarky is zero for all $\pi_n \in (0, \underline{\pi})$. Indeed, if $v_0 > w_{aut}$, the mass of agents living in the autarky state is always zero along the optimal contract path. If $v_0 = w_{aut}$ instead, Lemmas 2 and 3 imply that the mass of agents living in the autarky state at period t is $(1 - \pi_n)^t$, which converges to zero in the long run.[18]

Importantly, at least for a specific parameterization of the model, our numerical simulations below suggest that the aforementioned stronger result holds (i.e., zero mass of agents at the lower bound on the set of feasible continuation values in the limit). This is in sharp contrast with Thomas and Worrall [1990], who consider the problem without (PC) and, thus, without the set of constrains $w_j \geq w_{aut}$ for all j. In this case, v converges to its lower bound, $-\infty$, almost surely (immiseration result). It also differs from Phelan [1995], who considers a problem without (PC) but with a lower bound and an upper bound (smaller than w_{max}) on the set of continuation values. In his model, these states are recurrent along the optimal contract path, and the limit distribution has a positive mass of agents, smaller than one, at its lower bound.

The next proposition shows that for every $\pi_n > 0$, a non-degenerate invariant cross-

[16] Note that w_{max} is an absorbing state but is never reached along the optimal contract path.

[17] Indeed, suppose that $w_n \leq v$. Equation (2) in Appendix B, used to prove Proposition 1, and the strict concavity of W imply that $w_j = v$ and $\varsigma_j = \xi_j = 0$ for all j. In this case, a well-known result states that the principal would like to spread continuation values, $w_1 < v < w_n$, yielding a contradiction.

[18] Note that since $W'(w_{aut}) > 0$ for π_n small enough, both the financial intermediary and the agents are better off by signing an optimal contract with $v_0 > w_{aut}$ rather than $v_0 = w_{aut}$. Hence, none of the agents would ever be in autarky.

sectional distribution exists.[19]

Proposition 2. *The Markov process* $\{v_t\}$ *implied by the optimal choices of* $\{w_j\}_{j=1}^n$ *has a non-degenerate invariant distribution.*

Proof. Let $w_j(v)$ be the optimal continuation value when the promised value was v and shock j was drawn. Let P be a transition function that maps elements of the compact set $[w_{aut}, w_{max}]$ into Borel sets of $[w_{aut}, w_{max}]$. In particular,

$$P(v, A) = \sum_{j=1}^{n} \pi_j \mathbf{1}_{\{w_j(v) \in A\}}, \tag{1}$$

where A is a Borel set of $[w_{aut}, w_{max}]$. Note that $\mathbf{1}$ is the indicator function.

By the Theorem of Maximum, w_j is continuous in v, and thus, P has the Feller property. Theorem 12.10 in Stokey and Lucas [1989] implies that an invariant distribution exists. Non-degeneracy follows from the fact that none of the states reached is absorbing. □

Note that this proposition is silent on whether the invariant distribution is unique and stable. Hence, convergence from any initial value, v_0, toward this invariant distribution is not guaranteed. This proposition could be strengthen if one shows that the transition function, P, defined in (1), satisfies monotonicity. Hence, if w_j is bounded above by some $\overline{w} < w_{max}$, Theorem 12.12 in Stokey and Lucas [1989] guarantees convergence from any initial value towards the unique invariant distribution associated with (1).

For instance, if $w_j(v)$ is non-decreasing in v, which is satisfied in the numerical example below, then P satisfies monotonicity. Although we could not prove it generally, the result that $w_j(v)$ is non-decreasing in v also seems plausible in other contexts.[20]

Moreover, all results reported and discussed in this paper would follow if we impose an upper bound smaller than w_{max} instead. This can be motivated by allowing limited commitment on the side of the principal. In particular, the financial intermediary can renege on the contract at a given (normalized) fixed cost $C \in (0, \infty)$ and then be ex-

[19]Appendix C.2 discusses to what extent Propositions 1 and 2 can be generalized if we assume that the agents and the principal do not have the same discount factors.

[20]See, for instance, the discussion in Farhi and Werning [2007], page 383.

cluded from the financial market forever. As in Phelan [1995], this assumption of limited commitment generates an upper bound $\overline{w} < w_{max}$ on the space of continuation values such that $W(\overline{w}) = -C$.

The main difference between this paper and Phelan [1995] is the presence of (PC) in a context with private information.[21] To further understand the role of (PC), the next section solves numerically for the optimal contract when preferences are CARA and continuation values are bounded below by w_{aut} and above by $\overline{w} < w_{max}$. The focus is on the contrast between economies with and without (PC).

5 Numerical Example

In this section, we numerically solve the model with CARA preferences, i.e., $u(c) = -\frac{\exp(-\gamma c)}{\gamma}$ with $\gamma > 0$.[22] The objective is to provide a simple numerical example that complements the analytical results from previous sections. In particular, we show that the mass of agents living in autarky can be zero in the limit, and we perform some comparative statics on π_n.

To highlight the importance of ex post participation constraints, we also consider the case in which (PC) is left out of the financial intermediary problem but the set of constraints $w_j \geq w_{aut}$ for all j is kept in the problem. As explained above, this case is akin to Phelan [1995], who assumes that the possibility to walk away from the contract occurs at the beginning of a period. Hence, this numerical exercise also highlights the importance of the timing assumed in this paper for the results.

We consider a simple parameterization aiming to generate clean figures that are easy to visualize. We set $\gamma = 1$, $\delta = 0.8$, $n = 3$, $\{\pi_1, \pi_2, \pi_3\} = \{0.4, 0.4, 0.2\}$ and $\{\theta_1, \theta_2, \theta_3\} = \{0, 1, 2\}$. This parametrization implies that $w_{aut} = -0.5742$. As in Phelan [1995],

[21] There are, of course, other differences. For example, regarding preferences, Phelan [1995] assumes CARA preferences, whereas we assume that δ is high enough in this section.

[22] To solve the problem numerically, we use the value function iteration method with interpolation. We consider a tolerance of 10^{-6} for convergence. In all experiments, we allow for an equally spaced grid between w_{aut} and $\overline{w}$ with 2001 gridpoints. Because w_{aut} varies with some parameters of the model, the step size between adjacent gridpoints varies with the experiment. In each iteration, we use the sequential quadratic programming algorithm embedded in the *fmincon* command in MATLAB. In addition, we interpolate the value function using the shape-preserving piecewise cubic method embedded in the *interp1* command in MATLAB.

we assume an upper bound $\overline{w} < w_{max}$ on the space of continuation values such that $W(\overline{w}) = -C$. Otherwise, the numerical solution would be imprecise in the neighborhood of w_{max}. We always choose C such that $\overline{w} = -0.04$. Keeping in mind that it is impossible to exhaust all possible parameterizations of the model, we focus the discussion on the differences between the optimal contracts with and without (PC).

In Appendix C.1, we plot the optimal contracts and value functions for both models, with and without (PC). Because participation constraints tend to bind only for lower promised values v, the optimal contracts with and without (PC) prescribe similar allocations for higher values of v. Hence, Figure 1 (top graphs) reproduces the behavior of optimal continuation values for each type j, as functions of v, near their lower bound w_{aut}. It also plots forty-five-degree lines (dashed-lines) and horizontal lines (dotted-lines) marking the value of autarky. In addition, given the same initial promised value v_0 set to maximize the principal's life-time profits in the model with (PC), Figure 1 (bottom graphs) shows the trajectories of promised values over time after the realization of a sequence of the lowest endowment shock θ_1. The left graphs account for ex post participation constraints in the problem, whereas the right ones ignore them.[23]

[23]Note that the optimal continuation values in the model with (PC) display kinks a bit above the lower bound w_{aut}, although barely visible for w_1 in Figure 1. These kinks are located at the threshold value of v, such that (PC) ceases to bind at $j = 1$ for higher promised values. Above this threshold value, the possibility that (PC) might bind tomorrow at $j = 1$ is encoded in the value function of the principal, generating other kinks in the optimal contract, such as the clearly visible one for w_1. It is worth to mention that the clearly visible kinks for w_1 in the models with and without (PC) are not located at the same v. In the latter case, the kink is associated with the threshold value of v, such that $w_1 \geq w_{aut}$ ceases to bind for higher promised values.

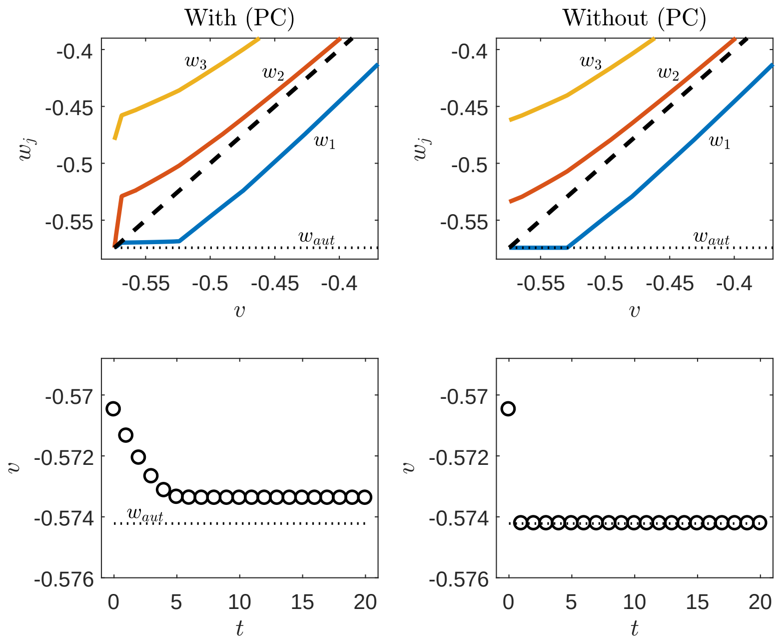

Figure 1: Optimal continuation values near w_{aut}. Top plots consider the optimal choices of w_j for all j as functions of v near $v = w_{aut}$. Full lines, dashed lines and dotted lines represent w_j for each endowment shock j, the forty-five degree line and the autarky state, respectively. Bottom plots consider the path of promised values over time after the realization of a sequence of the lowest endowment shock θ_1. Left and right plots consider the model with and without (PC), respectively.

Figure 1 establishes in the context of this simple example that once (PC) is accounted for, the mass of agents living in autarky is zero in the limit, which is in sharp contrast to the so-called immiseration result.

Consider the optimal contract in the right plots without (PC). The numerical solution shows that for the lowest type, below a certain threshold value, w_{aut} is the optimal continuation value. Hence, $w_j \geq w_{aut}$ binds at $j = 1$ in the neighborhood of $v = w_{aut}$. Given that w_1 as a function of v is bounded above by the forty-five degree line, the autarky state is reached with positive probability, after realizing a finite sequence of the lowest endowment θ_1.[24] Similarly, once in autarky, agents leave it with positive probability, after drawing θ_2 or θ_3. In the long run, a positive mass of agents lives at the autarky state.

In contrast, once (PC) is accounted for in the left plots, our numerical example shows that

[24] In our numerical example, the autarky state is reached after one period, but convergence could take longer if the initial promised value were set higher.

autarky cannot be reached with positive probability. Indeed, w_1 is a strictly increasing function of v, although not clearly visible in the figure. Moreover, (PC) binds at $j = 1$ slightly above $v = w_{aut}$, but $w_j \geq w_{aut}$ does not bind. In this case, after realizing a sequence of the lowest shock θ_1, promised values converge to a lower bound strictly above the autarky value in finite time.[25] In the long run, the mass of agents living in autarky is zero. Interestingly, as an agent gets closer to the autarky state, not only are w_2 and w_3 as functions of v above the forty-five degree line, but they also become steeper. Thus, in case θ_2 or θ_3 is drawn, this agent moves farther away from the autarky state, meaning that even more consumption is postponed.

Again, given the same parameterization, these differences between optimal contracts arise from the fact that autarky is a costlier state once (PC) is accounted for. First, the principal cannot spread continuation values for types other than $j = n$ in order to provide incentives at the autarky state. Second, autarky is a more persistent state as the agent leaves it whenever θ_n is drawn, which occurs with probability π_n. In contrast, once (PC) is left out of the problem, the principal can promise values higher than w_{aut} for all types other than $j = 1$, and thus, agents leave autarky whenever any endowment other than θ_1 is drawn, which happens with probability $1 - \pi_1 > \pi_n$.

To inspect how the optimal contract changes with different values of π_n, we allow π_3 to vary from 0.2 to 0.8, with a step size of 0.02, and we let $\pi_1 = \pi_2 = (1 - \pi_3)/2$. The top graphs in Figure 2 plot the continuation values (near the lower bound w_{aut}) prescribed by the optimal contract for $\pi_3 = 0.4$ and $\pi_3 = 0.6$ with (PC) present in the model.[26] These two cases summarize how optimal contracts differ when π_n is higher than a given threshold, $\pi_3 \in (0.36, 0.38)$ in our example. The bottom graph shows how the mass of agents at $v = w_{aut}$ changes with π_3 in the long run. Crosses (circles) represent the model with (without) ex post participation constraints.

[25] Note that, in our numerical example, this lower bound is only a bit above the autarky value. Therefore, it could be possible that this small difference is simply a numerical error. To address this concern, we compute lower bounds for specifications with 2001, 5001, 10001 and 20001 gripoints, as well as different interpolation methods, such as linear, piecewise cubic, cubic convolution and spline. We find that lower bounds vary little in between -0.57358 and -0.57338, still strictly above the autarky value of -0.57422. Hence, it is unlikely that the convergence to a value strictly above the autarky value is stemming from a numerical error.

[26] Recall that the value of autarky changes with π_j, $j = 1, ..., n$. In these cases, the autarky values are equal to -0.4645 and -0.3548, respectively.

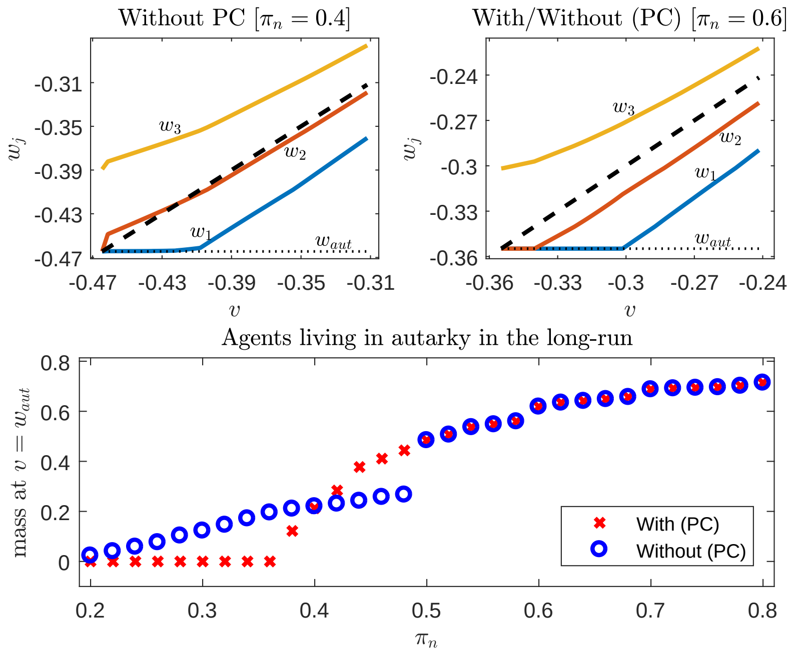

Figure 2: Comparative statics: π_n. Top graphs consider the optimal choices of w_j for all j as functions of v near $v = w_{aut}$ for $\pi_n = 0.4$ and $\pi_n = 0.6$. Full lines, dashed lines and dotted lines represent w_j for each endowment shock j, the forty-five degree line and the autarky state, respectively. Bottom graph plots the limit mass of agents at $v = w_{aut}$ as a function of π_n. Crosses (circles) represent the model with (without) ex post participation constraints.

Figure 2 highlights that for higher values of π_n, the model with (PC) can generate a positive mass of agents living in autarky in the long run. In this case, the autarky state can be reached in finite time with positive probability. Indeed, if π_n takes intermediate values, i.e., $\pi_3 \in [0.38, 0.48]$ in our example, w_1 becomes flat below a certain threshold promised value. In addition, if π_n takes high values, i.e., $\pi_3 \in [0.50, 0.80]$, both w_1 and w_2 become flat below certain threshold promised values. Hence, w_{aut} is the optimal continuation value below these thresholds, which is achieved in finite time given that w_1 as a function of v is bounded above by the forty-five degree line.

Interestingly, in the latter case, optimal contracts with and without (PC) are identical. Hence, (PC) never binds along the optimal contract path. Moreover, in the absence of (PC), although it is feasible to spread continuation values for lower types, i.e., $w_1 < w_2$, the principal optimally chooses not to do so. Intuitively, given that π_n is high, this choice does not severely impair risk sharing.

Appendix C.2 discusses to what extent our analytical and numerical results can be generalized if we assume that the agents and the principal do not have the same discount factors. In the context of this simple numerical example, our results are robust to a wide range of discount factors. In particular, there exists a discount factor of the principal above that of the agents, such that the aggregate consumption is equal to the aggregate endowment in the limit. Hence, a market-clearing interest rate (embedded in the principal's discount factor) along with a stationary equilibrium arises without changing our main conclusions.

Finally, in the online appendix, we consider a more standard calibration procedure, with constant relative risk aversion (CRRA) utility, to show that models with and without (PC) generate similar moments of the joint distribution of consumption and income in the long-run, at least with i.i.d. endowment shocks.[27] This quantitative result is remarkably robust. The presence of (PC) has little impact on the moments computed in several specifications that consider both partial and general stationary equilibriums. Intuitively, the presence of (PC) alters substantially the optimal contract design only near the autarky value, preventing agents from reaching it but not pushing them too far from it. Hence, even if the mass of agents living in autarky is high in the calibrated model without (PC) but zero in the model with (PC), the consumption dynamics and risk sharing provisions are similar across models in the long-run.

6 Conclusion

This paper studies the implications of the interaction between private information and one-sided commitment for risk sharing contracts. To the best of our knowledge, our results emphasize a novel force at play when designing these contracts that prevents agents from being "impoverished". Namely, the interaction between incentive compatibility con-

[27] Fernandes and Phelan [2000] show that serially correlated shocks increase the dimensionality of the state space. In particular, the number of state variables is $n+1$ with shocks that follow a first-order Markov process rather than one with shocks that are i.i.d. In this case, both the number of constraints and the number of programs that need to be computed increase substantially with the number of shocks. Hence, the assumption that $n \geq 3$, which is crucial to deliver our main conclusions, poses some computational challenges under the empirically relevant case of persistent income shocks. See also Doepke and Townsend [2006].

straints due to private information and ex post participation constraints severely impairs the amount of risk sharing that can be provided when agents are at the autarky state but still have access to financial markets. Indeed, we show that once at the autarky state, the financial intermediary cannot spread continuation values to provide incentives for agents unless they have been hit by the highest possible endowment shock. Hence, the only possibility to leave autarky is to be hit by such a highly favorable shock. If the probability of such an event is small, as in an environment with many income shocks (or with an unequal distribution of shocks), autarky is a persistent state. These factors make autarky a relatively costly state, and thus, the optimal contract prevents the agents from reaching it.

We make several simplifying assumptions that allow us to derive and expose analytical results in a standard environment that encompasses prominent contributions in the literature. We assume, for instance, that endowment shocks are independently and identically distributed, that the nature of punishment after a default episode is living in autarky forever, a partial equilibrium setup in which the principal and the agents have the same discount factors, and so on. Note, however, that the limited possibility to spread continuation values at the lower bound on the set of feasible continuation values, which is the main driving force behind our results, is derived solely by manipulating the promise keeping, incentive compatibility, and ex post participation constraints. Importantly, similar constraints must be satisfied in any contracting environment with private information regarding income and one-sided commitment allowing agents to renege on the contract after the realization of the income shock. Hence, the aforementioned novel force at play, which steers risk sharing contracts toward avoiding immiseration, is likely to be relevant in more-general contracting environments.

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Appendix A: Continuous Types

In this appendix we show that, if θ follows a continuous distribution F, with $f = F' > 0$ and support $[\underline{\theta}, \overline{\theta}]$, i.i.d. over time and agents, rather than a discrete distribution with finite support, then autarky becomes an absorbing state, such that the principal cannot spread continuation values at all.

The remaining ingredients of the model are the same. Under this stochastic process, the principal's problem can be rewritten as:

$$W(v) = \max_{\{b(\theta), w(\theta) \in [w_{aut}, w_{max}]\}_\theta} \int_{\underline{\theta}}^{\overline{\theta}} \left[-(1-\delta)b(\theta) + \delta W(w(\theta))\right] f(\theta) d\theta$$

subject to

$$\text{(PK)} \quad \int_{\underline{\theta}}^{\overline{\theta}} [(1-\delta)u(\theta + b(\theta)) + \delta w(\theta)] f(\theta) d\theta = v,$$

$$\text{(IC)} \quad (1-\delta)u(\theta + b(\theta)) + \delta w(\theta) \geq (1-\delta)u(\theta + b(\hat{\theta})) + \delta w(\hat{\theta}), \text{ for all } \theta, \hat{\theta} \in [\underline{\theta}, \overline{\theta}] \times [\underline{\theta}, \overline{\theta}],$$

$$\text{(PC)} \quad (1-\delta)u(\theta + b(\theta)) + \delta w(\theta) \geq (1-\delta)u(\theta) + \delta w_{aut}, \text{ for all } \theta \in [\underline{\theta}, \overline{\theta}],$$

where $w_{aut} = \int_{\underline{\theta}}^{\overline{\theta}} u(\theta) f(\theta) d\theta$. This problem is basically the same as the problem described in Section 2, except that we substitute summations by integrals in order to deal with continuous types. The remainder of this appendix shows that w_{aut} is an absorbing state.

First, note that incentive compatibility requires that $\hat{\theta} = \theta$ solves the following problem:

$$U(\theta) = \max_{\hat{\theta} \in [\underline{\theta}, \overline{\theta}]} \left\{ (1-\delta)u(\theta + b(\hat{\theta})) + \delta w(\hat{\theta}) \right\}, \quad \text{for all } \theta \in [\underline{\theta}, \overline{\theta}].$$

A standard lemma implies that (IC) is verified if and only if:

$$\text{(LIC)} \quad \dot{U}(\theta) = (1-\delta)u'(\theta + b(\theta)) \text{ almost everywhere,}$$

$$\text{(M)} \quad b \text{ is non-increasing,}$$

where $\dot{U}$ is the derivative of U with respect to θ. The first requirement is the local incentive compatibility constraint, which follows from applying the envelope theorem to the problem above (see Milgrom and Segal [2002]). The second requirement is the monotonicity condition that guarantees that (IC) is globally satisfied.

Second, note that given incentive compatibility, (PC) can be rewritten as

$$U(\theta) \geq (1-\delta)u(\theta) + \delta w_{aut}, \text{ for all } \theta \in [\underline{\theta}, \overline{\theta}],$$

and define Ω to be the set of endowments for which the participation constraint is binding. Formally,

$$\Omega = \{\theta \mid U(\theta) = (1-\delta)u(\theta) + \delta w_{aut}\}.$$

The following proposition shows that the interaction among (IC), (PC) and (PK) eliminates the possibility of risk-sharing when $v = w_{aut}$. Hence, w_{aut} is an absorbing state, meaning that once in autarky, the agent remains in autarky almost surely. Algebraically, at $v = w_{aut}$, $w(\theta) = w_{aut}$ and $b(\theta) = 0$ almost everywhere. Therefore, $W(w_{aut}) = 0$.

Proposition 3. *If $v = w_{aut}$, then $w(\theta) = w_{aut}$ and $b(\theta) = 0$ almost everywhere.*

Proof. Note that at $v = w_{aut}$, (PC) is binding almost everywhere. Otherwise, (PK) would imply that

$$w_{aut} = \int_{\underline{\theta}}^{\overline{\theta}} [(1-\delta)u(\theta + b(\theta)) + \delta w(\theta)] f(\theta) d\theta > \int_{\underline{\theta}}^{\overline{\theta}} [(1-\delta)u(\theta) + \delta w_{aut}] f(\theta) d\theta,$$

which contradicts $w_{aut} = \int_{\underline{\theta}}^{\overline{\theta}} u(\theta) f(\theta) d\theta$. Hence, almost every $\theta \in \Omega$.

Let $\mathring{\Omega}$ be the interior of Ω, which is clearly not empty when $v = w_{aut}$. Note that $\dot{U}(\theta) = (1-\delta)u'(\theta)$ for all $\theta \in \mathring{\Omega}$. Hence, for each $\theta \in \mathring{\Omega}$, (LIC), $u'' < 0$ and (PC) imply that $b(\theta) = 0$ and $w(\theta) = w_{aut}$. □

In an environment with continuous types, almost no one draws the highest type. Hence, we conjecture that an extrapolation of Proposition 1 implies that, although an absorbing state, autarky will never be reached in equilibrium by agents who are outside the autarky state. Indeed, the same intuition applies. The impossibility of spreading continuation values in autarky and thus providing some risk sharing should make the autarky state even costlier in an environment with continuous types.

Appendix B: Proof of Proposition 1

In this appendix, we prove Proposition 1. The first-order conditions (FOCs) of the Lagrangian in Section 4 with respect to w_j and b_j are:

$$\pi_j[W'(w_j)+\mu]=\lambda_{j+1,j}-\lambda_{j,j-1}+\lambda_{j-1,j}-\lambda_{j,j+1}-\varsigma_j-\xi_j, \quad \text{and}$$
$$\pi_j[1-\mu u'(\theta_j+b_j)]=(\lambda_{j,j-1}+\lambda_{j,j+1}+\varsigma_j)u'(\theta_j+b_j)-\lambda_{j+1,j}u'(\theta_{j+1}+b_j)-\lambda_{j-1,j}u'(\theta_{j-1}+b_j),$$

respectively. Moreover, $\mu=-W'(v)$ (envelope theorem; see Milgrom and Segal [2002]).

By summing the FOCs with respect to w_j in j and substituting $\mu=-W'(v)$, one obtains:

$$\sum_{j=1}^{n}\pi_j W'(w_j)+\sum_{j=1}^{n}(\varsigma_j+\xi_j)=W'(v). \tag{2}$$

By using Lemmas 2 and 3, evaluate the equation above at $v=w_{aut}$. After rearranging the terms:

$$W'(w_{aut})=W'(w_n)+\frac{1}{\pi_n}\sum_{j=1}^{n}(\varsigma_j+\xi_j),$$

where w_n, with a slight abuse of notation, is the optimal continuation value for type-n at $v=w_{aut}$. Also, the multipliers are evaluated at $v=w_{aut}$.

We complete the proof in two steps. First, we show that at $v=w_{aut}$, the optimality conditions imply that $\lim_{\pi_n\to 0}\sum_{j=1}^{n}(\varsigma_j+\xi_j)>0$, such that[28]

$$\lim_{\pi_n\to 0}W'(w_{aut})=\infty.$$

Second, we show that the condition above implies that if $v>w_{aut}$, then w_{aut} is not reachable within a neighborhood of $\pi_n=0$, i.e., there exists $\underline{\pi}(v)>0$ such that $w_j>w_{aut}$ for all j and for all $\pi_n<\underline{\pi}(v)$.

[28] Recall that strict concavity of W, $w_n\in(w_{aut},w_{max})$ and $\lim_{v\to w_{max}}W'(v)=-\infty$ imply that $W'(w_n)>-\infty$. Since this result is valid for all distributions of $\{\pi_j\}_{j=1}^{n}$, including those with $\pi_n\to 0$, then $\lim_{\pi_n\to 0}W'(w_n)>-\infty$.

Step 1. *At* $v = w_{aut}$*,* $\lim_{\pi_n \to 0} \sum_{j=1}^{n} (\varsigma_j + \xi_j) > 0$.

Proof. We prove that $\sum_{j=1}^{n} (\varsigma_j + \xi_j) > 0$ for all distributions $\{\pi_j\}_{j=1}^{n}$, including those with $\pi_n \to 0$. Suppose by contradiction that at $v = w_{aut}$, $\sum_{j=1}^{n} (\varsigma_j + \xi_j) = 0$. Hence, $\varsigma_j = \xi_j = 0$ for all j. Note that the FOCs with respect to w_j and b_j (including $j = n$) become:

$$\pi_j[W'(w_j) + \mu] = \lambda_{j+1,j} - \lambda_{j,j-1} + \lambda_{j-1,j} - \lambda_{j,j+1}, \quad \text{and}$$
$$\pi_j[1 - \mu u'(\theta_j + b_j)] = (\lambda_{j,j-1} + \lambda_{j,j+1})u'(\theta_j + b_j) - \lambda_{j+1,j}u'(\theta_{j+1} + b_j) - \lambda_{j-1,j}u'(\theta_{j-1} + b_j),$$

respectively. By summing the FOCs with respect to w_j in j and substituting $\mu = -W'(w_{aut})$, one obtains:

$$\sum_{j=1}^{n-1} \pi_j W'(w_j) + \pi_n W'(w_n) = W'(w_{aut}). \tag{3}$$

Whether $\pi_n > 0$ or $\pi_n \to 0$ does not matter for the arguments below. We break the analysis into two cases: $W'(w_{aut}) < 0$ and $W'(w_{aut}) \geq 0$.

<u>Case 1</u>: $W'(w_{aut}) < 0$. Since W is strictly concave, $w_{aut} \leq w_1$ and $w_j \leq w_{j+1}$, equation (3) implies that $w_j = w_{j+1} = w_{aut}$ for $j = 1, ..., n-2$. The FOCs with respect to w_j (excluding $j = n$) become:

$$\lambda_{j+1,j} - \lambda_{j,j-1} + \lambda_{j-1,j} - \lambda_{j,j+1} = 0.$$

Given that $\lambda_{1,0} = \lambda_{0,1} = 0$, a simple iterative argument implies that $\lambda_{j+1,j} = \lambda_{j,j+1}$ for all j. Moreover, $\lambda_{j+1,j} = \lambda_{j,j+1} = 0$.[29] Hence, the FOCs with respect to b_j (excluding

[29] Suppose this is not the case; then, the strict concavity of u, $b_{j+1} \leq b_j$ and $\lambda_{j+1,j} > 0$ imply that

$$(1-\delta)[u(\theta_j + b_j) - u(\theta_j + b_{j+1})] > (1-\delta)[u(\theta_{j+1} + b_j) - u(\theta_{j+1} + b_{j+1})] = \delta(w_{j+1} - w_j).$$

Hence,

$$(1-\delta)u(\theta_j + b_j) + \delta w_j > (1-\delta)u(\theta_j + b_{j+1}) + \delta w_{j+1},$$

and thus, $\lambda_{j,j+1} = 0$, a contradiction.

$j = n$) become:

$$u'(\theta_j + b_j) = -\frac{1}{W'(w_{aut})} > 0.$$

Hence, since $u'' < 0$, $b_j > b_{j+1}$, for $j = 1, ..., n-2$. But from (IC), $w_j = w_{j+1}$ implies $b_j = b_{j+1}$, a contradiction.

Case 2: $W'(w_{aut}) \geq 0$. Since W is strictly concave, $w_{aut} \leq w_1$ and $w_j \leq w_{j+1}$, equation (3) implies that $w_1 = w_{aut}$. Given that $\lambda_{1,0} = \lambda_{0,1} = 0$, the FOC with respect to w_1 and the arguments in footnote 29 imply that $\lambda_{2,1} = \lambda_{1,2} = 0$. Hence, the FOC with respect to b_1 becomes:

$$u'(\theta_1 + b_1) = -\frac{1}{W'(w_{aut})} \leq 0,$$

in contradiction to $u' > 0$. □

Step 2. *For each $v > w_{aut}$, $\lim_{\pi_n \to 0} W'(w_{aut}) = \infty$ implies that there is $\underline{\pi}(v)$ such that $w_j > w_{aut}$ for $j = 1, ..., n$ and for all $\pi_n < \underline{\pi}(v)$.*

Proof. Suppose by contradiction that at $v > w_{aut}$, $w_j = w_{aut}$ is an optimal choice for some j. Since $w_{aut} \leq w_1$ and $w_j \leq w_{j+1}$, it must be the case that $w_1 = w_{aut}$. Consider the FOC with respect to w_1 after plugging $\mu = -W'(v)$ and evaluating at $w_1 = w_{aut}$:

$$\xi_1 + \varsigma_1 = \pi_1[W'(v) - W'(w_{aut})] + \lambda_{2,1} - \lambda_{1,2}.$$

Given that the maximization problem is well-defined, $\lambda_{2,1} < \infty$ for all distributions $\{\pi_j\}_{j=1}^n$, including those with $\pi_n \to 0$. Hence, $\lim_{\pi_n \to 0} W'(w_{aut}) = \infty$ and $\lim_{\pi_n \to 0} W'(v) < \infty$ (recall that $v > w_{aut}$ and W is strictly concave) imply that $\xi_1 + \varsigma_1 < 0$ in a neighborhood of $\pi_n = 0$, a contradiction. □

Appendix C: Additional Results

Within the context of the simple numerical example described in Section 5, this appendix reports the optimal contracts and value functions in the models with and without (PC). Then, with some algebra and the simple numerical example, we discuss to what extent our conclusions can be generalized if we assume that the agents and the principal do not have the same discount factors.

C.1 Numerical Example: Optimal Contracts and Value Functions

In this section, within the context of the simple numerical example from Section 5, we show the optimal contracts and the principal's value functions that arise in the models with and without (PC).

Figure 3 plots the optimal contract (continuation values and transfers) for each type, along with forty-five-degree lines (dashed lines) and horizontal lines (dotted lines) highlighting the autarky state. Because participation constraints tend to bind for lower values of v, the optimal contracts with and without (PC) are similar for higher values of v. Figure 1 in the main text plots in larger-scale optimal continuation values as functions of v near its lower bound, w_{aut}.

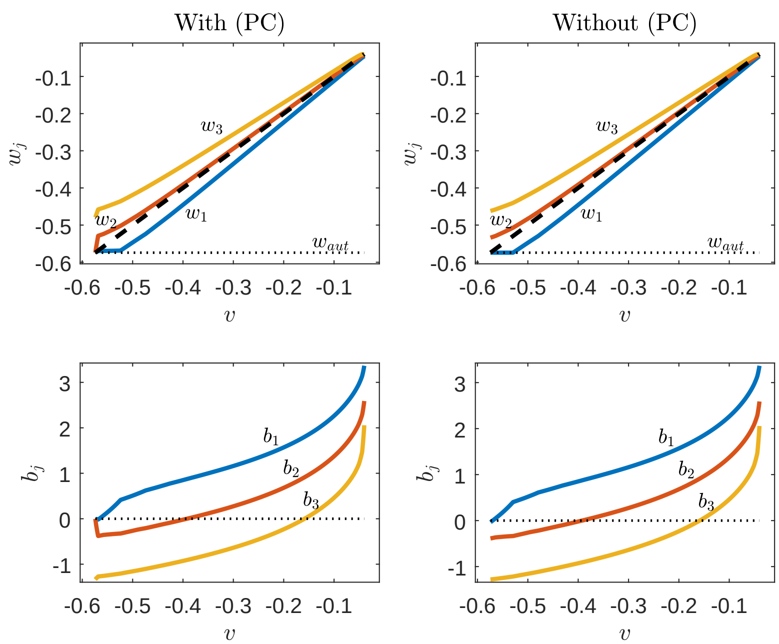

Figure 3: Optimal contract. The top and bottom plots consider the optimal choices of w_j and b_j for all j as functions of v, respectively. The left and right plots consider the model with and without (PC), respectively. Full lines, dashed lines and dotted lines represent the optimal contract for each endowment shock j, the forty-five-degree line and the autarky state, respectively.

The left graph of Figure 4 plots the value functions of the principal in the models with and without (PC), represented by the full and dashed lines, respectively. Because differences between them are not visible, the right graph reproduces at a larger scale their shape near the lower bound w_{aut}. Note that slightly above w_{aut}, the slope of the value function is positive in the model with (PC) but negative in the model without (PC). Given that spreading continuation values becomes markedly costly near w_{aut} once (PC) is accounted for, the principal can increase his profits by promising more consumption to the agent in the future.

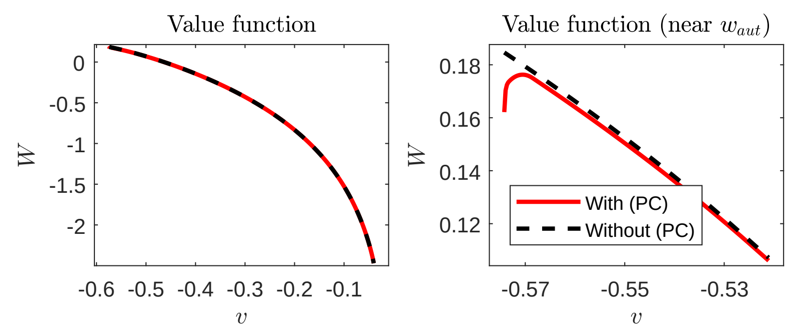

Figure 4: Value functions of the principal. Full and dashed lines represent the models with and without (PC), respectively. The left graph plots the value functions, whereas the right graph plots them at a larger scale near $v = w_{aut}$.

C.2 Different Discount Factors

Assume that the discount factor of the principal, say β, differs from that of the agents, δ.[30] Because Lemmas 1 and 2 do not rely on the preferences of the principal, they are still valid even if $\beta \neq \delta$. Hence, the impossibility of spreading continuation values at $v = w_{aut}$ for lower realizations of the endowment shocks, which is the main driving force behind our results, is still present if discount factors are allowed to differ freely. In addition, if we assume that the agents are more patient than the principal, i.e., $\beta < \delta$, an inspection of the proof of Lemma 3 reveals that this lemma remains valid.

In this section, we discuss to what extent Propositions 1 and 2 can be generalized if $\beta < \delta$. It turns out that they are fairly robust. Intuitively, if the agents are more patient,

[30] Other papers in the literature also allow for different discount factors in related environments. Wang [2005], for instance, considers a model of dynamic risk sharing with private information and costly state verification. Opp and Zhu [2015] study the dynamics of long-term contracts when the agent is impatient in a general setting that nests Thomas and Worrall [1988], among others.

the force at play to postpone consumption near autarky is reinforced. This does not mean that our conclusions would not be valid if the opposite holds. Even if $\beta > \delta$, due to Lemmas 1 and 2, the main driving force behind our analytical results still applies. On the one hand, the impossibility of properly spreading continuation values at the autarky state introduces a force to backload consumption. On the other hand, more-impatient agents introduce a motive to frontload consumption. We provide a numerical example showing that the force to backload dominates near the autarky state. Hence, our conclusions can still be valid even if the principal is more patient.

A close inspection of the online appendix reveals that the intermediate steps used to derive a Lagrangian functional for the principal's problem still apply. Hence, the Lagrangean would be analogous to that in the main text, except that β substitutes δ in the principal's objective function. Following the steps outlined in Appendix B, one can sum the first-order conditions with respect to w_j, substitute the envelope condition $\mu = -W'(v)$, and use Lemma 2 to evaluate the resulting equation at $v = w_{aut}$ to obtain

$$W'(w_{aut}) = \frac{\pi_n \frac{\beta}{\delta}}{1 - (1-\pi_n)\frac{\beta}{\delta}} W'(w_n) + \frac{1}{1-(1-\pi_n)\frac{\beta}{\delta}} \sum_{j=1}^{n} (\varsigma_j + \xi_j), \tag{4}$$

where w_n, with a slight abuse of notation, is the optimal continuation value for type-n at $v = w_{aut}$. Multipliers are also evaluated at $v = w_{aut}$.

If $\delta = \beta$, the equation above collapses to the one in the main text. Assume $\beta < \delta$ instead, such that Lemma 3 is still valid. In this case, one can follow similar steps as those in Appendix B to show that

$$\lim_{\pi_n \to 0} W'(w_{aut}) = \frac{1}{1-\frac{\beta}{\delta}} \sum_{j=1}^{n} (\varsigma_j + \xi_j) > 0.$$

Since this limit is not infinity, this result does not generalize Proposition 1. However, as we argue in the main text, it is the possibility to make positive profits by increasing continuation values that prevents the principal from promising the autarky value in the next period. Hence, this force at play behind Proposition 1 is still present in this case.

Proposition 2 states that a non-degenerate invariant distribution exists. The part of the proof regarding the existence of an invariant distribution easily generalizes to different discount factors. The non-degeneracy follows from Lemma 3, which guarantees that none of the states reached is absorbing.

In principle, as equation (4) highlights, different values of β, δ and π_n could make the slope of profits, W', positive near $v = w_{aut}$, even if the principal is more patient than the agents. Indeed, as we show below, this is true in our simple numerical example for a wide range of discount factors. In this context, the main conclusions of this paper still apply, even if $\beta > \delta$. As we argued above, the motive to postpone consumption near the atuarky state can dominate the motive to anticipate it due to impatience.

Figure 5 (top graphs) reproduces the behavior of optimal continuation values, as functions of v, near w_{aut} for $\beta = 0.7$ (left graphs) and $\beta = 0.9$ (right graphs). Recall that we set $\delta = 0.8$. The figure also plots forty-five-degree lines (dashed lines) and horizontal lines (dotted lines) highlighting the autarky state. Note that continuation values are still strictly increasing functions of promised values, although not clearly visible in the right graph. Figure 5 (bottom graphs) also shows the trajectory of promised values over time after the realization of a sequence of the lowest endowment shock θ_1. The initial promised value v_0 was such that it maximizes the principal's profits in the benchmark parametrization with $\beta = 0.8$. Again, promised values converge to a lower bound strictly above the autarky value in finite time. These numerical results generalize for a wide range of values for β.

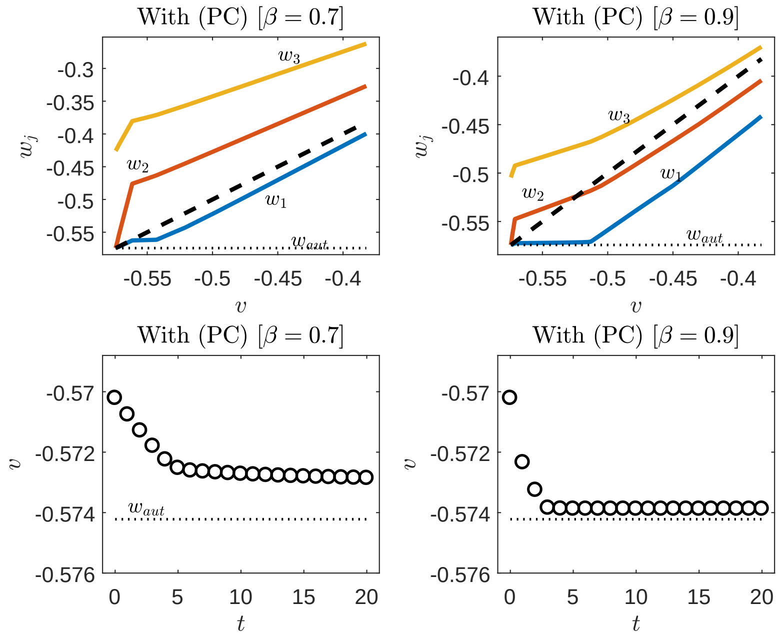

Figure 5: Optimal continuation values near w_{aut}. The top plots consider the optimal choices of w_j for all j as functions of v near $v = w_{aut}$. Full lines, dashed lines and dotted lines represent w_j for each endowment shock j, the forty-five-degree line and the autarky state, respectively. The bottom plots consider the path of promised values over time after the realization of a finite sequence of the lowest endowment shock θ_1. The left and right plots consider the model with $\beta = 0.7$ and $\beta = 0.9$, respectively.

Hence, even if the principal is more patient than the agents, the conclusions from Lemma 3 and Proposition 1 still apply in the context of this simple numerical example. Namely, some intertemporal trade occurs between the financial intermediary and the agents who draw θ_n, and the optimal contract prevents the agents from reaching the autarky state.

Indeed, Figure 6 (top graph) plots the numerical right-derivative of W at $v = w_{aut}$ as a function of β, ranging from 0.7 to 0.9, with step size 0.01. Recall that we set $\pi_n = 0.2$. In all cases, the slope of $W(w_{aut})$ is positive, implying that the mass of agents living in autarky is zero according to our numerical simulations. This slope increases as the financial intermediary becomes more impatient. Hence, as the discussion above emphasizes, the force at play behind our main results is stronger for lower values of β.

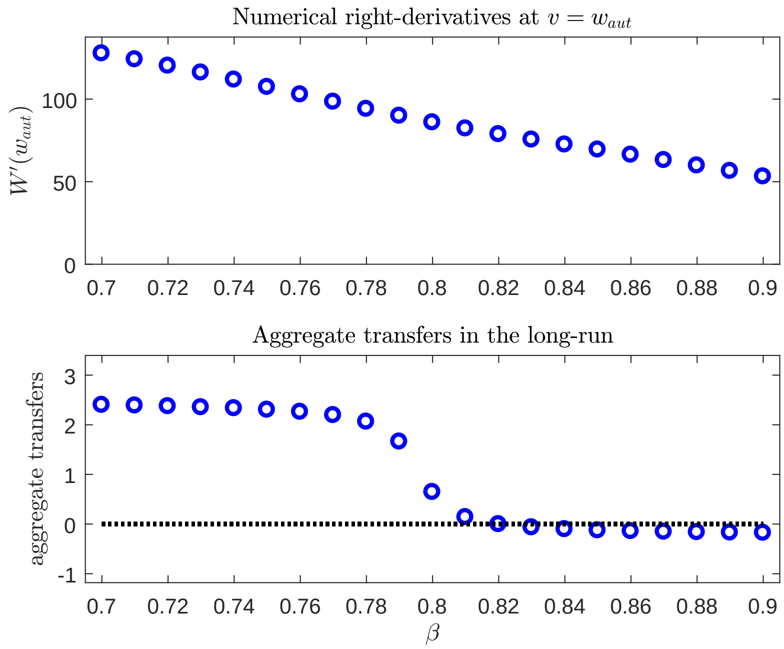

Figure 6: Comparative statistics: β. The top graph plots the numerical right-derivative of W at $v = w_{aut}$ as a function of β. The bottom graph plots the limit aggregate transfers as a function of β.

Figure 6 (bottom graph) also plots aggregate transfers, computed using the limit invariant joint distribution of types and values. Transfers decrease with the principal's degree of patience. Moreover, there exists β above δ, such that aggregate transfers are zero. In particular, $\beta \approx 0.82$. Hence, if we allow the interest rate embedded in the principal's discount factor to adjust to equalize aggregate consumption and aggregate endowment, a market-clearing interest rate along with a stationary equilibrium arises in the context of this simple numerical example without changing our main conclusions. Again, as the main driving force behind our results is still present if discount factors are allowed to differ freely, this numerical result is likely to be valid more generally.